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Theſaurarium Mathematicæ:

OR THE
TREASURY
OF THE
MATHEMATICKS.

CONTAINING

Variety of useful Practices in *Arithmetick*,
Geometry, *Trigonometry*, *Aſtronomy*, *Geography*,
Navigation and *Surveying*.

AS ALSO

The *Meſuration* of *Board*, *Glaſs*, *Tiling*,
Paving, *Timber*, *Stone* and *Irregular Solids*.

LIKEWISE

It teacheth the Art of *Gauging*, *Dialling*, *Fortifi-*
cation, *Military-Orders* and *Gunnery*: Explains the *Loga-*
rithms, *Sines*, *Tangents*, and *Secants*: Sheweth their uſe in
Arithmetick, &c. To which is annex'd a Table of 10000
Logarithms, *Log-Sines* and *Log-Tangents*.

Illustrated with ſeveral *Mathematical Sculptures* on Copper-Plates.

Originally Compos'd by J. TAYLOR, Gent. K

And now carefully Revis'd and Corrected.

To which is added, The Uſe and Practice of ſeveral *Propoſi-*
tions and *Problems* throughout the whole Work, as alſo the Deſcription and
Uſe of both *Globes*, and ſome of the chiefſt *Mathematical Inſtruments* both
for Sea and Land. With many other conſiderable Additions and Improvements.

By W. Alingham, Teacher of the *Mathematicks*.

Printed by J. L. for W. Freeman, at the Bible againſt
the *Middle-Temple-Gate* in *Fleetſtreet*. 1707.

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69e

To the READER.

AT the Request of my Friend I undertook the Revisal and Improvement of this Book, the doing whereof has cost me a great deal of Time and Pains, but I hope it will redound to thy great Benefit and Satisfaction.

I have nicely Examined the several Problems and Propositions throughout the whole Work, and have made such Corrections, Alterations and Improvements as I thought necessary, and do hope and think that this is now render'd the most useful Book that was ever yet Published on the Subject, in so Plain, Compendious and Practical a manner, but that I must submit to the Judicious and Candid Reader.

I have added many new Problems and Propositions with their Uses under their respective Heads; As also one Entire Tract of Cosmography (which may serve as a very necessary Introduction to the Astronomical and Geographical Part of this Book) A Description and Use of the Globes, after a New, very Easie and Compendious Method. And lastly, A Description and the Use of several of the Principal and most Useful Mathematical Instruments, which will be of great Advantage to the Reader being found together; and have endeavour'd to render this Treasury of the Mathematicks as Useful and Practical as I cou'd; and shall only wish the Reader Pleasure in the Perusal thereof and Success in all his Studies, and remain

his hearty Well-wisher
and Servant,

Chanel Row,
Nov. 1st. 1706.

W. Alingham.

The P R E F A C E.

HO W admirably profitable the Study of the Mathematicks has been to these British Islands, and to all other Parts of the Universe in which any kind of good Learning has been esteem'd and practis'd, is well known to all wise and judicious Men. And indeed it is an undeniable Truth, that among all humane Arts and Sciences whatsoever, the Noble Science Mathematical has obtain'd the greatest evidence of certainty, as being the Queen of Truth that imposeth nothing on her Subjects but what she proves by most infallible Demonstrations.

Now this Prerogative results from the Verity and Perspicuity of its Principles, which consist of Definitions, Postulats and Axioms. Hence comes it to pass that all Propositions that are prov'd by those most infallible Precepts are call'd certain demonstrative Truths; for which cause it has been the endeavour of sundry Philosophers, to make the force of their Arguments (as far as the quality of their discourse would admit) amount unto Mathematical Demonstrations, as being the most convincing Proof of a Proposition that by humane reasoning can be given.

Now having for divers Years (amongst my other Studies) been conversant in the Study of the Mathematicks, and for my own private use compil'd this Treatise, never in the least intending it should have appear'd in publick in this nice and Critical Age; but it by chance falling into the Hands of some of my Mathematical Friends and Acquaintance, I have at their Request condescended to Publish it, tho' not without a great Aversion in my own mind to expose my self in any publick thing.

But this Difficulty being overcome, I shall give the Impartial Reader to understand, that I have faithfully compiled this Treatise from the best of Authors (and my own Experience) that I have contracted their various Works into this little Cabinet or choice Compendium of the Mathematicks, in which he will find the whole Subject clearly and intelligibly handled: I have us'd a plain and easie Method: I have labour'd to be as plain and perspicuous as possible: I have apply'd such Examples to each as may best demonstrate their

The P R E F A C E.

their Operation, be most easie for memory, and applicable to practice; here is indeed Multum in parvo, the whole Marrow of the Mathematicks in this Tract afforded thee, which is as a true and Golden Key to unlock the choicest Mysteries in those Arts contain'd.

Thus, Reader, I have laid my Labours before thee, and must entreat thee to use me as thou wouldst be done by, which is the Spontaneous act of every good Man.

But if this shall chance to fall into the Hands of any curious conceited Person, who thinks himself wiser than the rest of the World, and so he beginneth enviously to carp hereat, and like a Country Cur bark at my back side, to him I shall in modesty only say, — Facilius est unicuivis nostrum aliena curiosè observare: quam propria negotia rectè agere. 'Tis much easier for those captious Readers to carp than to copy.

If the Collections of Authors shall offend any, and so procure the same censure with the Jackdaw, as our learned Poet hath long since caution'd against —

*Ne, si fortè suas repetitum venerit olim
Grex avium plumas, moveat cornicula risum,
Furtivis nudata coloribus.*

I would have such contentious Readers know, that I have robbed no Man of the Honour of his Works, but have given to each his due; only I have borrow'd some choice things of them, which is no more than what the most learned have always done.

Thus, Courteous and Impartial Reader, 'tis only for thee that I have taken these Pains, and have submitted to the Publication hereof, and it is to thee the future parts of my Study shall be serviceable, hoping that thou wilt find success in all thy Studies according to thy desire and endeavour; which are and shall be the hearty wishes of him who is

Thine and Urania's

Servant,

John Taylor.

T O

To the R E A D E R.

Being desired to Peruse this *Mathematical Treasury*, accordingly, to gratifie the Request of my Friend, I did, and I must confess, with no small Satisfaction to my self to see so much Practical Matter of useful *Mathematical Arts* so neatly and compendiously digested into this Portable Volume: 'twill be useful not only to *Learners* and meer *Tyro's*, but to others also who have made some considerable Progress in these Studies. 'Tis well *Methodiz'd*, very *Concise*, yet *Plain* and *Perspicuous*, so that any Person of a pregnant Fancy, may without a Tutor (in some reasonable time) wade through the whole or any Part thereof, and such as would be more expeditious may take the Assistance of a Teacher to instruct them.

The Author is wholly a Stranger to me, but to give him his due, in my opinion he has discharg'd himself like a Master in these Arts, and an Ingenious *Mathematician*, to whom I return Thanks for this his generous Offer in presenting his *Mathematical Treasury* to the Publick, and remain

A true Lover of the Mathematicoal Sciences, and all such that really delight in those pleasing (but useful) Speculations,

Henry Coley.

Courteous Reader,

I Have Perused this *Treatise*, and find that the Author has in every respect discharged himself like an *Artist*; the Work throughout the whole, is very *Plain* and *Easie*, nothing being omitted that might render it Intelligible to the meanest Capacity: And indeed, I know not any *Treatise* of this nature extant that is more Practically handled, so that I doubt not but that it will be very serviceable to the Publick; and that thou in particular mayst find Incouragement in the Perusal thereof, is the hearty Wish of him who is

Thine and Truth's Servant,

John Hawkins, *Philomath.*

The

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A D-

ADVERTISEMENT S.

AT the House late the Lord *Weymouth's*, in *Chancel-Row, Westminster*, are Taught *Arithmetick, Geometry, Trigonometry, Navigation, Surveying, Measuring, Fortification, Throwing of Bombs*, with several other Parts of the *Mathematicks*; also *Merchants* and other Accompts.

By *W. Alingham*.

He also Surveys and Measures any Lordship or Manor, makes a *Map* or *Plan* of the same. He likewise draweth any sort of *Dial*, Measures all sorts of Works and Timber, Re-draws or Copies any old Surveys.

Youth Boarded and made fit for any sort of Business, either by Sea or Land, and also for all sorts of Accompts.

ALL Gentlemen, or other Persons that shall have occasion for any sort of *Mathematical Instruments*, either for Sea or Land, may be furnished by *Henry Wynne*, *Mathematical Instrument-maker*, near *Serjeants-Inn* in *Chancery-Lane*.

Some few Errata's having escaped the Press, the Reader is desired to Correct the same with his Pen as followeth.

Page 60 Line 14. read Figure 94. p. 65 l. 21 r. Figure 95. p. 84 l. 17 r. Sum for Difference. p. 105 l. 17 for HE r. HL p. 164 l. 11 for N.W. by W. r. N.W. by N. p. 193 in the Margin for 45 r. 54. p. 194 l. 22 for 0.0 r. 7.50. p. 202 l. 18 for 24 r. 42. p. 255 l. 5. for A to A. p. 259 l. 7 r. from P to A. p. 260 l. 25 r. from P to C. p. 261 l. 27 r. which make MT equal to 150. p. 267 l. 22 r. Cazemate.

Arithmetick.

CHAP. I. Of ARITHMETICK.

SECT. I.

ARITHMETICK is an Art of numbering well : For as Magnitude, or Greatness is the Subject of *Geometry*, so is Multitude, or Number that of *Arithmetick*.

Omnia quaecunque à primæva rerum natura constructa sunt, Numerorum videntur ratione formata. Hoc enim fuit principale in animo conditoris exemplar. Boetius Arith. lib. 1. cap. 2.

I shall not in this place trouble you with the first Rudiments of Arithmetick, as Numeration, Addition, Subtraction, Multiplication and Division : Because they are already largely handled by many, as Mr. *Leybourn*, Mr. *Wingate*, and divers others, and also that then this Book would swell to too big a Bulk for the Pocket, and so my design would be frustrated ; I shall therefore only propose and operate some
B principal

Arithmetical Propositions.

principal Propositions, that are of Special Moment in Arithmetick, and which most immediately concern the other following Parts of this Treatise.

S E C T. I.

§ I. *The Explication of some Arithmetical Propositions.*

P R O P. I.

To three Numbers given, to find a fourth in a direct Proportion.

To operate this proportion Multiply the third Term, by the second Term, and their Product divide by the first Term, the Quotient shall be a fourth Term required. *Examp. I.* Admit the Circumference of a Circle whose Diameter is 14 Parts be 44 Parts, what is the Circumference of that Circle, whose Diameter is 21 Parts? Now according to the Rule if you multiply the third Term 21, by the second Term 44, it produceth 924; which divided by the first Term 14, the Quotient is 66, and so the Circumference of the Circle, whose Diameter is 21, will be 66 Parts, and so for any other in a direct Proportion.

P R O P. II.

To three Numbers given, to find a fourth in an Inversed Proportion.

To operate this Proposition, multiply the first Term by the second Term, and their Product divide by the third Term, the Quotient is the fourth

Arithmetical Propositions.

3

§ 1.

fourth Term required: *Examp.* Admit that 100
Pioneers, be able in 12 Hours to cast a Mote of
a certain Length, Breadth and Depth; in what
time shall 60 Pioneers do the same? Now if ac-
cording to the Rule, you multiply the first
Term 100, by the second Term 12, their Pro-
duct is 1200; which divided by the third Term
60, the Quotient is 20, so I say that in 20
Hours, 60 Pioneers shall do the same, and so
for any other in an Inversed Proportion.

P R O P. III.

*To three Numbers given, to find out a fourth in
a Duplicate Proportion.*

The nature of this Proposition is to discover
the Proportion of Lines to Superficies, and Su-
perficies to Lines; for like Plains are in a Du-
plicate Ratio; that is, as the Quadrate of their
Homologal sides; therefore to operate any Ex-
ample in this Proportion, square the third Term,
and its Square multiply by the second Term,
their Product divide by the Square of the first
Term, the Quotient is the 4th. Term sought.
Examp. Admit there be two Geometrical Squares;
now if the side of the greater Square be 50 Feet,
and require 3000 Tiles to pave it; what Num-
ber shall the lesser Square require, whose side is
30 Feet? To operate this according to the Rule,
I square the third Term 30, whose Square is 900:
Then I multiply it by the second Term 3000,
its Product is 2700000, which divided by 2500,
the Square of the first Term 50, the Quotient
is 1080, and so many Tiles will pave the lesser
Square, whose side is 30 Feet.

B 2

P R O-

§ 1.

P R O P. IV.

To three Numbers given, to find a fourth in a Triplicate Proportion.

The Nature of this Proposition is to discover the Proportion of Lines to Solids, and Solids to Lines; for like Solids, are in a Triplicate Ratio, that is to the Cubes, of their Homologal sides: Therefore to operate any Question in this Proportion, Cube the third Term, and this Cube multiply by the second Term, and their Product divide by the Cube of the first Term; the Quotient is the fourth Term sought. *Examp.* Admit an Iron Bullet whose Diameter is 4 Inches, weigh 9 Pounds; what is the Weight of that Bullet whose Diameter is 6 Inches? Now to operate this Proportion; first according to the Rule, I Cube the third Term 6 whose Cube is 216, then I multiply its Cube by the second Term 9, the Product is, 1944, which divided by 64, the Cube of the first Term; the Quotient is $30\frac{3}{4}$ Pounds which is equal unto 307.6 Ounces: Which is the Weight of the propounded Shot; and so for any other.

P R O P. V.

To two Numbers given, to find out a third, fourth, fifth, sixth, &c. Numbers in a continual Proportion.

To operate this Proportion, you must multiply the second Number by it self, and that Product divide by the first Term, the Quotient is a third proportional; Again you must multiply

Arithmetical Propositions.

5

§ 1.

ply the third Term by it self, and its Quadrate divide by the second Term, the Quotient is a fourth proportional, and so after this manner a fifth, sixth; or as many more Proportionals as you please may be found: *Examp.* Let it be required to find six Numbers in a continual Proportion to one another; as 4 to 8. To operate this first according to the Rule, I multiply the second Term 8 by it self, the Product is 64, which divided by the first Term 4, the Quotient is 16: So is 4, 8, and 16 in a continual Proportion; And so observing the Rules prescribed, proceed in your Operation until you have found your six Numbers in a continual Proportion; which in this Example will be 4, 8, 16, 32, 64, and 128, and so will you have form'd six Numbers in a continual Proportion.

P R O P. VI.

Between two Numbers given, to find out a mean Arithmetical Proportional.

This Proposition might be performed without the help of the Rule of Proportion: Nevertheless because it conduceth to the Resolution of the next ensuing Proposition, I insert it in this Place. To operate it this is the Rule: Add half the Difference of the given Terms, to the lesser Term, so that Aggregate is the Arithmetical mean required; *Examp.* Admit 20 and 50 to be the two Numbers propounded: Now to operate this Proposition, first according to the Rule, I find that the difference of the two given Terms 20, and 50, is 30, whose half is 15, which being added to the lesser Term 20, it makes 35,

B 3

16

§ 1. so is 35, a mean Arithmetical Proportion betwixt 20, and 50, given.

P R O P. VII.

Between two Numbers given, to find out a mean Musical Proportional.

Boetius hath this Rule for it, wherefore take his own Words: * saith he, "*Differentiam*

* In his second Book of his *Arithmetick* and the 38 Chapter; where he saith that this Proportion hath, *Magnam vim in Musici Modulaminis temperamenti, & in Speculatione naturalium quæstionum, i. e. Great Force in Musical Composition, (or in the Composure of Musick) and in the Discovery of the Secrets of Nature.*

" *terminorum in minorem terminum multiplica, & post*
 " *junge terminos, & juxta eum*
 " *qui inde confectus est, com-*
 " *mitte illum numerum, qui ex*
 " *differentiis & termino minore*
 " *productus est, cujus cum*
 " *latitudinem inveneris, addas*
 " *eam minori termino, & quod*
 " *inde colligitur medium terminum pones.* That is, mul-

tiply the difference of the Terms, by the lesser Term, and add likewise the same Terms together: This done, if you divide the Product, by the Sum of the Terms, and to the Quotient thereof, add the lesser Term; the last Sum is the Musical mean desired: *Examp.* Admit the two Numbers given be 6, and 12. I say that if the difference of the Terms which is 6, were multiplied by the lesser Term 6, it would produce 36; then if you add the two Terms 6, and 12, together, their Sum would be 18: Now if you divide 36, by 18, the Quotient is 2. Lastly, if to the Quotient 2, you add the lesser Term 6, the Sum thereof will be 8, which is a Mean Musical Proportional required.

P R O P.

P R O P. VIII.

How to find the Square-Root of any whole Number, or Fraction. § I.

Defin. To extract the Root of any Square Number propounded, is to find out another Number, which being multiplied by it self, produceth the Number propounded. Now for the more easie and ready Extraction of the Square-Root of any Number given, This Table here under annexed will be useful; which at first sight giveth all single square Numbers, with their respective Roots.

ROOT.	1	2	3	4	5	6	7	8	9
SQUAR.	1	4	9	16	25	36	49	64	81

The Explication of the TABLE.

In the uppermost Rank of this Table is placed the respective Root of every single Square-number, and in the other the single Square-numbers themselves; so that if the Root of 25 were demanded, the Answer would be 5, so the Square Root of 49 is 7, of 81 is 9; and so for the rest, and so contrarily the Square of the Root 5 is 25, of 7 is 49, of 9 is 81, &c.

Example: If the square Root of 20736 were required, first they being wrote down in order as you see, draw the crooked Line, * then to prepare this or any other Number for Extraction,

B 4

* As you use to do in Division; to represent the Quotient.

§ 1. ction, make a Point over the place of *Unites*; and so on every other Figure towards the Left-hand, as you see in the Margin. Then find the Root

$$\begin{array}{r} \cdot \quad \cdot \quad \cdot \\ 2 \ 0 \ 7 \ 3 \ 6 \end{array} \left(\begin{array}{r} 1 \ 4 \ 4 \\ 1 \end{array} \right)$$

$$\begin{array}{r} 24 \) \ 1 \ 0 \ 7 \\ \underline{ 9 \ 6} \end{array}$$

$$\begin{array}{r} 284 \) \ 1 \ 1 \ 3 \ 6 \\ \underline{ 1 \ 1 \ 3 \ 6} \end{array}$$

: : : :

of the first Square 2, which is 1; place it in the Quotient, and also under 2; then draw a Line, and subtract 1 from 2, there remains 1; which place under the Line, then to the last remainder 1, bring down the next Square 07; and then there will be this Number 107, which Number I call a *Resolvend*: Then double the

Root in the Quotient 1, whose double is 2, which 2 place under the place of Tens in the *Resolvend*, under 0; so is this 2 called a *Divisor*; and 10 called a *Dividend*.

Then demand how often the *Divisor* 2, can be had in the *Dividend* 10, it permitteth but of 4, which place in the Quotient, and under 7 the place of *Unites* in the *Resolvend*, and there will appear this Number 24; Then multiply this 24 by 4, (the last Square placed in the Quotient) it produceth 96, which place orderly under 24, as you see, and this 96 is called an *Ablatitium*; (but some call it a *Gnomon*;) then draw a Line under it, and subtract 96, the *Ablatitium*, out of the *Resolvend* 107, there remains 11, which place orderly under the last drawn Line, then thereunto bring down the next Square 36, so will there be a new *Resolvend*

Arithmetical Propositions.

9

§ 1.

solvend 1136; then double the whole Root 14 in the Quotient, whose double is 28: Place it under the Resolvend 1136 as was a-fore directed; so shall 28 be a new Divisor, and 113 be a Dividend; then I find the Divisor 28 can be had in the Dividend 113, 4 times, which four place in the Quotient, and under the place of *Unites* in the Resolvend, so there appeareth this Number 284, which Number multiplied by 4, the last Figure in the Quotient produceth a new *Ablatitium* 1136; which place orderly under the Resolvend 1136, and then draw a Line, then subtract the *Ablatitium* 1136, from the Resolvend 1136; and the remainder is 00, or nothing: And thus the Work of Extraction being finished, I find the Root of the square Number 20736, to be 144; and so must you have proceeded gradually step by step, if the Number propounded, had consisted of 4, 5, 6, or more Squares; still observing the aforegoing Rules and Directions.

N O T E.

But when a whole Number hath not a Root exactly expressible by any rational or true Number, then to find the fractional part of the Root very near; To the given whole Number annex Pairs of Cyphers, as 00, 0000, or 000000, then esteem the whole Number, with the Cyphers both annexed thereunto, as one intire whole Number: And extract the Root thereof according to the foregoing Directions, then as many Points as were placed over the Integers, so many of the first Figures in the Quotient must

§ 1. must be taken for Integers; and the remainder for the Roots fractional part in Decimal parts, and so you may proceed infinitely near the true Root of a Number.

To extract the Square-Root of a Vulgar or Decimal Fraction, and a Mixt-Number.

First, if the Fraction propounded be not in its least Terms, reduce it, and then by the Rules aforegoing, find the Root of the Numerator for a new Numerator; and of the Denominator for a new Denominator; so shall this new Fraction be the Square-root of the Vulgar Fraction propounded, so the Square-root of $\frac{16}{25}$ is $\frac{4}{5}$.

But many times the Numerator and Denominator of a Vulgar Fraction hath not a perfect Square-root; to find whose Root infinitely near, you must reduce it into a Decimal Fraction, whose Numerator must consist of an equal Number of Places, to wit, 2, 4, 6, &c. Then extract the Square-root of that Decimal, as if it were a whole Number, and the Root that proceedeth from it is a Decimal Fraction, expressing the Square-root of the Fraction proposed, infinitely near: So the Root of $\frac{1}{16}$ (whose Decimal is .81250000) will be found to be $\frac{9013}{10000}$ which is very near, for it wanteth not $\frac{1}{10000}$ of an Unite of the exact Square-root, of $\frac{1}{16}$ propounded.

The manner of extracting the Square-root of a Mixt-Number.

Now having a Mixt Number propounded whose Root is required, to find which reduce it into an improper Fraction, and then extract the

Arithmetical Propositions.

II

§ I.

the Root thereof as before. Suppose the Number propounded be $75\frac{24}{34}$; its improper Fraction is $6\frac{79}{34}$, whose Square-root I find to be $8\frac{2}{3}$ or $8\frac{2}{3}$, very near, &c. But if it had not an exact Square-root, then reduce the Fractional part of the given Mixt-number into a Decimal Fraction, of an even Number of Places, and then annex this Decimal to the Integers, and so extract the same as a whole Number; and observe that so many Points as were set over the Integers, so many of the first Figures in the Quotient must be esteemed Integers; and the remainder for the Root's Fractional part.

P R O P. IX.

How to find the Cube-Root of any whole Number, or Fraction.

Defin. To extract the *Cube-Root* of any Number propounded, is to find out another Number, which being multiplied by it self, and that Product by the Number again, shall produce the Number propounded; Now for the more easie and ready Extraction of the Cube-root of any Number propounded, this Table hereafter annexed will be useful, which at first sight giveth the Cube-root of any whole Number under 1000; which are called single Cube-numbers.

ROOT.	1	2	3	4	5	6	7	8	9
CUBE.	1	8	27	64	125	216	343	512	729

The

The Explication of the TABLE.

In the uppermost Rank of the Table is placed the respective Roots of every single Cube, and in the other the respective single Cube-numbers; for if the Cube-root of 512 were desired, the Answer would be 8, of 64 is 4; and so of the rest: And if the Cube of the Root 7 were desired, it would be found 343; of 9 it would be 729, &c.

Examp. Admit the Cube-root of the Number 262144 were required, first they being wrote down in order as you see, draw the crooked Line. * Then place a Point

* As you use to do in Division,
to represent the Quotient.

over the place of Unites, and another over the place of Thousands; and so on, still intermitting two places between every adjacent Point; and observe, that as many Points as in that Order are placed over any Number propounded, of so many Figures doth the Root consist: So that in this Example, there being two Points, therefore the Root consisteth of two places, as you see in the Quotient; Now first find the Root of the first Cube 262; which permitteth but of 6, place 6 in the Quotient, and subscribe its Cube 216, under 262, and then draw a Line under it, and subtract 216 out of 262, and the remainder is 46, which place in order under the last drawn Line as you see: Then to the Remainder 46, bring down the next Cube-number 144, so will there appear 46144, which I call a Resolvend: Then draw a Line under it, and square the Number in the Quotient 6, whose

Square

Arithmetical Propositions.

13

§ 1.

Square is 36; then triple it, and it will be 108, then subscribe this triple Square 108, under the Resolvend, so that the place of Unites in the triple Square 8, may stand under 1 the place of Hundreds in the Resolvend: Then triple the Root in the Quotient 6, whose Triple is 18; then subscribe the Triple 18, under the Resolvend, so that the place of Unites 8 in the Triple, may stand under 4 the place of Tens in the Resolvend, and so draw a Line underneath it, and add the triple Square 108, and the Triple 18 together in such order as they stand; their Sum is 1098, which may be called a Divisor, and the whole Resolvend 46144, except 4 the place of Unites a Dividend; then draw another Line.

$$\begin{array}{r}
 262144 \quad (64 \\
 216 \\
 \hline
 46144 \\
 \hline
 108 \\
 18 \\
 \hline
 1098 \text{ Divisor} \\
 \hline
 432 \\
 288 \\
 64 \\
 \hline
 46144
 \end{array}$$

Then seek how many times 1098 the Divisor can be had in 4614 the Dividend, it permitteth but of 4, which subscribe in the Quotient; now multiply the triple Square 108, by 4, it produceth 432, which in order subscribe under the triple Square 108: Then square 4, the Figure last placed in the Quotient, whose Square is 16; and multiply it by 18 the Triple, it produceth 288, which subscribe under the Triple

§ 1.

Triple orderly, then subscribe the Cube of 4 (last placed in the Quotient) which is 64, in order under the Resolvend. Then draw a Line underneath it, and add the three Numbers, viz. 432, 288, and 64, together in such order as they are placed, their Sum is 46144: Then draw another Line under the Work, subtracting the said total 46144, from the Resolvend 46144, there remains 00, or nothing, which remainder subscribe under the last drawn Line, thus the Work being finish'd I find the Cube-root of 262144 the Number propounded, to be 64: And thus you must have proceeded orderly step by step, if the Number propounded had arisen to some 3, 4, 8, 10, or more places, observing the Direction prescribed until all had been completed.

N O T E.

But when a whole Number hath not a Cube-root expressible by any true or rational Number, then to proceed infinitely near the exact Truth, Annex to the Number a certain Number of Tenaries of Cyphers as 000, 000000, 000000000, &c. then esteeming the whole Number with the Cyphers annexed as one intire whole Number, extract the Root thereof, as is afore taught. Then as many Points as were placed over the whole Number, so many places of Integers will there be in the Root, and the rest expresseth the Root's Fractional part very near.

To

Arithmetical Propositions.

15

§. I.

To Extract the Cube-Root, of any Vulgar or Decimal or Mixt Fraction consisting of a Whole Number and a Fraction.

To Extract the Cube-root of any Vulgar Fraction, you must first reduce it into his least terms, and then, according to the former Directions, Extract the Cube-root of the Numerator, the Root found shall be a new Numerator; so likewise the Root of the Denominator shall become a new Denominator; so shall this new Fraction be the Cube-root of the Fraction propounded, so I find the Cube-root of $\frac{8}{125}$ to be $\frac{2}{5}$, and so for any other Vulgar Fraction.

But many times the Numerator, and Denominator hath not a true Root: Then

The manner of Extracting the Cube-Root of a Decimal Fraction.

to find the Root thereof infinitely near, you must reduce the Fraction given, into a Decimal, whose Numerator is Tenaries of Places, and then extract the Root according to the former Directions, so shall the Root found, be a Decimal Fraction expressing nearly the Cube-root of the Fraction propounded, so I find the Root of $\frac{8}{125}$ or $\frac{2}{5}$, whose Decimal is, 666666666, to be, $\frac{873}{1000}$ very near the Root of $\frac{8}{125}$ or $\frac{2}{5}$ propounded.

The manner of extracting the Cube-Root of a Mixt Number.

Now having a Mixt Number propounded, whose Root is required, first reduce it into an Improper Fraction, and then extract the Cube root thereof, as is afore directed, so the Cube-root of $12\frac{10}{27}$ Improper $\frac{343}{27}$, will be found to be $\frac{7}{3}$ or $2\frac{1}{3}$.

But

§ 1.

But if it hath not an exact Cube-root, then reduce the Fractional part of the given mixt Number into a Decimal Fraction, which shall consist of Tenaries of places, then to the whole Number annex the Decimal Fraction, and extract the Cube-root of the whole, and observe that so many Points as are over the Integers, so many of the first places in the Quotient must be esteemed Integers, and the rest expresseth the Fractional part of the Root in Decimal parts of a Fraction, so the Cube-root of $2\frac{1}{2}$, whose Decimal is 2, 375000000, &c. will be found to be 1, 334, or $1\frac{334}{1000}$, and is very near the true Root, and so for any other mixt Number of this nature.

Having thus given you the Practice of extracting the square and Cube-root, I shall give you the reason of pointing them.

The Square of any Number under 10 will be less than 100, and the Squares of all Numbers from 10 under 100 will be less than 1000; also the Squares of all Numbers, from 100 to under 1000, will be less than 1000000, &c.

And on the contrary 'tis evident, that the Square-root of any Number under 100 will be less than 10, and the Square-root of all Numbers from 100 to under 10000 will be less than 100; and of all Numbers, from 10000 to under 1000000, will be less than 1000, &c.

So that the Square-root of all Numbers under 100 will consist but of one place, under 10000 they will have but two places, and under 1000000 the Root will consist but of three places at the most; from whence may be seen the reason of pointing every other Figure before you begin

Arithmetical Propositions.

17

§ 1.

Begin the Operation: For the Root encreasing a Place, encreases the Square of such Number two Places.

So also it holds in the Cube, where 'tis evident, that $9 \times 9 \times 9$ will be less than 1000, that is, the Cube of any Number under 10 will be less than 1000; and the Cube of all Numbers, from 10 to under 100, will be less than 1000000, &c. So on the contrary 'tis evident, that the Cube-roots of all Numbers, from 1000 to under 1000000, will be under 100, &c. Whence you see, that the Root encreasing one Place, augments the Cube of such Number three Places: For suppose 1, 10, 100, 1000, &c. the Roots, their Cubes are 1, 1000, 1000000, 1000000000, which encreases by 3 Places.

The same would also follow, if it were 9, 90, 900, 9000, &c. that is, their Cubes would exceed each other by 3 Places.

From which is evident the reason of pointing the 1st, 4th, 7th, &c. Places in a proposed Cube, to find its Root: For the first three Places will give but one Place for its Root. If the Cube have four Places in it, the Root will have two Places; and the said Root will have no more than two, if there be five or six Places in the given Cube: But if there be seven Places, then will the Cube-root of such Number consist of three Places.

NOTE, This way of arguing will not only hold in the Square and Cube, but also in high Powers.

Having given you this short Discourse, to show the reason of pointing them, in order to their Extraction, I shall now give a Taste of the Use

C

of

of these Operations in the Solution of some useful, and necessary Problems.

S E C T. II.

- § 2. *The Extraction of the Square-Root apply'd to the Solution of several Questions in Arithmetick, Geometry, Trigonometry, Navigation, Embattelling of Armies, &c.*

P R O P. I.

TWO Numbers, viz. 24 and 384, given to find a mean Proportion.

Multiply the two Numbers together, and extract the Square-root of the Product, it gives the mean Proportional required.

P R O P. II.

Let the Area of a Triangle, Rectangle, Circle, &c. be 71824 Foot.

Extract the Square-root of the given Area, it gives 268 Feet, the side of a Square equal to the given Area.

P R O P. III.

The side of a Square is 26 Foot; What's the side of another six times as big?

Find the Area of the Square whose side is given, this Area multiply by 6, then extract the Square-root of the Product, it gives 637 the side of a Square six times as big.

NOTE I: If you would have a Square but $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. as big. Take $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, of the Area found

Arithmetical Propositions.

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found, and extract the Square-root thereof, and
it will give you the Area of a Square that is but
 $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, &c.$ so big. § 2.

NOTE 2. So if you would have a Square
any number of times as big, or any part of a
time as little as any Figure, 'tis but multiply-
ing the given Area by the Number of times,
you would have it as big; or taking the part
of a time, you would have it as little, and ex-
tracting the Square-root of the Product, or
Quote, will give you the Area required.

P R O P. IV.

There is a Cable, or Rope, whose Diameter
is 7 Inches, what's the Diameter of another
four times as strong.

And here note, That that Rope which has its
section four times as much when cut thro' is
four times as strong; and be but half the Super-
ficies, 'tis but half as strong.

Now the Proportion of the Area's of Circles
to each other, are as the Squares of their Dia-
meters. Square 7 therefore it makes 49, which
multiply by 4 it gives 196: The Square-root of
which is 14, the Diameter of the Cable four times
as strong.

NOTE 1. Whence 'tis observable, That a
Rope four times as strong as another, has its
Diameter but only double to it.

NOTE 2. The like Operation will serve to
find the Strength of Timber; that is, to aug-
ment or diminish any Strength in a given Ra-
dio. As for Example:

C 2

PROP.

P R O P. V.

Suppose the side of a large Beam, or Girder be 30 Inches; What's the side of another square Beam, or Girder, whose Strength is but $\frac{1}{2}$ of the former?

Square 30, the Product 900 divide by 6, the Quote is 150; the Square-root of which is 12, 2 the side of the Girder, or Beam, that will bear but $\frac{1}{2}$ of the Weight of the former.

P R O P. VI.

In Triangle BAC (Plate 9th, Fig. 96.) the BA is 48 Foot, and the Perpendicular CA 40 Foot; What's the Length of the Hypotenuse BC.

The reason of the Solution of this Proposition is given in the 47th of the 1st Book of *Euclid* and in the 9th *Theorem* of my *Epitome* of *Geometry*, where 'tis prov'd in a right angled Triangle, that the Square-root of the Sum of the Squares of the Base and Perpendicular is equal to the Hypotenuse.

Square therefore 48 and 40, and add those two Squares together, it makes 3904; the Square-root of which is 62. 5 near for the Hypotenuse BC.

This is commonly the last Case in right Angled, plain Triangles, and in Navigation finds the Distance sailed by having the Difference of Latitude (which is the Perpendicular) and Departure (which is the Base) given: For the Square-root of the Sum of the Squares of the

Arithmetical Propositions.

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Difference of Latitude, and Departure is the Distance sailed. § 2.

This Proposition is also of use in Fortification: For it finds the Length of a Scaling Ladder that will reach from the Edge of a Moat to the Top of a Rampire, when the Breadth of the Moat and Height of a Rampire is given.

It's also of great use to Artificers, in finding the Length of *Hyp-Rafters*, Strings of Stairs, &c.

P R O P. VII.

Again, if the *Hypotenuse* and *Base* be given, then the *Perpendicular* is easily found: For if from the Square of the *Hypotenuse* you take the Square of the *Base*, and extract the Square-root of the Remainder, it will give the Length of the *Perpendicular*. So if the *Hypotenuse* and *Perpendicular* be given, 'tis but subtracting the Square of the *Perpendicular* from the Square of the *Hypotenuse*, and extracting the Square-root of the Remainder for the Length of the *Perpendicular*.

This Proposition is useful in Navigation: For by it the Difference of Latitude may be found, having the Distance sailed and Departure given; or by it the Departure may be found, having the Distance sailed and Difference of Latitude given.

P R O P. VIII.

To place any Number of Soldiers. Suppose 2496 in Square *Battalia*; that is, to place them in such order that there may be as many Rank as in File. C 3 Extra

§ 2.

Extract the Square-root of the Number proposed, it gives 364 the Number to be placed in Rank and File.

P R O P. IX.

To place any Number of Men, suppose 3591 in double Battalia, that is, so that the Men in Rank may be double to those in File.

Extract the Square-root of 17956, which is half the Number of Men proposed, it gives 134 the Number that must be placed in Files: This doubled gives the Number that must be placed in Rank, and these two Numbers multiply'd together, will make up the whole Army if the Work be right, else not.

P R O P. X.

Any Number of Men. Suppose 48224 with their Distance in Rank and File, that is, that their Distance in File should be 8 Foot, and in Rank 3 Foot, to order them in a square Battalia of Ground, 'tis required to know how many Men must be in Rank and how many in File.

Say as 8 to 3 so is 48224 to 18084, the Square-root of which is 167 near: Then to find the Number of Men to be placed in Rank, divide 48224 by 167, the Quote 288 is the Number of Men to be placed in Rank, beside 128 Men over and above.

P R O

Arithmetical Propositions.

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§ 2.

P R O P. XI.

The Use of the Extraction of the Cube-root.

Suppose the side of a Cube be 84 Foot ;
What's the side of another double to it ?

Cube 84 the Number given, it makes 592704 ;
then extract the Square-root of 1185408, it
doubled, and 'twill give 105 the side of a Cube
double to the former.

After the same manner may you find the
side of a Cube three, four, or more times as
big, or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, or any other part of a time so
little, by multiplying the Cube of the given
side by the Number of times proposed, or ta-
king such part of the said Cube as you would
have it to be less, and extracting the Square-
root of such Product or Quote.

P R O P. XII.

To find the side of a Cube equal to any gi-
ven Solid, whether *Parallelepipedon*, *Sphere*, *Cy-
linder*, *Cone*, &c.

Admit the Solidity of a *Parallelepipedon* were
5451776, What's the side of a Cube equal thereto ?

Extract the Cube-root of 5451776 the Content
of the given Solid, what comes out is (176) the
side of the Cube required.

From this Proposition the Proportion of the
Diameter of a Sphere to the side of a Cube
equal thereto may be found. For the Solidity
of a Sphere, whose Diameter is 1, is .523608805
the Cube-root of which .806 is near.

C 4

So

§ 2.

So that if the Diameter of a Sphere be 1 the side of the Cube equal thereto is 806.

And therefore as 1 to .806 so is any Diameter of a Sphere to the side of the Cube equal thereto.

To show all the Uses of the Square and Cube root would fill a large Volume. These are enough to let the Reader see the Necessity of learning this useful Operation.

To make this Treatise more perfect, I have added a Table of the Squares of all whole Numbers under 1000, and the Cubes of all Numbers under 200; which will be necessary on several Occasions.

A TABLE

A TABLE shewing the Squares of all Numbers under 1000.

Root	Square	Root	Square	Root	Square	Root	Square
1	1	40	1600	79	6241	118	13924
2	4	41	1681	80	6400	119	14161
3	9	42	1764	81	6561	120	14400
4	16	43	1849	82	6724	121	14641
5	25	44	1936	83	6889	122	14884
6	36	45	2025	84	7056	123	15129
7	49	46	2116	85	7225	124	15376
8	64	47	2209	86	7396	125	15625
9	81	48	2304	87	7569	126	15876
10	100	49	2401	88	7744	127	16129
11	121	50	2500	89	7921	128	16384
12	144	51	2601	90	8100	129	16641
13	169	52	2704	91	8281	130	16900
14	196	53	2809	92	8464	131	17161
15	225	54	2916	93	8649	132	17424
16	256	55	3025	94	8836	133	17689
17	289	56	3136	95	9025	134	17956
18	324	57	3249	96	9216	135	18225
19	361	58	3364	97	9409	136	18496
20	400	59	3481	98	9604	137	18769
21	441	60	3600	99	9801	138	19044
22	484	61	3721	100	10000	139	19321
23	529	62	3844	101	10201	140	19600
24	576	63	3969	102	10404	141	19881
25	625	64	4096	103	10609	142	20164
26	676	65	4225	104	10816	143	20449
27	729	66	4356	105	11025	144	20736
28	784	67	4489	106	11239	145	21025
29	841	68	4624	107	11449	146	21316
30	900	69	4761	108	11664	147	21609
31	961	70	4900	109	11881	148	21904
32	1024	71	5041	110	12100	149	22201
33	1089	72	5184	111	12321	150	22500
34	1156	73	5329	112	12544	151	22801
35	1225	74	5476	113	12769	152	23104
36	1296	75	5625	114	12996	153	23409
37	1369	76	5776	115	13225	154	23716
38	1444	77	5929	116	13456	155	24025
39	1521	78	6084	117	13689	156	24336

Root	Square	Root	Square	Root	Square	Root	Square
157	24649	200	40000	243	59049	286	81796
158	24964	201	40401	244	59536	287	82369
159	25281	202	40804	245	60025	288	82944
160	25600	203	41209	246	60516	289	83521
161	25921	204	41616	247	61009	290	84100
162	26244	205	42025	248	61504	291	84681
163	26569	206	42436	249	62001	292	85464
164	26896	207	42849	250	62500	293	85849
165	27225	208	43264	251	63001	294	86436
166	27556	209	43681	252	63504	295	87025
167	27889	210	44100	253	64009	296	87616
168	28224	211	44521	254	64516	297	88209
169	28561	212	44944	255	65025	298	88804
170	28900	213	45369	256	65536	299	89401
171	29241	214	45796	257	66049	300	90000
172	29584	215	46225	258	66564	301	90601
173	29929	216	46656	259	67081	302	91204
174	30276	217	47089	260	67600	303	91809
175	30625	218	47524	261	68121	304	92416
176	30976	219	47961	262	68644	305	93025
177	31329	220	48400	263	69169	306	93636
178	31684	221	48841	264	69696	307	94249
179	32041	222	49284	265	80225	308	94864
180	32400	223	49729	266	70756	309	95481
181	32761	224	50176	267	71289	310	96100
182	33124	225	50625	268	71824	311	96721
183	33489	226	51076	269	72361	312	97344
184	33856	227	51521	270	72900	313	97969
185	34225	228	51984	271	73441	314	98596
186	34596	229	52441	272	73984	315	99225
187	34969	230	52900	273	74529	316	99856
188	35344	231	53361	274	75076	317	100489
189	35721	232	53824	275	75625	318	101124
190	36100	233	54289	276	76176	319	101761
191	36481	234	54756	277	76729	320	102400
192	36864	235	55225	278	77284	321	103041
193	37249	236	55696	279	77841	322	103684
194	37636	237	56169	280	78400	323	104329
195	38025	238	56644	281	78961	324	104976
196	38446	239	57121	282	79524	325	105625
197	38809	240	57600	283	80089	326	106276
198	39204	241	58081	284	80656	327	106929
199	39601	242	58564	285	81225	328	107584

Root	Square	Root	Square	Root	Square	Root	Square
329	108241	372	138384	415	172225	458	209764
330	108900	373	139129	416	173056	459	210681
331	109561	374	139876	417	173889	460	211600
332	110224	375	140625	418	174724	461	212521
333	110889	376	141376	419	175561	462	213444
334	111556	377	142129	420	176400	463	214369
335	112225	378	142884	421	177241	464	215296
336	112896	379	143641	422	178084	465	216225
337	113569	380	144400	423	178929	466	217156
338	114244	381	145161	424	179776	467	218089
339	114921	382	145924	425	180625	468	219024
340	115600	383	146689	426	181476	469	219961
341	116281	384	147456	427	182329	470	220900
342	116964	385	148225	428	183184	471	221841
343	117649	386	148996	429	184041	472	222784
344	118336	387	149769	430	184900	473	223729
345	119025	388	150544	431	185761	474	224676
346	119716	389	151321	432	186624	475	225625
347	120409	390	152100	433	187489	476	226576
348	121104	391	152881	434	188356	477	227529
349	121801	392	153664	435	1892	478	228484
350	122500	393	154449	436	190096	479	229441
351	123201	394	155236	437	190969	480	230400
352	123904	395	156025	438	191844	481	231361
353	124609	396	156816	439	192721	482	232324
354	125316	397	157609	440	193600	483	233289
355	126025	398	158404	441	194481	484	234256
356	126736	399	159201	442	195364	485	235225
357	127449	400	160000	443	196249	486	236196
358	128164	401	160801	444	197136	487	237169
359	128881	402	161604	445	198025	488	238144
360	129600	403	162409	446	198916	489	239121
361	130321	404	163216	447	199809	490	240100
362	131044	405	164025	448	200704	491	241081
363	131769	406	164836	449	201601	492	242064
364	132496	407	165649	450	202500	493	243049
365	133225	408	166464	451	203401	494	244036
366	133956	409	167281	452	204304	495	245025
367	134689	410	168100	453	205209	496	246016
368	135424	411	168921	454	206116	497	247009
369	136161	412	169744	455	207025	498	248004
370	136900	413	170569	456	207936	499	249001
371	137641	414	171396	457	208849	500	250000

Root	Square	Root	Square	Root	Square	Root	Square
501	251001	544	295936	587	344569	630	396900
502	252004	545	297025	588	345744	631	398161
503	253009	546	298116	589	346921	632	399424
504	254016	547	299209	590	348100	633	400689
505	255025	548	300304	591	349281	634	401956
506	256036	549	301401	592	350464	635	403225
507	257049	550	302500	593	351649	636	404496
508	258064	551	303601	594	352836	637	405769
509	259081	552	304704	595	354025	638	407044
510	260100	553	305809	596	355216	639	408321
511	261121	554	306916	597	356409	640	409600
512	262144	555	308025	598	357604	641	410881
513	263169	556	309136	599	358801	642	412164
514	264196	557	310249	600	360000	643	413449
515	265225	558	311364	601	361201	644	414736
516	266256	559	312481	602	362404	645	416025
517	267289	560	313600	603	363609	646	417316
518	268324	561	314721	604	364816	647	418609
519	269361	562	315844	605	366025	648	419904
520	270400	563	316969	606	367236	649	421201
521	271441	564	318096	607	368449	650	422500
522	272484	565	319225	608	369664	651	423801
523	273529	566	320356	609	370881	652	425104
524	274576	567	321489	610	372100	653	426409
525	275625	568	322624	611	373321	654	427716
526	276676	569	323761	612	374544	655	430025
527	277729	570	324900	613	375769	656	430336
528	278784	571	326041	614	376996	657	431649
529	279841	572	327184	615	378225	658	432964
530	280900	573	328329	616	379456	659	434281
531	281961	574	329476	617	380689	660	435600
532	283024	575	330625	618	381924	661	436921
533	284089	576	331776	619	383161	662	438244
534	285156	577	332929	620	384400	663	439569
535	286225	578	334084	621	385641	664	440896
536	287296	579	335241	622	386884	665	442225
537	288369	580	336400	623	388129	666	443556
538	289444	581	337561	624	389376	667	444889
539	290521	582	338724	625	390625	668	446224
540	291600	583	339889	626	391876	669	447561
541	292681	584	341056	627	393129	670	448900
542	293764	585	342225	628	394384	671	450241
543	294849	586	343396	629	395641	672	451584

Root	Square	Root	Square	Root	Square	Root	Square
673	452929	716	512656	759	576081	802	643204
674	454276	717	514089	760	577600	803	644809
675	455625	718	515524	761	579121	804	646416
676	456976	719	516961	762	580644	805	648025
677	458329	720	518400	763	582169	806	649636
678	459684	721	519841	764	583696	807	651249
679	461041	722	521284	765	585225	808	652864
680	462400	723	523729	766	586756	809	654481
681	463761	724	524176	767	588289	810	656100
682	465124	725	525625	768	589824	811	657721
683	466489	726	527076	769	591361	812	659344
684	467856	727	528529	770	592900	813	660969
685	469225	728	529984	771	594441	814	662596
686	470596	729	531441	772	595984	815	664225
687	471969	730	532900	773	597529	816	665856
688	473344	731	534361	774	599076	817	667489
689	474721	732	535824	775	600625	818	669124
690	476100	733	537289	776	602176	819	670761
691	477481	734	538756	777	603729	820	672400
692	478864	735	540225	778	605284	821	674041
693	480249	736	541696	779	606841	822	675684
694	481636	737	543169	780	608400	823	677329
695	483025	738	544644	781	609961	824	678976
696	484416	739	546121	782	611524	825	680625
697	485809	740	547600	783	613089	826	682276
698	487204	741	549081	784	614656	827	683929
699	488601	742	550564	785	616224	828	685584
700	490000	743	552049	786	617795	829	687241
701	491401	744	553536	787	619368	830	688900
702	492804	745	555025	788	620943	831	690561
703	494209	746	556516	789	622520	832	692224
704	495616	747	558009	790	624100	833	693889
705	497025	748	559504	791	625681	834	695556
706	498436	749	561001	792	627264	835	697225
707	499849	750	562500	793	628849	836	698896
708	501264	751	564001	794	630436	837	700569
709	502681	752	565504	795	632025	838	702244
710	504100	753	567009	796	633616	839	703921
711	505521	754	568516	797	635209	840	705600
712	506944	755	570025	798	636804	841	707281
713	508369	756	571536	799	638401	842	708964
714	509796	757	573049	800	640000	843	710649
715	511225	758	574564	801	641601	844	712236

Root	Square	Root	Square	Root	Square	Root	Square
845	714025	884	781456	923	851929	962	925444
846	715716	885	783225	924	853776	963	927369
847	717409	886	784996	925	855625	964	929296
848	719104	887	786769	926	857476	965	931225
849	720801	888	788544	927	859329	966	933156
850	722500	889	790321	928	861184	967	935089
851	724201	890	792100	929	863041	968	937024
852	725904	891	793881	930	864900	969	938961
853	727609	892	795664	931	866761	970	940900
854	729316	893	797449	932	868624	971	942841
855	731025	894	799236	933	870489	972	944784
856	732736	895	801025	934	872356	973	946729
857	734449	896	802816	935	874225	974	948676
858	736164	897	804609	936	876096	975	950625
859	737881	898	806404	937	877966	976	952576
860	739600	899	808201	938	879844	977	954529
861	741321	900	810000	939	881721	978	956484
862	743044	901	811801	940	883600	979	958441
863	744769	902	813604	941	885481	980	960400
864	746496	903	815409	942	887364	981	962361
865	748225	904	817216	943	889249	982	964324
866	749956	905	819025	944	891136	983	966289
867	751689	906	820836	945	893025	984	968256
868	753424	907	822649	946	894916	985	970225
869	755161	908	824464	947	896809	986	972196
870	756900	909	826281	948	898704	987	974169
871	758641	910	828100	949	900601	988	976144
872	760384	911	829921	950	902500	989	978121
873	762129	912	831744	951	904401	990	980100
874	763876	913	833569	952	906304	991	982081
875	765625	914	835396	953	908209	992	984064
876	767376	915	837225	954	910116	993	986049
877	769129	916	839056	955	912025	994	988036
878	770884	917	840889	956	913939	995	990025
879	772641	918	842724	957	915849	996	992016
880	774400	919	844561	958	917764	997	994009
881	776161	920	846400	959	919681	998	996004
882	777924	921	848241	960	921600	999	998001
883	779689	922	850084	961	923521	1000	1000000

A TABLE

A TABLE shewing the Cube of all Numbers from 1 to 200.

Root	Cube	Root	Cube	Root	Cube	Root	Cube
1	1	38	54874	75	421875	112	1404928
2	8	39	59319	76	438976	113	1442897
3	27	40	64000	77	456533	114	1481544
4	64	41	68921	78	474552	115	1520875
5	125	42	74088	79	493039	116	1560896
6	216	43	79507	80	512000	117	1601613
7	343	44	85184	81	531441	118	1643032
8	512	45	91125	82	551368	119	1685155
9	729	46	97336	83	571787	120	1728000
10	1000	47	103823	84	592704	121	1771561
11	1331	48	110592	85	614125	122	1815848
12	1728	49	117649	86	636056	123	1860867
13	2197	50	125000	87	658503	124	1906624
14	2744	51	132651	88	681472	125	1953125
15	3375	52	140608	89	704969	126	2000376
16	4096	53	148877	90	729000	127	2048383
17	4913	54	157464	91	753571	128	2097152
18	5832	55	166375	92	778688	129	2146689
19	6859	56	175616	93	804357	130	2197000
20	8000	57	185193	94	830584	131	2248091
21	9261	58	195112	95	857375	132	2299968
22	10648	59	205379	96	884736	133	2352637
23	12167	60	216000	97	912673	134	2406104
24	13824	61	226981	98	941192	135	2460375
25	15625	62	238328	99	970299	136	2515456
26	17576	63	250047	100	1000000	137	2571353
27	19683	64	262144	101	1030301	138	2628072
28	21952	65	274625	102	1061208	139	2685619
29	24389	66	287496	103	1092727	140	2744000
30	27000	67	300763	104	1124864	141	2803221
31	29791	68	314432	105	1157625	142	2863288
32	32770	69	328509	106	1191016	143	2924207
33	35939	70	343000	107	1225043	144	2985984
34	39306	71	357911	108	1259712	145	3048625
35	42877	72	373248	109	1295029	146	3112136
36	46658	73	389017	110	1331000	147	3176523
37	50655	74	405224	111	1367631	148	3241792

Root	Cube	Root	Cube	Root	Cube	Root	Cube
149	3307949	162	4251528	175	5359375	188	6644672
150	3375000	163	4330747	176	5451776	189	6751269
151	3442951	164	4410944	177	5545233	190	6859000
152	3511808	165	4492125	178	5639752	191	6967871
153	3581577	166	4574296	179	5735339	192	7077888
154	3652264	167	4657463	180	5832000	193	7189057
155	3723875	168	4741632	181	5929741	194	7301384
156	3796416	169	4826809	182	6028568	195	7414875
157	3869893	170	4913000	183	6128487	196	7529536
158	3944312	171	5000211	184	6229304	197	7645373
159	4019679	172	5088448	185	6331625	198	7762392
160	4096000	173	5177717	186	6434856	199	7880599
161	4173281	174	5268024	187	6539203	200	8000000

CHAP. II.

*The Explication and Use of the
Tables of LOGARITHMS.*

S E C T. I.

The Explication of the Tables of the Logarithms, and of Parts proportional. § I.

Defin. **T**HE *Logarithms* were first invented, found out and framed, by that never to be forgotten and thrice Honourable Lord, the Lord *Nepeir*: Which Numbers, so found out and framed by his diligent Industry, he was pleased to call *Logarithms*; which in the Greek signifies the Speech of Numbers, I shall not here trouble you with the manner of the Construction of those Tables of *Logarithms*, but shall first lay down some brief and general Rules, that thereby the better you may understand those Tables, and then I shall explain their manifold Uses, in sundry Examples Arithmetical, &c.

P R O P. I.

Any Number given under 10000, or 100000, to find the Logarithm corresponding thereunto.

I. If the Number propounded consist of one place, whose *Logarithm* is required to be found,

D

as

§ 1. as suppose (5,) look for 5 in the top of the left hand Column under the Letter * N, and

* Signifies the Number, or Figure sought.

† Signifies the Logarithm answering to the Number opposite.

right against 5, and in the next Column under LOG.

† you will find this Number or Rank of Figures, 0698970, which is the *Logarithm* of the

Number 5 required.

2. If the Number consisteth of two places, as if it were 57, look 57 under N, and opposite to it, and under LOG. you will find this Number 1. 755875, which is the *Logarithm* of 57, the Number propounded.

3. If the Number propounded consist of three places, as 972, look for 972, under N, and opposite to 972, and under (0) the Column, you shall find this Number 2. 987666, which is the *Logarithm* of 972, the Number which was propounded.

4. But if the Number consists of four places, as 1685, look the three first Figures 168, under the Column N; and opposite to that, and under 5 at the top of the Page, you will find this Number 3. 226599, which is the *Logarithm* of 1685, the Number propounded.

 Note this Rule well; for this explains the Use of the Table of proportional Parts, printed at the end of this Book.

5. But if the Number given be above 10000, and under 100000, you may find its *Logarithm* by the Table of Parts proportional, printed at the

latter end of this Book. Thus, if the *Logarithm* of 35786 be sought, first seek the Log. of 3578, which will be 553649, and the common Difference under D is 121; with this Difference 121, Enter the Table of Parts proportional,

tional, and finding 121 in the first Column under D, you may then lineally under 6, find the Number 72, which add to the Log. of 3578, that is 553649, it produceth 553712, which is the Log. of 35786 the Number propounded: Now because the Number propounded 35786, ariseth to the place of X. M. therefore there must be the Figure 4 prefixed before its *Logarithm*, and then it will be thus 4. 553712, which 4 is called the Index, as shall be hereafter shewed.

Now before we proceed to find Numbers corresponding to *Logarithms*, it will be necessary to explain the meaning of the first Figure to the left hand of any *Logarithm* placed. Mr. Briggs Definition:

calleth it a Characteristick or Index, which doth represent the Distance of any the first Figure of any whole Number

The Rule to find the Characteristick or Index appertaining to any Logarithm.

from Unity, whose Index is 0, a Cypher; so the Index of 10 is 1, and so to 100 whose Index is 2, and so to 1000 whose Index is 3, and so to 10000 whose Index is 4; and so if you persist further the Characteristick is always one less in Dignity than the Places or Figures of the Number propounded.

PROP. II.

To find the Logarithm of a Vulgar Fraction, and a Mixt Number.

First, as is before shewed, if it be a Vulgar Fraction, find the Log. of the Numerator, and the Log. of the Denominator, then subtract the

D 2

Log.

§ 1. *Log.* of the Numerator from the *Log.* of the Denominator, the remainder The *Log.* 7 is ————. 0. 84509
 is the *Log.* of The *Log.* 5 is ————. 0. 69897
 the Fraction
 propounded : The *Log.* $\frac{7}{5}$ is ————. 0. 14612
 Now if you
 would find the *Logarithm* of $\frac{7}{5}$, do as is prescribed, whose *Log.* I find to be 0. 146121. Now to find the *Log.* of a Mixt Number, reduce it into an Improper Fraction, and then do as before, so the *Log.* of $15\frac{2}{3}$, Improper $\frac{77}{5}$, is 1, 187, 52, and so do for any other Mixt Number.

P R O P. III.

A Logarithm propounded to find the Whole, or Mixt Number, corresponding thereunto.

For the more speedy finding the Number, answering unto the *Logarithm* propounded observe, that if the Index be 0, then the Number sought may be found between 1 and 10; If 1, between 10 and 100; if 2, between 100 and 1000; if 3, between 1000 and 10000, and so on, still observing the Rules of the Characteristick, or Index; therefore look in the Table until you find the *Logarithm* proposed, and against it in the Margin, according to the foregoing Directions under N, you shall find the Number belonging thereunto. This Rule holds in force in Mixt Numbers also.

Thus.	0.	845098	Are the Logarithms of,	7
	1.	556302		36
	2.	130334		135
	3.	980276		9556

NOTE.

of LOGARITHMS.

37

§ 1.

NOTE.

But if you cannot find the *Logarithm* exactly in the Table, as in many Operations it so happeneth, you must then take the nearest *Logarithm* Number to the *Logarithm* propounded, and so take the Number belonging thereto for the desired Number.

SECT. II.

Of the Admirable Use of the Logarithms in Arithmetick. § 2.

PROP. I.

To multiply one Number by another.

ADmit 90 to be multiplied by 42, what is the Product? First find the *Log.* of the Multiplicand 90, Then the *Log.* of the Multiplier 42. Add these two *Log.* together, it gives 3.7748, which is the *Log.* of 3780, the Product of 90, and 42.

The <i>Log.</i> of 90 is —	1. 95424
The <i>Log.</i> of 42 is —	1. 62324
The <i>Log.</i> of 3780 is —	3. 57748

D 3

PROP.

NOTE

The Use of the Logarithms

P R O P. II.

To divide one Number by another.

Admit the Dividend (or Number to be divided) be 648, and the Divisor 72, what is the Number that the Quotient shall consist of? To find which, first write down the *Logarithm* of the Dividend 648, and also write down the *Logarithm* of the

Divisor 72. The *Log.* of 648 is — 2. 81157

Then sub- The *Log.* of 72 is — 1. 85733

stract the *Log.*

of the Divi- The *Log.* of 9 is — 0. 95424

for, out of the

Log. of the Di-

vidend, the remainder is 0. 95424, the *Logarithm* of 9; so I conclude that the Divisor 72 is contained in the Dividend 648, 9 times, and so do for any other.

P R O P. III.

To find the Square-Root of a Number.

Admit it be required to extract the Square Root of the Number 144; to perform which first write down the *Log.* of 144, which is 2. 15836. Then take the half thereof, which is 1. 07918, it is the *Log.* of 12, the Root of 144 propounded, and so do for any other.

NOTE on the contrary, by doubling the *Log.* of any Number, you have the Geometrical Square thereof: And by tripling the *Log.* of any Number you have the Cube of it.

P R O P.

P R O P. IV.

To find the Cube Root of any Number.

§ 2.

Admit it be required to extract the Cube-Root of 1728 ; to perform which, first write down the *Log.* of 1728, which is 3. 23754, then take the third part thereof, which is 1. 07918, the *Log.* of 12, the Cube-root of 1728 the Number proposed, and so for any other. Note on the contrary, if you multiply the *Log.* of any Number propounded by 3, it produceth the *Log.* of the Cube thereof.

P R O P. V.

A Summ of Money being forborn for any Number of Years, to find how much it will amount unto, reckoning Interest on Interest, according to any Rate propounded.

Admit 300 Pounds Sterling be put out for 4 Years, for Compound Interest at 6 *l.* per Cent. what will it amount to when the four Years are expired? To find which subtract the *Log.* of 300 *l.* the Principal, whose *Log.* is 2. 477121, out of the *Log.* of 318 *l.* Principal and Interest for a Year, whose *Log.* is 2. 502427, the remainder is 0. 025306, which being multiplied by 4, the Number of Years of its Continuance, produceth 0. 101224, which added to the *Log.* of the Principal 300 *l.* to wit, to 2. 477121, makes 2, 578345, which is the *Log.* of 378 *l.* 14s. 10d. 2q. very near, and so much will 300 *l.* amount to.

D 4

P R O P.

The Use of the Logarithms

P R O P. VI.

A Summ of Money being to be paid hereafter, to find what it is worth in ready Money.

Admit 100 Pounds Sterling to be paid at 30 Years end; I demand how much it is worth in ready Money? After the rate of Interest of 6 *l.* per Cent. To find which, substract the *Logarithm* of 100 the Principal, whose *Log.* is 2. 000000 from the *Log.* of 106 Principal and Interest, whose *Log.* is 2. 025306, the remainder is 0. 025306, which multiplied by 30 the Number of Years to succeed, produceth 0. 759180, which substracted out of 2. 000000, leaveth 1. 240820, which is the *Log.* of $17\frac{4}{5}\%$, which sheweth the said 100 *l.* is worth but 17 *l.* 8 s. 2 d. 3 q. fere,

P R O P. VII.

A Yearly Rent, or Annuity, to continue any Number of Years, to find what it is worth in ready Money, at any Rate of Interest propounded.

What is 100 Pound *per Annum* to continue 30 Years, worth in ready Money at 6 *l.* per Cent. To find which, first substract the *Log.* of 100 *l.* the Principal, which is 2. 000000 from the *Log.* of 106 *l.* Principal and Interest for a Year, whose *Log.* is 2. 025306 the remainder is 0. 025306: Then multiply 0. 025306 by 30 the Number of Years of its Continuance, it produceth the Number 0. 759180; Then divide 100 *l.* by 6 the rate of Interest, and the Quotient is $16\frac{6}{6.67}$, &c. which $16\frac{6}{6.67}$ is the proportional

tional Parts of 100*l.* the Principal; then add the *Log.* thereof, which is 1. 221829 to the former *Log.* 0. 759180 it produceth 1. 981009, which is the *Log.* of $95 \frac{721}{10000}$ parts the Arrearages with the said Summ for that time; then from those Arrearages $95 \frac{721}{10000}$ subtract the parts proportional of 100, to wit, $16 \frac{6667}{10000}$, the remainder is $79 \frac{5543}{10000}$, which is the bare Arrearages for that proportional part; Then take the *Log.* of $79 \frac{5543}{10000}$, which is 1. 897929, out of the which take the *Log.* found by Multiplication of Years, to wit, 0. 759180, there remains 1. 138749, which is the *Log.* of the Value of the Arrearages in ready Money; then to the *Log.* 1. 138749, add the *Log.* of 100*l.* Principal, 2. 000000, it produceth this Number 3. 138749; the *Log.* of $1376 \frac{48}{100}$ reduced is 1376*l.* 9*sh.* 7*d.* $\frac{80}{100}$ or $\frac{4}{5}$ fere: And so much is the said Annuity worth in ready Money.

C H A P. III.

*The Explication of the SINES,
TANGENTS and SECANTS.*

S E C T. I.

§ I. *Of Right Sines, Tangents, Secants, Co-Sines, Tangents, and Secants: Of any Arch, or Angle of a Triangle.*

P R O P. I.

To find the right Sine, or Tangent, of any Arch or Angle of a Triangle, containing any Number of Degrees and Minutes.

 And here 'tis necessary to understand, that every Circle is supposed to be divided into 360 equal parts, which are called Degrees, and every of those Degrees into 60 Minutes, and every Minute into 60 Seconds, and every Second into 60 Thirds, &c. so that a Semi-Circle contains 180 Degrees, and a Quadrant 90 Degrees; Now an Arch or Angle of a Triangle, is the Intersection of its two sides, and the Measure thereof, is an Arch of a Circle, which cutteth each of the two sides equidistant from the Angular Point, (which is the Center.) Now the Logarithm, Sine, or Tangent of any such Arch of a Triangle, containing any Number of Degrees or Minutes of the Quadrant, may be found in the Tables, printed at the End of this Book, where they are plainly expressed, and are found as directed in the precedent Rules.

IF the Angle or Arch of the Triangle propounded be less than 45 Deg. the Sine, or Tangent, belonging thereunto, is found in the Column under the Title SINE, or TANGENT, at the top of the Table; and if there be any Minutes annexed unto the Degrees, you must find them out in the first Column under M. signifying Minutes, and opposite to those Minutes, and under the Title aforesaid, you shall have the Logarithm of the Sine or Tangent, of the Arch or Angle required.

But

Tangents and Secants.

43

§ 1.

But if the Arch or Angle of a Triangle exceed 45 Degrees, you must then look for the Sine or Tangent belonging thereunto, in the bottom of the said Table; and if thereunto are Minutes annexed, you must look for them in the first Column to the Right-hand under M, and so opposite to those Minutes in the Column above the Title, Sine, or Tangent; there have you the Log. of the Sine, or Tangent, of the Arch or Angle, of the Triangle propounded.

Examp. Suppose it were required to find the Log-Sine or Log-Tangent of an Angle of 25 D. 37 M. whose Log-Sine, whereof according to the former Directions, I find to be 9. 635833, and Tangent thereof to be 9. 680768, and so for any other under 45 Degrees.

Again, suppose it were required to find the Log-Sine or Log-Tangent of an Angle of 64 D. 23 M. the Sine whereof I find to be this Number 9. 955065, and the Tangent thereof 10. 319231, and so for any other Arch, or Angle of a Triangle, above 45 Degrees.

P R O P. II.

To find the Co-Sine or Co-Tangent of any Arch, or Angle propounded.

Defin. The Co-Sine or Co-Tangent of an Angle or Arch, is the remaining part of the Angle propounded to a Quadrant or 90 Degrees; and is by some called the Complement of an Angle. Thus the Arch or Angle of 64 D. 23 M. taken out of 90 D. leaves 25 D. 37 M. for its Complement; on the contrary, if 25 D. 37 M. were taken out of 90 Degrees, there would remain

44 *The Explication of the Sines, &c.*

§ I. main 64 D. 23 M. for its Complement. So you see that these two Angles are the Complements of each other, because they two are equal to a Quadrant or 90 Degrees.

Now the Logarithm of the Complement may be exactly found with ease; for the Sines and Tangents of every Degree, and Minute of the Quadrant in one Column is joyned with his Complement in the next Column; so that without substracting the Angle from 90 D. you may readily find the Complement thereof either the Arch in Degrees and Minutes, or the Log-Sine, or Tangent thereof, as you have occasion: Thus the Log. of the Sines Complement before mentioned, to wit, 64 D, 23 M. Compl. is 25 D. 37 M. is 9. 635833, Tang. is 9. 680768; so 64 D. 23 M. is the others Compl. whose Sine is 9. 955065, and his Tang. is 10. 319231; so for any other.

P R O P. III.

To find the Secant of any Arch or Angle propounded.

In this little Book I have not room to set down the Tables of Artificial Secants at large, as I have done with the Sines and Tangents: Nevertheless I will not here omit to shew how they may be easily found out, by the Tables of Sines. The Method is thus, substract the Logarithm Sine, of the Sines Compl. of an Angle, from the double *Radius* of the Tables, and the remainder shall be the Secant required: As if I desire the Secant of 25 D. 37 M. I find the Logarithm-Sine of his Complement to be 9. 955065, which substracted from the double *Radius*, that

is 20. 000000, there remains 10. 044935, which is the Secant of it, and so the Secant of 64 D. 23 M. is 9. 955065, which is the Complement of the former; because they both are equal to 20. 000000, the double *Radius*, and so may any other be found out.

CHAP. IV. Of GEOMETRY.

Defin. **T**HE End and Scope of Geometry is to measure well: For as Number or Multitude is the Subject of Arithmetick, so is Magnitude that of Geometry: To measure well, is therefore to consider the Nature of every thing that is to be measured; to compare such like things one with another, and to understand their Reason and Proportion, and also their Similitude: And this is the End and Scope of Geometry *.

* *Quod quaeritur cognoscendi illius gratia, quod semper est, non de ejus quod oritur, quando-*

que de interit. Geometria, ejus quod est semper, Cognitio est. Ac tollet igitur (o Generose vir) ad veritatem, animum: atque ita, ad Philosophandum praeparavit cogitationem, ut ad supera convertamus: quae nunc, contra quam decet, ad inferiora dejicimus, &c. Plato lib. 7. de Rep.

I shall not trouble you with the Definitions of Geometry, they being largely handled by many, and herein every one meanly conversant in the Study of the Mathematicks is acquainted, but shall immediately fall in hand with the principal Propositions which chiefly concern the other following parts of this Treatise.

SECT.

S E C T. I.

§ 1. *The Explication of some Geometrical Propositions.*

P R O P. I.

To erect a Perpendicular on any part of a Line assigned.

Fig. 1. **L**ET the Line be A, B, and on the Point D, 'tis required to raise a *Perpendicular* to A, B: To operate which, first open your Compasses to any convenient distance, and placing one Foot thereof in D, with the other, make the two Marks C and E equidistant from D; then open the Compasses to some other convenient distance, and set one Foot in E, and describe the *Arch* FF; then likewise in C, describe the *Arch* GG, then through the Intersections of these two *Arches*, and to the Point D, draw HD *perpendicular* to AB, as was required.

P R O P. II.

To erect a Perpendicular on the End of a Line.

Fig. 2. Let the given Line be AB, and on the End thereof at B, 'tis required to raise a *Perpendicular* Line: To perform which, open your Compasses to any distance as BD, then on B as a Center, describe the *Arch* D, E, F, then from D to E place BD; then placing one Foot in E, describe the *Arch* CF, then remove your Compasses to F, and draw the *Arch* CE. Lastly,

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Lastly, through their Intersection draw CB, which is a *Perpendicular* to AB, on the end B, as required.

§ 1.

P R O P. III.

From a Point above to let fall a Perpendicular on a Line.

Let the Line given be BA, and 'tis required from the Point above at C, to let fall a *Perpendicular* to the said Line: To perform which, place one Foot of your Compasses in C, and open them beyond the given Line AB, and describe the Arch EF; divide EF into two parts in D; Lastly, draw CD, which shall be perpendicular unto AB, falling from the Point above at C, as was so required. Fig. 3.

P R O P. IV.

To draw a right Line Parallel to a right Line, at any distance assigned.

Let the distance assigned be OE, and the Line given be AB, and 'tis required to draw CD *Parallel* to AB, at the distance OE: To perform which, take in your Compasses the distance OE, and on A describe the Arch H, and on B the Arch K; then draw CD, so as it may justly touch the two *Arches*, but cut them not; so shall CD be parallel to AB, at the assigned distance OE, as was required. Fig. 4.

P R O P.

§ 1.

P R O P. V.

To protract an Angle of any Quantity of Degrees propounded.

Fig. 5. Let it be required to *protract*, or lay down an Angle of 40 Degrees: To perform which first draw a right Line as AB, then open your Compasses to 60 Degrees, in your Line of Chords: And with that Distance on A, describe the Arch EF; then take 40 Degrees in your Compasses out of your Line of Chords, and place it on the Arch, from F to H. Lastly draw a Line through the Point H, and from A, draw AC so shall the Angle CAB contain 40 Degrees, as required.

P R O P. VI.

To measure an Angle already protracted.

Fig. 5. Let the Angle given be CAB, and 'tis required to find the Quantity thereof: To perform which, take in your Compasses 60 Degrees from your Line of Chords; and on A describe the Arch EF; then take in your Compasses the Distance FH, and apply it to your Line of Chords, and you will find the Angle CAB to contain 40 Degrees.

P R O P. VII.

To divide an Angle into two equal parts.

Fig. 6. Let the Angle given be BAC, and 'tis required to divide it into two equal parts: To perform which, do thus; first take in your Compasses any convenient Distance, and placing one

one Foot in A, describe the *Arch* FKHE; then on H, describe the *Arch* KK; and on K, the *Arch* HH. Lastly, through the *Intersections* of these two *Arches*, draw the Line AD, to the *Angular Point* A; so shall the *Angle* BAC, be divided into two equal parts, viz. BAD, and DAC, as required.

§ 1.

PROP. VIII.

To divide a right Line into any Number of equal or unequal parts; or like to any divided Line propounded.

Let the Line AB be given to be divided into 5 equal parts, as the Line CD. To perform which, do thus; first on the Point C, draw out a Line, making an *Angle* with CD at pleasure, then make CF equal to AB, and join their Extremities FD; then draw *Parallel Lines* to FD, through all the 5 Points of CD (by the 4 Prop. foregoing) which shall divide AB into 5 equal parts, as required: This way is to be observed, when the Line given to be divided is greater than the divided Line propounded.

Fig. 7.

CASE II.

But if AB be shorter than the given divided Line CD, take the Line AB in your Compasses, and on D strike the *Arch* F; then draw the *Tangent* CF, then take the nearest distance from the first Division of CD to the *Tangent-Line* CF, which distance shall divide AB into 5 equal parts, as the given divided Line CD, as required.

Fig. 8.

These two Propositions, well understood, do demonstrate many other Propositions; and thereon is grounded the Use of the Sector, which is the most universal Instrument for Mathematical Operations.

E

PROP.

P R O P. IX.

How to protraſt or lay down any of the Regular Figures, called Polygons.

To perform which, divide 360 Degrees, (the Number of Degrees in a Circle) by the Number of the *Poligon* his ſides: As if it be a *Pentagon* by 5, if a *Hexagon* by 6, &c. the Quotient is the Angle of the *Center*; its *Complement* to 180 D. (or a *Semi-circle*) is the Angle at the *Figure*, half whereof is the Angle of the *Triangle* at the *Figure*: Now I will ſhew how to delineate any *Polygon* three ways, viz. 1 by the Angle at the *Center*, 2 by the Angle at the *Figure*, 3 by the Angle of the *Triangle* at the *Figure*: I have hereunto annexed a Table, which gives at the firſt Sight, (without the trouble of Diviſion) 1. the Quantity of the Angle at the *Center*; 2. the Quantity of the Angle at the *Figure*; and 3. the Quantity of the Angle at the *Triangle* of the *Figure*, from a *Triangle* to a *Decagon*.

Names of the Polygons.	Sides	Angles at the Center	Angles at the Figure	Angles at the Trian.
		D M	D M	D M
Triangle	3	120 00	60 00	30 00
Square	4	90 00	90 00	45 00
Pentagon	5	72 00	108 00	54 00
Hexagon	6	60 00	120 00	60 00
Heptagon	7	51 43 $\frac{1}{2}$	128 34 $\frac{1}{2}$	64 17 $\frac{1}{4}$
Octagon	8	45 00	135 00	67 30
Nonagon	9	40 00	140 00	70 00
Decagon	10	36 00	144 00	72 00

CONSTRUCTION I.

First by the *Angle at the Center*, to delineate Fig. 9.
a *Hexagon*, whose Angle at the Center is 60 Degrees; first lay down an Angle of 60 Deg. (by Prop. the 5. foregoing) making its sides of a convenient Length at pleasure; then take such a distance from O the Center of the Figure, equally on both sides, as may make the third side equal to the side of the *Polygon* given, which here is 100 parts:

* Then divide the third side equally into two equal parts, and draw a Line through it, from $\odot$ the Center: Set each half of the side of the *Polygon* 100, to wit, 50; on each from the middle of the third Line: † Thus having placed the side of the *Hexagon* PP, 100 parts in order; describe the whole *Hexagon* PPPPPP, as was required.

* But this in a *Hexagon* need not be done, because the 3 sides of the *Triangle* are equal; but in all other *Polygons* it must be done.

† But if the third Line do exceed or be short of the side of the *Polygon* propounded; then by *Parallels* on each side, cut the sides of the *Triangle*, till you have found by those *Intersections* where to set the Line proposed in any *Polygon*, &c.

CONSTRUCTION II.

Now by the *Angle of the Figure*, to delineate Fig. 10.
any regular *Polygon*, let it be required to protract a *Hexagon*, whose side as afore is 100 parts; first I draw a Line, and make it 100 of those parts: Then I find in the precedent Table the Angle of a *Hexagon* at the Figure to be 120 Degrees. Then on each side of the drawn Line I lay down an Angle of 120 Deg. (according to the 5 precedent Propositions) and so work 6 times,

- § 1. times, (or as many times as your *Polygon* hath sides) making each side 100 parts, and each Angle 120 Degrees; so shall you have enclosed the *Polygon* PPPPPP, as required.

CONSTRUCTION III.

- Fig. 11. To protract or lay down a *Hexagon*, or any other regular *Polygon*, by the Angle of the Triangle, do thus; first draw the side of the *Hexagon* PP, make it 100 parts. I find in the precedent Table, that the Angle of the Triangle is 60 Deg. then at each end of the Line PP, I

 Observe these Rules well; for you will find them of infinite Use in Fortification, &c.

lay down an Angle of 60 Deg. (by Prop. 5. precedent) and continue the two Lines PO and PO until they intersect each other in O: Then on O, as a Center (OP being *Radius*) describe a Circle, and within it describe the *Hexagon* PPPPPP, as you see in the Figure: And so may you delineate any other *Polygon*, whose Angles from a *Triangle* to a *Decagon*, are all specified in the precedent Table.

P R O P. X.

To divide a Line according to any assigned Proportion.

- Fig. 12. Admit the right Line given to be AB, and 'tis required to divide the same into two parts, bearing Proportion the one to the other as the Lines E, and F doth: To perform which, first draw the Line CD equal to the given Line AB; then draw the Line HC from C, to contain an Angle at pleasure. Then from C to G place the

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the Line F, and from G to H place the Line E: Then draw the Line HD. And lastly, draw GK parallel to HD, (by the 4 Prop. precedent) so is the Line DC equal to AB, and divided into two parts, bearing such Proportion to each other, as the two given Lines E and F, as was required.

§ 1.

P R O P. XI.

To two Lines given, to find a third proportional to each of them.

Admit the two given Lines be A and B, and 'tis required to find a third Proportional to A, as A to B: First make an Angle at pleasure, as HIK; then place the Line B from I unto P, and the Line A from I unto L, and draw PL; then also place the Line A from I unto M, and draw QM, parallel unto LP, (by 4 Prop.) so shall the Line IQ be a third Proportional unto the two given Lines A and B, as was required. For as B is to A, so is A unto the Proportional found IQ. Fig. 13.

P R O P. XII.

To three Lines given to find out a fourth Proportional unto them.

Admit the three given Lines to be A, B, and C; and 'tis required to find a third proportional to them, which shall have such Proportion unto A as B hath unto C. To perform which, first make an Angle at pleasure as DKG, now seeing the Line C, hath such Proportion to B, as the Line A, unto the Line sought: Therefore

E 3

fore

§ 1. fore place the Line C from K unto H and B, from K to F, and draw FH. Again, place the Line A from K to I, and draw IE parallel unto FH, (by 4 Prop.) until it cutteth DK in E; so have you the Line KE, a fourth Proportional, as was required: For as C is unto B, so is A unto the found Line KE.

P R O P. XIII.

To find a mean proportional Line between any two right Lines given.

Fig. 15. Let the two given Lines be A and B, between which it is required to find a mean proportional Line. To perform which, first joyn the two Lines A and B together, so as they make the right Line CED; Then describe thereon a Semicircle CFD; then on the Point E erect the Perpendicular EF, (by 1 Prop.) to cut the Limb of the Semicircle in F, so shall EF be a mean proportional Line between the two given Lines A and B, as required.

P R O P. XIV.

To find two mean proportional Lines between any two right Lines given.

Fig. 16. Let the two given Lines be A and B, between which 'tis required to find two mean Proportionals. To perform which, first make an Angle containing 90 Deg. making the sides CD, and CE of a convenient Length: Then from C place the Line B unto F, and the Line A from C unto G, and draw FG; which divide

And this of all other the Inventions of Plato, Apollonius, Sporus, Architas, Diocles, Nicomedus, and many other famous Geometricians and Philosophers, I like best for the ready Performance of this Conclusion, whose several Methods I could here describe, but for Brevity sake do omit them.

divide equally in H, and describe the *Semicircle* FKG. Then take the Line B in your Compasses, and placing one Foot in G, with the other make a Mark in the Limb of the *Semicircle* in K, then draw ST in such sort that it may justly touch the *Semicircle* in K, and may cut through the two sides of the Angle, equidistant from the Center of the *Semicircle* H; so shall SF and TG be two *mean Proportionals* betwixt the two given Lines A and B, as required. § 1.

P R O P. XV.

To make a Geometrical Square equal to divers Geometrical Squares.

Let there be given the 5 sides of five *Geometrical Squares*, viz. A, B, C, D, E; and 'tis required to make one *Geometrical Square* equal to the said five Squares: To perform which, first make a Right Angle as ABC, making its contained sides of a convenient Length. Then from B place A to D, and from B place B to E, and draw ED; then place ED from B to F, and C from B to G, and draw GF; then place GF from B to H, and D from B to I, and draw IH. Lastly, from B unto K place IH, and from B unto L place the Line E, and draw LK. So shall LK be the side of a Square, equal to the five Squares propounded. Fig. 17.

P R O P. XVI.

To make a Circle equal to divers Circles propounded.

Fig. 18. Let the two Circles propounded be A and B, and 'tis required to make a third Circle, equal to the said Circles propounded. To perform which, first take the Diameter of the lesser Circle A, and place it as a *Tangent* on the *Diameter* of the greater Circle B, at *right Angles*, as ECD. Then draw the *Diagonal* ED, which divide equally in F, on which as a Center describe the Circle K, making ED the *Diameter*, of which Circle K shall be equal unto the two given Circles A and B, as required *.

* So after the same manner, may divers Circles be added into one by the help of the former Proposition well understood.

S E C T. II.

Of Planometry, or the way to measure any plain Superfice.

PLANOMETRY is that part of the *Mathematicks* derived from that noble Science *Geometry*, by which the *Superficies* or *Planes* of things are measured, and by which their *Superficial Content* is found, which is done most commonly by the *Squares* of such Measures, viz. a *Square Inch*, *Square Foot*, *Square Yard*, *Square Pace*, *Square Perch*, &c. That is, whose side is an *Inch*, *Foot*, *Yard*, *Pace*, or *Perch Square*. So that the *Content* of any Figure is said to be found, when

you

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you know how many such *Inches, Feet, Yards, Paces, &c.* are contained therein. Thus the End and Scope of *Geometry* is to measure well. § 2.

P R O P. I.

To find the Superficial Content of a Geometrical Square.

Let the side of the Square AA be 4 *Perch*, Fig. 19. what is the *Area*, or Superficial Content thereof? To find which multiply its side 4, by its self, it produceth 16, which is the Content of that Square AAAA propounded.

P R O P. II.

To find the Superficial Content of a Parallelogram, or long Square.

Multiply the Length in Parts by the Breadth Fig. 20. in Parts, the Product is the Content thereof. So in the *Parallelogram*, or long Square ABCD, the Length of the side AB, or CD, is 20 *Paces*, and the Breadth AC, or BD, is 10 *Paces*, and this Superficial Content is required. I say therefore if according unto the Rule, you multiply the Length 20 by the Breadth 10, it produceth 200 *Paces*, which is the Content of the *Parallelogram* or long Square ABCD.

P R O P.

P R O P. III.

To find the Superficial Content of any Right-lined Triangle.

Fig. 21. Although right-lined Triangles are of several Kinds and Forms ; as first in respect unto their Angles, they are either *Right-angled*, or *Oblique-angled*, i. e. *Acute-angled*, or *Obtuse-angled*. Secondly, in respect of their sides, they are either an *Equilateral*, *Isoceles*, or *Scalenium Triangle*: But seeing they are all measured after the same manner, I shall give one general Rule, which

The Rule. is this : *Multiply the Length of the Base by the Length of the Perpendicular ; the half of this Product is the Area or Superficial Content.* So if the Content of the Triangle ABC be required: To find which, first from the Angle B let fall the *Perpendicular* DB, on the *Base* AC, (by Prop. 3. § 1.) let therefore the Length of the *Perpendicular* BD be 24, and the *Base* AC 44 parts. Now if the *Base* AC 44 were multiplied by BD 24, the Product is 1056, half whereof is 528, the Content of the Triangle ABC, propounded.

P R O P. IV.

To find the Superficial Content of a Rhombus.

Fig. 22. First let fall a *Perpendicular* from one of the *Obtuse-angles* unto its opposite side, (by Prop. 3. § 1.) and then multiply the Length of the side thereof by the Length of the *Perpendicular*, their Product is the Content thereof.

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So in the *Rhombus* ABCD, the side AC, or BD is 16 *Inches*, and the *Perpendicular* KC is 14 *Inches*, which multiplied into the side 16, produceth 224 *Inches*; which is the *Area*, or Superficial Content of the *Rhombus* ABCD, propounded. § 2.

P R O P. V.

To find the Superficial Content of a *Rhomboides*.

First let fall a *Perpendicular*, as in the former Fig. 23. Proposition, then the Length thereof multiply by the Length of the *Perpendicular*; the Product is the *Area*, or Superficial Content thereof. For in the *Rhomboides* EDAH, whose Length AH, or ED is 32 *Feet*, and the Length of the *Perpendicular* HK is 16 *Feet*, which multiplied together produceth 512 *Feet*, which is the *Area* or Superficial Content of the *Rhomboides* AHED, propounded.

P R O P. VI.

To find the Superficial Content of any *Polygon*, or many equal sided Superficies.

Fig. 94. First, from the Center unto the middle of either of the sides of the *Polygon* let fall a *Perpendicular*, (by 3. Prop. § 1.) then multiply the Length of half the *Periphery* by the *Perpendicular*, the Product shall be the Superficial Content of the *Polygon*. Fig. 24.

Admit the *Polygon* to be an *Hexagon* AAAAAA, whose side AA is 22 *Feet*, and the *Perpendicular* BE 19 *Feet*; now, if 66, half the *Periphery*,

- § 2. *riphery* be multiplied by 19, it produceth 125 Feet; which is the *Content* of the Polygon AA &c. as required.

P R O P. VII.

To find the Area or Superficial Content of an irregular Figure or Polygon.

This and the like Figures are practically measur'd by dividing it into Triangles; which done, *measure each Triangle severally, and add the several Contents together, so will the Sum be the Content of the said Figure.*

Suppose the Dimensions of an irregular Figure are as in Plate 9, what's the Content?

Work according to the preceding Direction, and you shall find 2375 Foot 6 Inches.

P R O P. VIII.

To find the Superficial Content of a Circle.

- Fig. 25. Multiply half the *Circumference* by one half of the *Diameter*, their Product is the *Superficial Content* thereof.

Admit the *Circumference* of a Circle ACBD be 44 Inches, what is the *Area* or *Content* thereof? (by the 9 Prop. § 2.) I find the *Diameter* to be 14 Inches, therefore I say if 22, half the *Circumference*, be multiplied by 7, half the *Diameter*, it shall produce 154 Inches; which is the *Superficial Content* of the Circle ACDB, as required.

P R O P.

PROP. IX.

By the Diameter of a Circle given, to find the Circumference.

Suppose the Diameter be 14, what is the Circumference? The *Analogy* or *Proportion* holds thus, as 7 to 22, so is 14 unto 44, the Circumference required.

PROP. X.

By the Circumference of a Circle given, to find the Diameter.

Suppose the Circumference of a Circle be 44, what is the Diameter? The *Analogy* or *Proportion* is as 22 to 7, so is 44 unto 14, the Diameter required.

But here note, These Proportions of 7 to 22, and the like, are coarse, and fit only for rough Calculations: And therefore where Exactness is required, the following Proportions must be used; which is,

As 113::355, or as 1: 3. 141592, &c. so is the Diameter to the Circumference.

The first of these is *Adrianus Metius* his Proportion, being the nearest in fewest Figures; the second is *Archimedes's* Numbers, and is very convenient, by reason the Labour of Division is saved, the first Term being Unity.

PROP.

§ 2.

P R O P. XI.

By the Content of a Circle given, to find the Circumference.

Suppose the Content of a Circle be 154, what is the Circumference, the Analogy, or Proportion?

As 7 unto 4 times 22, which is 88, so is 154 the Content of the given Circle; to the Square of the Circumference 1936, whose Root being extracted, as is taught (in Prop. 8. § 1. Chap. 1.) gives the Circumference 44, as required.

P R O P. XII.

By the Content of a Circle given, to find the Diameter.

Suppose the Superficial Content of a Circle be 154 parts, what is the Diameter thereof? To find which, this is the Analogy or Proportion.

As 22,

To 4 times 7, which is 28,

So is 154, the given Content,

To the Square of the Diameter 196, whose Root being extracted (by 8 Prop. chap. 1. § 1.) giveth the Diameter 14, as required.

P R O P. XIII.

By the Diameter of a Circle given to find the side of a Square equal thereto.

To find which, this is the Analogy or Proportion.

As

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§ 2.

As 1. 000000,
To 0. 886227,
So is the Diameter of the Circle propounded,
To the side of a Square, whose Superficial Content, is equal unto the Superficial Content of the Circle propounded.

P R O P. XIV.

By the Circumference of a Circle given, to find the side of a Square equal to it.

This is the Analogy, or Proportion.

As 1. 000000,
To 0. 282093,
So is the Circumference of the Circle propounded, to the side of a Square equal to the Circle.

P R O P. XV.

By the Content of a Circle given, to find the side of a Square equal to it.

To do which, extract the Square-Root of the Content propounded, (by Prop. 8. Chap. 1. §. 1.) So is the Root the side of a Geometrical Square, equal thereunto.

P R O P. XVI.

By the Diameter of a Circle given, to find the side of an inscribed Square.

This is the Analogy, or Proportion.

As 1. 000000,
To 0. 707107,

So

§ 2. So is the Diameter of the Circle propounded to the side of the inscribed Square.

P R O P. XVII.

By the Circumference of a Circle given, to find the side of an inscribed Square.

This is the *Analogy, or Proportion.*

As 1. 000000,

To 0. 225079,

So is the Circumference of the Circle propounded, to the side of the inscribed Square.

P R O P. XVIII.

To find the Superficial Content of an Oval, or Elleipsis.

Fig. 26. Let the Oval given be ABCD, and 'tis required to find the Area or Superficial Content thereof: To do which, multiply the Length AB 40 Inches, by the Breadth CD 30 Inches the Product is 1200. Which divide by 1. 27324 the Quotient is $942 \frac{48}{100}$ parts. Which is the Area or Superficial Content of the Oval ABCD propounded.

P R O P. XIX.

To find the Superficial Content of any Section, cut from a Circle, by Lines drawn from the Center.

Multiply half the Circuit of the Section, by the Semidiameter of the whole Circle, and the Product thence arising is the Area or Superficial Content thereof.

Sup

Suppose there be a Circle whose Diameter is § 2.
 14 parts, and the Circuit of the Quadrant ABC Fig. 27.
 is 11 parts, and the Content of the said Qua-
 drant is desired: To find which, multiply $5\frac{1}{2}$
 or 5.5 half the Circuit of the Quadrant by 7 the
 Semidiameter, the Product is $38\frac{1}{2}$, which is the
 Content of the Quadrant ABC propounded.

P R O P. XX.

To measure a Parabola, which is a Figure bound-
 ed by two Lines; one of which is curve, the
 other strait: It is generated by cutting a Cone
 with a plain Parallel to the side thereof.

The Rule for measuring it is, to multiply
 the Line AB by the Line DC, and the Product
 again by 2. This last Product divide by 3, the
 Quote is the Area of the Parabola: As

Suppose the Breadth of a Parabola be 29 Foot,
 and the Length or Height 45 Foot, what's the
 Area? Work according to the preceding Dire-
 ction, and you shall find 870 Foot 21 Inches,
 to be the Area.

S E C T. III.

Of Stereometry: Or the way how to measure § 3.
 any Regular Solid.

STereometry is that part of the Mathematicks,
 springing from Geometry, by which the
 Content of all solid Bodies are discovered by two
 Multiplications or three Dimensions, and is va-
 lued

- § 3. lued by the *Cube* of some famous Measure; as an *Inch-Cube*, a *Foot-Cube*, a *Yard-Cube*, or a *Perch-Cube*, &c.

P R O P. I.

To find the solid Content of a Cube.

Multiply the side into its self, and that Product by its side again; their Product is the solid Content thereof.

- Fig. 28. Suppose there be a *Cube* A, whose side is 2 Feet, and his solid Content is required: I say, if his side 2 be multiplied by its self, it produceth 4; which again multiplied by 2, it produceth 8 Feet, which is the solid Content of the *Cube* propounded.

P R O P. II.

To find the solid Content of a Parallelopipedon.

First get the *Superficial Content* of the End, (by Prop. 1, or 2, § 2.) which multiply into the Length, the Product is the *solid Content*.

- Fig. 29. Suppose there be a *Parallelopipedon* B, whose sides of the *Base* is 40 and 30 *Inches*, and Length 120 *Inches*, and his *solid Content* is demanded: I say if you multiply 30 by 40, the Product is 1200, which is the *superficial Content* at the *Base*; Which multiplied by the Length, 120 *Inches* produceth 144000 *Inches*, which is the *solid Content* of the *Parallelopipedon* B, propounded.

P R O P.

P R O P. III.

To find the solid Content of a Cylinder.

First get the *superficial Content* of the Circle at the *Base*, (by Prop. 8. § 2.) and by that multiply its *Length*, the *Product* is the *solid Content*.

Suppose there be a *Cylinder*, as D, whose Fig. 30. *Diameter* of the Circle at the *Base* is 7 parts, and the *Length* of the *Cylinder* is 14 parts, and 'tis required to find the *solid Content* thereof: First I find the *superficial Content* of the *Base* to be 38.5, which multiplied into 14 the *Length*, giveth 539 parts, which is the *solid Content* of the *Cylinder* propounded.

P R O P. IV.

To find the solid Content of a Pyramid.

First get the *superficial Content* of the *Base* of the *Pyramid*, (by some of the foregoing Propositions in *Planometria*) and then multiply that into $\frac{1}{3}$ of his *Altitude*, the *Product* is the *solid Content* thereof.

Suppose there be a *Pyramid* H, whose side of Fig. 31. the *Base* is $4\frac{1}{2}$ parts, or $4\frac{5}{10}$, and his *Altitude* 12 parts, and his *solid Content* is required: First I find (by Prop. 1. § 2.) the *superficial Content* of the *Base* to be $20\frac{25}{100}$ or $20\frac{1}{4}$; which multiplied by 4, (which is $\frac{1}{3}$ of the *Altitude* 12) produceth 81 parts, for the *solid Content* of the *Pyramid* propounded.

§ 3.

P R O P. V.

To find the solid Content of a Cone.

First find the *superficial Content* of the Circle at the *Base*, (by Prop. 7. § 2.) then multiply it by $\frac{1}{3}$ of its *Altitude* or *Heighth*, the *Product* is the *solid Content* thereof.

Fig. 32. Suppose there be a *Cone* as B, whose *Diameter* of the *Base* is 7, and his *Altitude* or *Heighth* is 15 parts, and his *solid Content* is required: First, I find the *superficial Content* of the *Base* to be $38\frac{1}{2}$ or 38.5; which multiplied into 5, ($\frac{1}{3}$ of its *Altitude* or *Heighth*) produceth 192.5, or $\frac{1}{2}$, which is the *solid Content* of the *Cone* propounded.

P R O P. VI.

By the Diameter of a Globe to find his solid Content.

This is the *Analogy*, or *Proportion*.

As 6 times 7, which is 42,

Is to 22;

So is the *Cube* of the *Diameter* of the *Sphere*, or *Globe* propounded,

To the *solid Content* thereof.

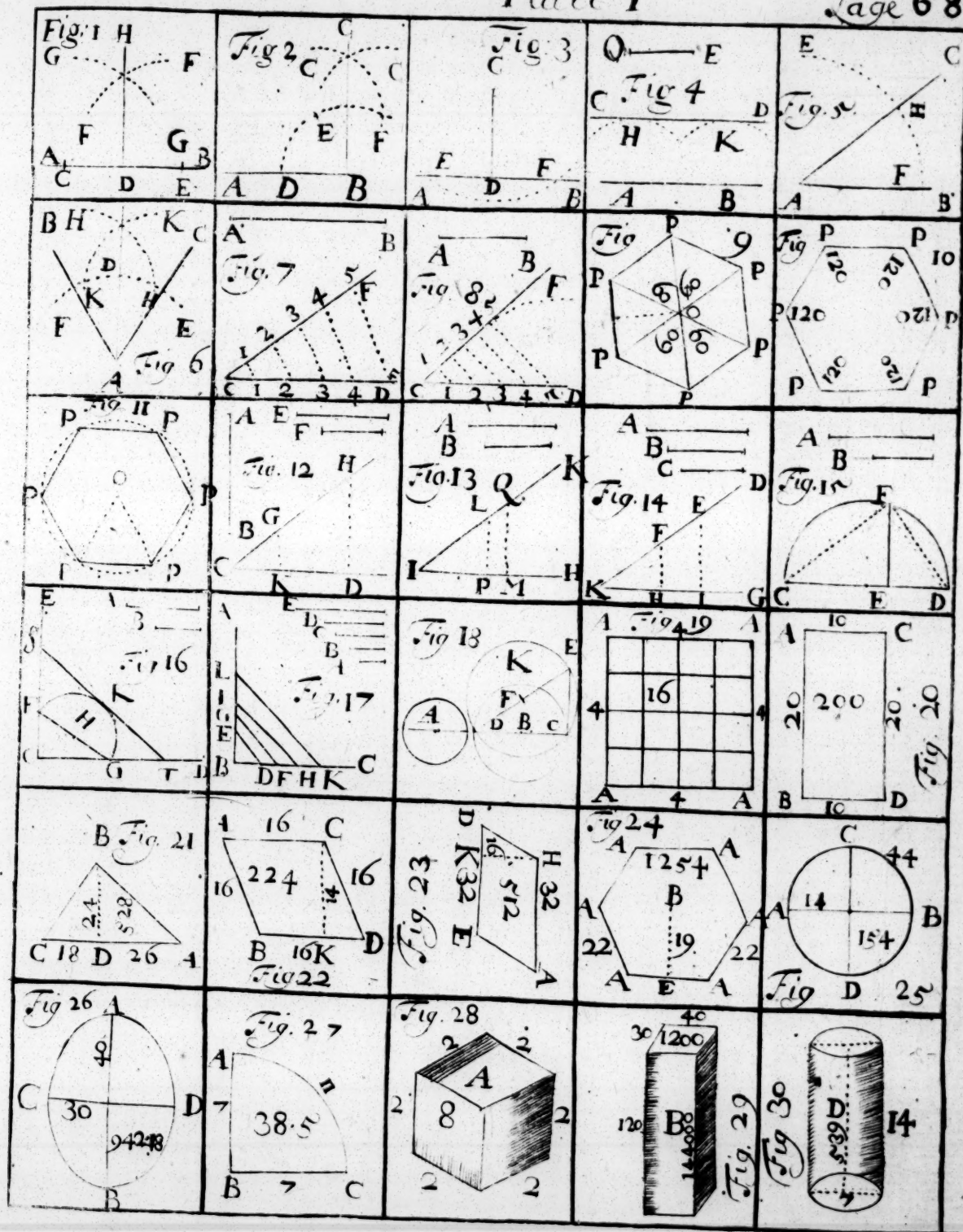
Suppose there be a *Sphere* or *Globe*, whose *Diameter* is 12 Inches, what is the *solid Content* thereof? Say, (see the *Globe R.*)

As 42,

Fig. 33. Is to 22,

So is 1728, the *Cube* of the *Diameter*,

To



Geometrical Propositions.

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To the solid Content $905 \frac{6}{7}$, or $\frac{1}{7}$ of the Globe, § 3.
or Sphere propounded: This,

and all other such like Propositions, are performed by the help of the first Proposition of the first Chapter of this Book.

 Note, That every Sphere is equal unto two Cones, whose Height and Diameter of the Base is the same with the Axis of the Sphere: And a Sphere is two thirds of a Cylinder, whose Height and Diameter of the Base is the

same with the Axis of the Sphere, according to the 9th Manifestation of the first Book of Archimedes of the Sphere and Cylinder; that is, the Cylinder having the Sphere inscribed, and the Cone inscribed are as 3. 2. 1.

P R O P. VII.

By the Circumference of a Sphere, or Globe, to find his solid Content.

This is the Analogy, or Proportion.

As 1. 000000,

To 0. 016887,

So is the Cube of the Circumference of the Globe, or Sphere propounded,
To the solid Content thereof.

P R O P. VIII.

By the Axis of a Globe, to make a Cube equal thereunto.

This is the Analogy, or Proportion.

As 1. 00000,

To 0. 80604,

So is the Axis of the Sphere propounded,
To the Cube-Root, which shall be equal to it.

Geometrical Propositions.

P R O P. IX.

By the Circumference of a Globe, to make a Cube equal thereunto.

This is the *Analogy*, or *Proportion*.

As 1. 000000,

To 0. 256556,

So is the *Circumference of the Globe propounded*,

To the *Cube-Root*, which shall be equal to the *Sphere or Globe propounded*.

P R O P. X.

By the solid Content of a Sphere or Globe, to make a Cube equal thereunto.

Extract the *Cube-Root* of the *solid Content* of the *Sphere or Globe*, (by Prop. 9. § 1. Chap. 1.) so shall the Root, so found, be the side of a *Cube* equal unto the *Globe or Sphere propounded*.

P R O P. XI.

A Segment of a Sphere being given to find the solid Content thereof.

To find which, first say, As the *Altitude* of the other *Segment* is to the *Altitude* of the *Segment* given, so is that *Altitude* of the other *Segment* increased by half the *Axis* unto a fourth. Then say, As 1, to 1, 0472, so is the *Product* of the *Quadrant* of half the *Chord* of the *Circumference* of that *Segment*, multiplied by that fourth, to the *solid Content* of the *Segment propounded*.

CHAP. V.

Of TRIGONOMETRY: Or the Doctrine of Triangles.

SECT. I.

*Some General Maxims belonging to Plain, or § 1.
Right-lined Triangles.*

TRIGONOMETRY is necessary in most parts of the *Mathematicks*, and herein indeed consisteth the most frequent Use of the *Logarithms*, *Sines*, *Tangents*, and *Secants*: It is conversant in the measuring of *Triangles*, *Plain* or *Spherical*; comparing their *Sides* and *Angles* together, according unto their known *Analogies*, or *Proportions*: So that any three parts of a *Triangle* being given, the three *Angles* excepted, the other parts may be found out and known. Now in the *Doctrine* of *Right-lined Triangles*, it will be necessary to know these *Maxims* following.

1. That a *Right-lined Triangle* is a Figure constituted by the Conjunction, or Intersection, of the three Right or Streight-lines in their Angles or Meeting-places: So that every *Triangle* hath six distinct Parts, viz. *Three Sides*, and *Three Angles*.

F 4

2. That

§ 1. 2. That all *Right-lined Triangles* are either
 Fig. 34. *Right-angled*, that is, which hath one *Right-Angle*, as ABC Fig. 34. Or *Oblique-angled*,
 whole three *Angles* are all *Acute*; that is, less
 than a *Quadrant*, or 90 Deg. or else they have
 One *Angle Obtuse*, or greater than a *Quadrant*:
 So all *Triangles* that have not one *Right-Angle*,
 Fig. 36. are called *Oblique-Triangles*; as Fig. 36. to wit,
 the *Triangle ABC*.

3. That the three *Angles* of any *Right-lined Triangle*, are equal unto two *Right-Angles*; or
 180 *Degrees*: So that any two of their *Angles*
 being known, the third *Angle* is also found,
 being the *Complement* of the other two, unto
 180 *Degrees*; but this is more readily found in
 a *Rectangled Triangle*; for the *Rectangle* being
 a *Quadrant*, or 90 *Degrees*, one of the acute
Angles therefore being given, the other is rea-
 dily known, being the *Complement* thereof unto
 90 *Degrees*.

Fig. 35. 4. That the three sides, comprehending the
Triangle, some call *Legs*, others *Sides*; but in
Rectangled Triangles, as in the *Triangle ABC*,
 I call AB the *Base*, BC the *Cathetus*, or *Perpen-*
dicular, and AC the *Hypothenuse*.

5. That the *Sines* of the *Angles* are propor-
 tional unto their opposite *Sides*, and their *Sides*
 to their opposite *Angles*: So that if the *Side* of a
Triangle were desired, put the *Sine* of the op-
 posite *Angle* in the first place. Also if an *An-*
gle be required, put the *Logarithm* of his oppo-
 site side in the first place.

6. That the *Angles* of any plain *Triangle* are
 measured by a *Scale* of *Chords*; but the *Sides*
 of any plain *Triangle* are measured by any
 Scale

Plain Trigonometry.

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Scale of equal Parts; as Inches, Feet, Yards, Poles, Miles, Leagues, &c. § 1.

7. That if an *Angle* propounded be greater than 90 Deg. and so not to be found in the Tables, take the *Complement* thereof unto 180 Deg. and work by the *Sine* or *Tangent* thereof, and the Work will be the same.

And here, for the more short and speedy Performance of these Conclusions in *Trigonometry*, I have annexed and used these following *Symbols*, which I would have you take notice of.

=	Equal, or Equal to.	X	Difference.
+	More.	S	Sine.
-	Less.	Sc	Co-sine.
x	Multiplied by.	T	Tangent.
°	Degrees, as 15°.	Tc	Co-tangent.
'	Minutes, as 40'.	Se	Secant.
cr.	A Side.	Sec	Co-secant.
cr.	Sides.	Co. Ar. Comp.	Arithmetic.
V	An Angle.	R	A Right-angle.
VV	Angles.	2 R	Two Right-angles.
Z	Sum.	Q	Square.

S E C T.

S E C T. II.

§ 2.

Of Plain Rectangled Triangles.

P R O P. I.

Two Angles and the Base of a Rectangled Triangle given, to find the other parts.

Fig. 34.

Admit the Triangle given be ABC : Now the Angle at B is an Angle of 90° , or a Right-angle; and the Angle at C is $57^\circ 35'$, and the Base AB is 736 parts.

Now first I find the Angle at A to be $32^\circ 25'$, it being the Complement of the Angle at C unto 90° : Secondly, to find the Cathetus or Perpendicular; this is the Analogy, or Proportion.

As $S, V, C, 57^\circ 35'$ ————— 9. 92643

To Log. AB . Base 736 parts ————— 2. 86687

So is $S. V. A. 32^\circ 25'$ ————— 9. 72922

————— 12. 59609

To Log. of the Cathetus, or Perpendicular BC } ————— 2. 66966
467 $\frac{4}{10}$ parts.

Observe
this for a
general
Rule in
Trigonome-
try.

Add the Log. of the third and second Terms together, and from their Sum deduct the Log. of the first Number, so is the Remainder the Log. of the fourth Term, or Number sought, as you see in the foregoing Example.

Thirdly, to find the Hypotenuse AC , the Analogy or Proportion holds thus.

As $S. V, C 57^\circ 35'$,

To

Plain Trigonometry.

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To Log. Base AB 736 parts;

So Radius or S. 90° ,

§ 2.
Fig. 34.

To Log. Hypotenuse AC $871\frac{8}{10}$ parts required:

Thus are the three required parts of the given Triangle ABC found, viz. the Angle A to be $32^\circ 25'$, the Cathetus BC to be $467\frac{4}{10}$ parts, and the Hypotenuse AC to be $871\frac{8}{10}$ parts, as was to required to be found.

P R O P. II.

The Hypotenuse, Base, and one of the Angles of a Rectangled Triangle given, to find the other parts thereof.

In the Triangle ABC the Hypotenuse AC is $871\frac{8}{10}$ parts, the Base AB is 736 parts, and the Angle at B is known to be a Right-angle, or 90° : First, to find the Angle at the Cathetus C, the Analogy or Proportion holds thus.

As Log. Hypothen. AC $871\frac{8}{10}$ parts

Fig. 34.

To Radius or S. 90° .

So Log. Base AB 736 parts,

To the S. V. at Cathetus C $57^\circ 35'$.

Secondly, Now having found the Angle at the Cathetus C, to be $57^\circ 35'$, I say the Angle of the Base A is $32^\circ 25'$, being the Compl. of the Angle C unto 90° .

Thirdly, to find the Cathetus BC, this is the Analogy or Proportion.

As Radius or S. 90° ,

To Log. Hypothen. AC $871\frac{8}{10}$ parts;

So S. V. at Base A $32^\circ 25'$.

Fig. 34.

To Log. Cathetus BC, $467\frac{4}{10}$ parts required:

It may also be found, as in the former Proposition.

P R O P.

Plain Trigonometry.

P R O P. III.

In a Rectangled Triangle, the Base, and Cathetus given to find the other parts thereof.

In the Triangle ABC the Base AB is 736 parts, and the Cathetus BC is $467\frac{4}{10}$ parts, and the Angle B between them is a Right Angle, or 90° ; and here you may make either side of the Triangle, Radius: But I shall make BC the Cathetus Radius, and then to find the Angle at the Cathetus C, this is the Analogy or Proportion.

Fig. 35.

As Log. Cathet. BC. $467\frac{4}{10}$ parts,
To Radius or S. 90° .

So Log. Base AB 736 parts;

To T.V. Cathet. C $57^\circ 35'$, as required.

Secondly, I find the other Angle at A to be $32^\circ 25'$, it being the Complement, to C $57^\circ 35'$, unto 90° .

Thirdly, To find out the Hypothenuse AC, this is the Analogy or Proportion.

As S.V. Cathet. C $57^\circ 35'$,

To Log. Base AB 736 parts;

So Radius or S. 90° ,

To Log. Hypothenuse AC $871\frac{8}{10}$ parts, required: But making the Base AB Radius, you may find the Hypothenuse AC by this Analogy or Proportion.

As Radius or S. 90° ,

To Log. Base AB 736 parts;

So Sc.V. Base A $32^\circ 25'$,

Fig. 35.

To Log. Hypothenuse AE $871\frac{8}{10}$ parts required; and thus you have all the parts of the Triangle propounded.

P R O P.

P R O P. IV.

The Base and Hypothenuſe, with the Angle between them given, to find the other parts of a Rect-angled Triangle.

In the Triangle ABC the *Base* AB is 736 parts, and the *Hypothenuſe* AC is $871\frac{8}{10}$ parts, and the Angle A included between them is $32^{\circ} 25'$. *First*, To find the Angles: And first remember, that the Angle B is a right Angle, or 90° . *Secondly*, That the Angle at C is the Complement to the Angle at A $32^{\circ} 25'$ unto 90° ; Fig. 35. and therefore is $57^{\circ} 35'$. Now theſe being known, you may find the *Cathetus* by this *Analogy* or *Proportion*.

As S. V. Cathet. C $57^{\circ} 35'$,

To Log. Base AB 736 parts;

So S. V. Base A $32^{\circ} 25'$,

To Log. Cathet. BC $467\frac{4}{10}$ parts required.

Thus I have ſufficiently explained all the *Cases* of Plain Rect-angled Triangles; for to theſe Rules they may be all reduced.

SECT.

S E C T. III.

§ 3.

Of Oblique-Angled Plain Triangles.

P R O P. I.

Two Angles, and a Side opposite, in an Oblique-Angled Triangled given, to find the other parts thereof.

IN the Triangle ABC the Angle at A is 50° , and at C is 37° , and the side AB is 30 parts, and opposite to the Angle C.

First, To find the Angle B, remember that (as 'tis said, in the third *Maxim* aforegoing) 'tis the Complement to the Angles A 50° , and C 37° to 180° , and therefore is the Angle at B 93° .

Fig. 36.

Secondly, Having thus found the Angles, the two unknown sides may be found by the Proportion they bear to their opposite Angles; for that Proportion holds also in these. Thus to find the side BC, this is the *Analogy* or *Proportion*.

As S. V. C $37^\circ 00'$,
To Log. side AB 30 parts;
So S. V. A. $50^\circ 00'$,
To Log. side BC $38 \frac{12}{100}$ parts required to be found.

But it may be more readily found, and performed in such Case as this, where you have a *Sine* or *Tangent* in the first place, by the *Arithmetical Complement* thereof, and so save the *Subtraction*.

Now

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Now the readiest way to find the *Arithmetical Complement* is that of Mr. Norwood, in his *Doctrine of Triangles*;

§ 3.
 The Rule to find the Complement Arithmetical of any Logarithm Number.

which is thus: Begin with the first Figure towards the left hand of any Number, and write down the Complement, or the Remainder thereof, ——— 9. 962398
 unto 9: And so do ——— 0. 037602
 with all the rest of the

Figures, as you see here done; saying 9, wants of 9, 0: And again 9, wants 0; 6, wants 3; 2, wants 7; 3, wants 6; 9, wants 0: Only when you come to the last Figure to the right hand, take it out of 10; so 8 wants 2 of 10. Thus you may readily find the *Co-Ar.* of any Sine, almost as soon as the Sine it self.

But if you want the Complement Arithmetical of any Tangent, you may take the *Co-tang.* which is exactly the *Co-Arith.* of the double Radius; so that the Tangent and *Co-tangent* of an Arch makes exactly 20. 000000.

Now if the Radius be in the first place, then there is no need of taking the *Co-Arith.* of the first Number, only you must cut off the first 1, to the left hand thus 1, and you will have the Logarithm of the Number desired.

Thirdly, Now to find the Side AC by the opposite Angle B, which is $93^{\circ} 00'$: (And seeing the Angle B exceeds 90° , you must work by the Complement to 180°) as in the seventh Work in page 61 is taught.

As

Fig. 36.

As S. V. C. $37^{\circ} 00'$.	—————	0. 22054
To Log. cr. AB 30 parts	—————	1. 47712
So is Sc. V. B. $87^{\circ} 00'$,	—————	9. 99940
To Log. cr. AC $49 \frac{78}{100}$ parts.	—————	1. 69706

Thus having found all the parts of the Triangle propounded, viz. The Angle B to be $93^{\circ} 00'$, the side AC to be $49 \frac{78}{100}$ parts, and the side BC to be $38 \frac{19}{100}$ parts, as was required to be found.

NOTE, 'Twill be doubtful sometime, whether the V fought be acute or obtuse; tho' in the Operation there will always come out the Sine of an acute Angle, as in the immediate preceding Operation, the *Log-sine* that came out was but 87 Deg. where the Angle C was really 93 Deg. Now when it happens that the Angle is within a Degree or less of 90 Deg. the Eye cannot distinguish whether 'tis more or less than a right Angle: And therefore, in such Case, to know which it is, take this general Rule. See if the Square of the Side subtending the doubtful Angle exceed the Sum of the other two Squares, then 'tis obtuse: If they be equal, 'tis right; if the Square of the said Side be less than the Sum of the Squares of the other two Sides, the Angle is acute.

P R O P. II.

Two sides, and an Angle opposite to one of them in an Oblique-angled Triangle given, to find the other parts thereof.

In the Triangle ABC the side AB is 30 parts, and the side AC is $49\frac{78}{100}$ parts, and the opposite Angle C, is $37^{\circ} 00'$.

First, To find the Angle at B, this is the A- Fig. 36. analogy or Proportion.

As Log. cr. AB 30 parts,

To S. V. at C $37^{\circ} 00'$;

So Log. cr. AC $49\frac{78}{100}$ parts;

To Sc. V. B $93^{\circ} 00'$, as was required to be found.

Now seeing that the Angle C is $37^{\circ} 00'$, and the Angle B is $93^{\circ} 00'$, which makes $130^{\circ} 00'$, therefore must the Angle A be $50^{\circ} 00'$; the Complement to 180° : So having found all the three Angles, you may find the other side CB $38\frac{12}{100}$ parts, as afore in the first Proposition, by his opposite Angle.

P R O P. III.

Two Sides of an Oblique-angled Triangle, with the Angle included between them given, to find the other parts thereof.

In the Triangle ABC, the side AC is $49\frac{78}{100}$ parts, the side DB is 30 parts, and the Angle A between them is $50^{\circ} 00'$; and 'tis required to find the other parts of the Triangle propounded. To resolve this Conclusion, let fall

G

a Per-

§ 3. a Perpendicular DB, from the Angle B, on the side AC; (by Prop. 3. § 1. Chap. 4.) and then proceed thus:

Fig. 37. First, Seeing the *Oblique-angled Triangle* ABC is divided into two *Rectangled Triangles*, viz. ADB and BDC. Now I will begin with the Triangle ADB, in which is given the Angle A $50^{\circ} 00'$, and the Angle D is a right Angle, or 90° , and the side AB 30 parts, and the sides AD and DB, and the Angle at B, are required.

First to find the Angle at B, remember that it is the *Complement* unto the Angle A $50^{\circ} 00'$, unto $90^{\circ} 00'$, and therefore must the Angle B be $40^{\circ} 00'$: Now for to find the *Cathetus* BD, (as in Prop. 1. and 2. § 2. Chap. 5.) by the Rule of Opposition, the *Analogy* or *Proportion* holds thus:

As Radius or S. 90° ,
To Log. Hypoth. AB 30 parts;
So S. V. at A $50^{\circ} 00'$,
To Log. Cath. BD $22\frac{98}{100}$ parts sought.

And again say,

As Radius or S. 90° ,
To Log. Hypoth. AB 30 parts;
So S. V. at B $40^{\circ} 00'$,
To Log. Base AD $19\frac{28}{100}$ parts sought.

Thus in the Triangle ADB, you have found the Angle B to be $40^{\circ} 00'$, the *Cathetus* BD to be $22\frac{98}{100}$ parts; and the *Base* AD to be $19\frac{28}{100}$ parts, as was required.

Now for the other Triangle, which is BDC, in which there is given the side BD, $22\frac{98}{100}$ parts, and

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and the Angle at D is a *Right-angle*, or 90° , § 3.
and the sides DC and CB, and the Angles B
and C, are required.

First to find the side DC subtract AD $19\frac{28}{100}$ Fig. 37.
parts, out of AC $49\frac{78}{100}$ parts; there remains

the *Base* DC, $30\frac{50}{100}$ parts: Thus have you the
two sides of the Triangle, to wit, the *Base* DC,
 $30\frac{50}{100}$ parts, and the *Perpendicular* BD $22\frac{98}{100}$ parts,
and the Angle D between them is a *Right-angle*
or 90° . Now you may find the Angle at B by
the *Tangent* (as in Prop. 3. § 2. Chap. 5.) thus

As Log. *Cath.* BD $22\frac{98}{100}$ parts,

To *Radius* or S. 90° ;

So Log. *Base* CD $30\frac{50}{100}$ parts,

To T. V. B. $53^\circ 00'$.

Secondly, For the Angle C, remember 'tis the
Complement of the Angle B, 53° to 90° ; and
therefore is the Angle C $37^\circ 00'$, required.

Thirdly, To find the *Hypoth.* BC, this is the
Analogy or *Proportion*.

As S. V. B. $53^\circ 00'$,

To Log. *Base* DC $30\frac{50}{100}$ parts;

So *Radius* or S. 90° ,

To Log. *Hypoth.* BC $38\frac{19}{100}$ parts: Thus have
you found all the required parts of the Trian-
gle ABC propounded, viz. the Angle C to be

$37^\circ 00'$, the Angle B to be

$53^\circ 00'$, * and the *Side* BC

$38\frac{19}{100}$ parts, as required to be
found.

* That is equal unto the two
Angles B 40° and B 53° as afore
in the former Proposition.

§ 3.

Another way to perform the same.

Fig. 37.	Side.	AB	$30\frac{00}{100}$	Take the Sum of the two sides, and the difference of the two sides; and work as followeth. Now to find the two Angles B and C, this is the Manner, and by this Analogy or Proportion,
	Side.	AC	$49\frac{78}{100}$	
	Sum.		$79\frac{78}{100}$	
	Difference.		$19\frac{78}{100}$	

they are found out and known.

As Log. Z. cr^s. AB and CA, $79\frac{78}{100}$ parts,To Log. X. cr^s. AB and CA, $19\frac{78}{100}$ parts;So T. of $\frac{1}{2}$ VV unknown, $65^{\circ} 00'$,To T. $\frac{1}{2}$ X. of VV, $28^{\circ} 00'$.

This Difference of Angles $28^{\circ} 00'$, add unto $65^{\circ} 00'$, (half the difference of the unknown Angles) and it shall produce $93^{\circ} 00'$, which is the greater Angle, and subtracted from it, leaves $37^{\circ} 00'$, which is the lesser Angle C, so have you the required Angles.

P R O P. IV.

The three sides of an Oblique-angled Triangle given, to find the Angles.

In the Triangle ABC, the side AC is $49\frac{78}{100}$ parts, the side AB is 30 parts, and the side BC is $38\frac{19}{100}$ parts; and the three Angles of the Triangle are required.

Fig. 37.	Side.	AB	$30\frac{00}{100}$	The Resolution of this Conclusion is thus: take the Summ and Differ. of the two sides AB and BC; and then work as follows: To find a Segment of the Base AC to wit, CE; say,
	Side.	BC	$38\frac{19}{100}$	
	Sum.		$68\frac{19}{100}$	
	Diff.		$08\frac{19}{100}$	

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§ 3.

As Log. Base AC $49\frac{78}{100}$ parts,
To Z. cr. AB and BC $68\frac{12}{100}$ parts;
So X. cr. AB and BC $8\frac{12}{100}$ parts,
To Log. of a Segment of the Base AC, to wit,
CE $11\frac{22}{100}$ parts.

This Segment of the Base CE $11\frac{22}{100}$ parts, being substracted from the whole Base AC $49\frac{78}{100}$ parts, the remainder is EA $38\frac{56}{100}$ parts, in the middle of which as at D, the Perpendicular DB Fig. 37. will fall from the Angle B; and so divide it into two Rectangled Triangles, to wit, ADB and CDB, whose Base DA is $19\frac{28}{100}$ parts, which taken from AC $49\frac{78}{100}$ parts, leaves the Base of the greater Triangle CD $30\frac{50}{100}$ parts.

Now having the two Bases of these two Triangles, and their Hypothenuses; to wit, CD $30\frac{50}{100}$ parts, DA $19\frac{28}{100}$ parts, CB $38\frac{12}{100}$ parts, and BA 30 parts, you may find all their Angles by the Rule of opposite sides, to their Angles as afore.

I. In the Triangle CDB.

To find the Angles, this is the *Analogy* or *Proportion*.

As Log. BC $38\frac{12}{100}$ parts,
To Radius or S. 90° ,
So Log. DC $30\frac{50}{100}$ parts,
To S. V. B. $53^\circ 00'$: whose Complement is the angle at C $37^\circ 00'$ unto 90 : or a Quadrant.

II. In the Triangle ADB.

To find the Angles, this is the *Analogy* or Fig. 37. *Proportion*.

G 3

As

§ 3.

As Log. AB 30 parts,

To Radius or S. 90° :So Log. AD $19^{\circ} 28'$ parts,To S. V. B $40^{\circ} 00'$.

The Complement whereof, unto $90^{\circ} 00'$, is the Angle at A $50^{\circ} 00'$.

Now in the first Triangle CDB, there is found the Angle C to be $37^{\circ} 00'$, and the Angle B to be $53^{\circ} 00'$.

In the second Triangle ADB, there is found the Angle A, to be $50^{\circ} 00'$, and the Angle D to be $40^{\circ} 00'$.

Now the two Angles at B, to wit, $53^{\circ} 00'$ and $40^{\circ} 00'$ makes $93^{\circ} 00'$, which is the Angle of the *Oblique-angled Triangle* ABC at B: Thus the three Angles of the said given Triangle ABC are found as was required, viz. the Angle A to be $50^{\circ} 00'$, the Angle B to be $93^{\circ} 00'$, and the Angle C to be $37^{\circ} 00'$, as sought.

Thus I have sufficiently, fully and plainly explained all the Cases of Plain, Right-lined Triangles, both Right and Oblique-angled: I shall now fall in hand with Spherical Triangles, both Right and Oblique-angled.

S E C T. IV.

Of Spherical Rectangled Triangles.

§ 4.

And here first it will be necessary also to understand those few general Maxims or Rules, that are of special Moment, in the Doctrine of Spherical Triangles.

1. **T**hat a Spherical Triangle is comprehended and formed by the Conjunction and Intersection of three *Arches* of a Circle, described on the *Surface* of the *Sphere* or *Globe*.

2. That those Spherical Triangles consist of six distinct parts, viz. three Sides and three Angles, any of which being known, the other is also found out and known.

3. That the three Sides of a Spherical Triangle, are Parts or *Arches* of three great Circles of a *Sphere*, mutually intersecting each other: And as plain or Right-lined Triangles are measured by a Measure, or *Scale* of equal parts: So these are measured by a *Scale* or *Arch* of equal Degrees.

4. That a *Great Circle* is such a Circle that doth bisect the *Sphere*, dividing it into two equal parts; as the *Equinoctial*, the *Ecliptick*, the *Meridians*, the *Horizon*, &c.

5. That in a Right-angled Spherical Triangle, the Side subtending the Right-angle we call the *Hypothenuse*, the other two containing the Right-angle we may simply call the *Sides*, and for Distinction either of them may be called the *Base* or *Perpendicular*.

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6. That

§ 4.

6. That the Summ of the Sides of a *Spherical Triangle* are less than two *Semicircles* or 360° .

7. That if two Sides of a *Spherical Triangle* be equal to a *Semicircle*; then the two Angles at the *Base* shall be equal to two *Right-angles*; but if they be less, then the two Angles shall be less; but if greater, then shall the two Angles be greater than a *Semicircle*.

8. That the Summ of the Angles of a *Spherical Triangle* is greater than two *Right-angles*, and less than six.

9. That every *Spherical Triangle* is either a *Right*, or *Oblique-angled Triangle*.

10. That the *Sines* of the Angles are in Proportion unto the *Sines* of their opposite Sides; and the *Sines* of their opposite Sides are in Proportion unto the *Sines* of their opposite Angles.

11. That in a *Right-angled Spherical Triangle*, either of the *Oblique-angles* is greater than the *Complement* of the other, but less than the *Difference* of the same *Complement* unto a *Semicircle*.

12. That a *Perpendicular* is part of the *Arc* of a *Great Circle*, which being let fall from any Angle of a *Spherical Triangle*, cutteth the opposite Side of the *Triangle* at *Right-angles*, and so divideth the *Triangle* into two *Right-angled Triangles*, and these two parts (either of the Sides or Angles) so divided must be sometimes added together, and sometimes substracted from each other, according as the *Perpendicular* falls within or without the *Triangle*.

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§ 4

PROP. I. Case 1.

A Side and an Angle adjacent thereunto being given, to find the other Side.

In the Triangle ABC, there is given the Side AB $27^{\circ} 54'$; and the Angle A $23^{\circ} 30'$, and the Side BC is required; to find which this is the *Analogy or Proportion.*

As the Radius or S. $90^{\circ} 00'$, ———— 10. 00000

To S. of cr. AB $27, 54$ ———— 9. 67018

So $\propto$ T. V. A $23, 30$ ———— 9. 63830 *Fig. 38.*

To cr. BC $11, 30$ ———— 9. 30848

NOTE, If in the right angled Spheric Triangle Figure 38 you allow the Base AB to be part of the Equinoctial, AC part of the Ecliptick, and CB part of a Meridian passing through the Sun's place, then doth the preceding Case find (CB) which is the Sun's present Declination, by having his greatest Declination and right Ascension given.

PROP. II. Case 2.

A Side and an Angle adjacent thereunto being given, to find the other Oblique-angle.

In the Triangle ABC, there is given the Side AB $27^{\circ} 54'$, and the Angle A $23^{\circ} 30'$, and the Angle at C is required; to find which, say by this *Analogy or Proportion,* *Fig. 38.*

As

§ 4.

*As the Radius or S. $90^{\circ} 00'$,**To Sc. of cr. AB $27, 54$.**So is S. V. at A $23, 30$,**To Sc. V. at C $69, 22$ required.*

This Proposition finds the V of the Sun's Position, that is the V the Meridian makes with the Ecliptick, from the forementioned Data.

P R O P. III. Case 3.

A Side and an Angle adjacent thereunto being given, to find the Hypotenuse:

Fig. 38. In the Triangle ABC, there is given the Side AB $27^{\circ} 54'$, and the Angle at A $23^{\circ} 30'$, and the Hypotenuse AC is required, which may be found by this Analogy or Proportion.

*As the Radius or S. $90^{\circ} 00'$,**To Sc. of V. at A $23, 30$;**So is Tc. cr. AB $27, 54$.**To Tc. Hypotenuse AC $30, 00$ required.*

NOTE, This Case finds the Sun's place in the Ecliptick from the same Data.

P R O P. IV. Case 4.

A Side and an Angle opposite thereunto being given, to find the other Oblique-angle.

Fig. 38. In the Triangle ABC, there is given the Side BC $11^{\circ} 30'$, and the Angle A $23^{\circ} 30'$, and the Angle C is required; to find which, say by this Analogy or Proportion,

*As Sc. cr. BC $11^{\circ} 30'$,**To Radius or S. $90, 00$;**So is Sc. V. at A $23, 30$,**To S. V. at C $69, 22$, as required.*

This

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This Case finds the Angle of the Sun's Position, by having the greatest and present Declination given.

§ 4.

P R O P. V. Case 5.

A Side and the opposite Angle given, to find the Hypotenuse.

In the Triangle ABC, there is given the Side Fig. 38. BC $11^{\circ} 30'$, and the Angle at A $23^{\circ} 30'$, and the Hypotenuse AC is required, which may be found by this *Analogy* or *Proportion*.

As S. V. at A $23^{\circ} 30'$,

To Radius or S. $90, 00$;

So is Ser. BC $11, 30$,

To S. Hypotenuse AC $30, 00$, as required.

This Case from the same things given, as in the last finds the Sun's place.

P R O P. VI. Case 6.

A side and the opposite Angle given, to find the other side.

In the Triangle ABC, there is given the Side BC $11^{\circ} 30'$, and the Angle at A $23^{\circ} 30'$, and the side AB is required; to find which, this is the *Analogy* or *Proportion*.

As Radius or S. $90^{\circ} 00'$,

To Tc. of V. at A $23, 30$;

So is T. cr. BC $11, 30$,

To S. of cr. AB $27, 54$, as was required.

This Case from the last *Data* finds the Sun's right Ascension.

Fig. 38.

P R O P.

P R O P. VII. Case 7.

The Hypothenuse, and an Oblique-angle given, to find the Side adjacent thereunto.

In the Triangle ABC, there is given the *Hypothenuse* AC $30^{\circ} 00'$, and the Angle A $23^{\circ} 30'$, and the Side AB is required, which is found by this *Analogy* or *Proportion*.

Fig. 38.

As the Radius or S. $90^{\circ} 00'$,

To Sc. V. at A $23, 30$;

So is T. *Hypoth.* AC $30, 00$,

To T. cr. AB $27, 54$, as was required.

This Case supposes the Sun's place and greatest Declination given, to find his right Ascension.

P R O P. VIII. Case 8.

The Hypothenuse, and an Oblique-angle given, to find the opposite Side.

In the Triangle ABC, there is given the *Hypothenuse* AC $30^{\circ} 00'$, and the Angle at A $23^{\circ} 30'$, and the Side BC is required, which is found by this *Analogy* or *Proportion*.

Fig. 38.

As the Radius or S. $90^{\circ} 00'$,

To S. *Hypoth.* AC $30, 00$;

So is S. V at A $23, 30$,

To the S. cr. BC $11, 30$, which was required.

Having the same things given as in the last, to find the Sun's present Declination.

P R O P.

Spherical Trigonometry.

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§ 4.

PROP. IX. Case 9.

The Hypothenuſe, and an Oblique-angle given, to find the other Oblique-angle.

In the Triangle ABC, there is given the Hypothenuſe AC $30^{\circ} 00'$, and the Angle A $23^{\circ} 30'$, now the Angle at C is required, which may be found by this *Analogy* or *Proportion*.

As the Radius or S. $90^{\circ} 00'$,

To Sc. Hypoth. AC $30, 00$;

So is T. of V. at A $23, 30$,

To Tc. of V. at C $69, 22$, as was required.

Fig. 38.

From the laſt *Data* is found the Angle of the Sun's Poſition.

PROP. X. Case 10.

The Sides given, to find the Hypothenuſe.

In the Triangle ABC, there is given the Side AB $27^{\circ} 54'$, and the Side EC $11^{\circ} 30'$, and the Hypothenuſe AC is required; to find which, ſay by this *Analogy* or *Proportion*.

As the Radius or S. $90^{\circ} 00'$,

To Sc. cr. EC $11, 30$;

So is Sc. cr. AB $27, 54$,

To Sc. Hypothenuſe AC $30, 00$, required.

Fig. 38.

Having the Sun's preſent Declination and right Aſcenſion given, by this Caſe you find the Sun's Place; and by the following Caſe, having the ſame *Data*, is found the Sun's greateſt Declination.

PROP.

Spherical Trigonometry.

P R O P. XI. Case II.

The Sides given, to find an Angle.

In the Triangle ABC, there is given the Side AB $27^{\circ} 54'$, and the Side BC $11^{\circ} 30'$, and the Angle at A is required, which may be found by this *Analogy* or *Proportion*.

Fig. 38. As the Radius or S. $90^{\circ} 00'$,
To S. cr. AB $27, 54$;
So is Tc. cr. BC $11, 30$,
To Tc. of V. at A $23, 30$, as required.

P R O P. XII. Case 12.

The Hypothenuse, and a Side given, to find the other Side.

Fig. 38. In the Triangle ABC, there is given the Hypothenuse AC $30^{\circ} 00'$, and the Side AB $27^{\circ} 54'$, and the Side BC is required, which may be found by this *Analogy* or *Proportion*.

As Sc. cr. AB $27^{\circ} 54'$,
To Radius or S. $90, 00$;
So is Sc. Hypothenuse AC $30^{\circ} 00'$,
To Sc. cr. BC $11^{\circ} 30'$, as required.

Having the Sun's Place, and right Ascension given, by this Case you find the Sun's present Declination.

P R O P.

P R O P. XIII. Case 13.

The Hypothenuse, and a Side given, to find the contained Angle.

In the Triangle ABC, there is given the *Hy* Fig. 38.
pothenuse AC $30^{\circ} 00'$, and the Side AB $27^{\circ} 54'$,
 and the Angle at A is required, which may be
 found by this *Analogy* or *Proportion*.

As the Radius or S. $90^{\circ} 00'$,

To T. cr. AB $27^{\circ} 54'$;

So is Tc. Hypoth. AC $30^{\circ} 00'$,

To Sc. of V. at A $23^{\circ} 30'$, as required.

By this Case, from the last mentioned *Data*,
 may the Sun's greatest Declination be found.

P R O P. XIV. Case 14.

The Hypothenuse, and a Side given, to find the opposite Angle.

In the Triangle ABC, there is given the *Hy* Fig. 38.
pothenuse AC $30^{\circ} 00'$, and the Side AB $27^{\circ} 54'$,
 now the Angle C is required, which may be
 found by this *Analogy* or *Proportion*.

As the S. Hypoth. AC $30^{\circ} 00'$,

To Radius or S. $90^{\circ} 00'$;

So is S. of cr. AB $27^{\circ} 54'$,

To S. of V. at C $69, 22$, as required.

This Case finds the Angle of the Sun's Posi-
 tion, from the foregoing *Data*.

P R O P.

P R O P. XV. Case 15.

The Oblique-angles given, to find either Side.

In the Triangle ABC, there is given the Angle A $23^{\circ} 30'$, and the Angle at C $69^{\circ} 22'$, and the Side BC is required, which may be found by this *Analogy* or *Proportion*.

Fig. 38.

As the S. of V. at C $69^{\circ} 22'$,

To the Radius or S. $90^{\circ} 00'$;

So is the Sc. of V. at A $23^{\circ} 30'$,

To the Sc. of cr. BC $11^{\circ} 30'$, as required.

Having the Sun's greatest Declination, and Angle of his Position given, by this Case you may find the present Declination.

P R O P. XVI. Case 16.

The Oblique-angles given, to find the Hypothenuſe

In the Triangle ABC, there is given the Angle A $23^{\circ} 30'$, the Angle C $69^{\circ} 22'$, and the Hypothenuſe AC is required, which may be found by this *Analogy* or *Proportion*,

Fig. 38.

As the Radius or S. $90^{\circ} 00'$,

To Tc. of V. at C $69^{\circ} 22'$;

So is Tc. of V. at A $23, 30$,

To Sc. Hypoth. AC $30, 00$, as required.

This Case, by the same things given as in the last, finds the Sun's Place.

These USES at the end of each Proposition, will make the Calculations more pleasant; and when you come to apply them to Astronomy, you will be much more ready at them.

S E C T.

S E C T. V.

Of Oblique angled Spherical Triangles.

P R O P. I. Case 1.

Two Sides, and an Angle opposite to one of them given, to find the other opposite Angle.

IN the Triangle ADE, there is given the Side Fig. 39.
AE $70^{\circ} 00'$, the Side ED $38^{\circ} 30'$, and the Angle A $30^{\circ} 28'$: Now the Angle at D is required, to find which, this is the *Analogy* or *Proportion*.

As S. cr. DE $38^{\circ} 30'$,
To S. V. at A $30^{\circ} 28'$;
So is S. cr. AE $70^{\circ} 00'$,
To S. V. at D $130^{\circ} 03'$, required.

 Note, That in these Operations, for the more Facility of the Learner, I omit Seconds, which doth belong unto the Angles, &c.

P R O P. II. Case 2.

Two Angles and a Side opposite to one of them given, to find the Side opposite to the other.

IN the Triangle ADE, there is given the An- Fig. 39.
gle at D $130^{\circ} 03'$, the Angle E $31^{\circ} 34'$, and the Side AE $70^{\circ} 00'$: Now the Side AD is required, which may be found by this *Analogy* or *Proportion*.

As S. V. at D $130^{\circ} 03'$,
To S. cr. AE $70, 00$;
So is S. V. at E $31, 34$,
To S. cr. AD $40, 00$, required.

. H

P R O P.

P R O P. III. Case 3.

Two Sides and an Angle included between them being known, to find the other Angles.

Fig. 39. In the Triangle ADE, there is given the Side AE $70^{\circ} 00'$, the Side AD $40^{\circ} 00'$, and the Angle A $30^{\circ} 28'$: Now the Angles D and E are required, which is thus found; take the Sum and Difference of the two Sides, and work as followeth, saying,

Side. AE $70^{\circ} 00'$,

Side. AD $40^{\circ} 00$

Sum. $\begin{array}{r} 110\ 00 \\ \hline \end{array}$ } $\frac{1}{2}$ Z. $55^{\circ} 00'$.

Diff. $\begin{array}{r} 30\ 00 \\ \hline \end{array}$ } $\frac{1}{2}$ X. $15\ 00$.

Angle. A $30\ 28$ } $\frac{1}{2}$ V. $15\ 14$.

As S. $\frac{1}{2}$ Z. cr^s. AE and AD, $55^{\circ} 00'$,

To S. $\frac{1}{2}$ X. cr^s. AE and AD, $15\ 00$;

So is Tc. $\frac{1}{2}$ V. at A, $15\ 14$,

To T. $\frac{1}{2}$ X. VV. D and E. $49\ 14\ 30''$.

 And here observe, that if the Sum

of the two contained Sides exceed a Semicircle, then subtract each Side severally from 180° , and proceed with those Complements, as with the sides given, the Operation produceth the Complements of the Angles sought, unto a Semicircle or 180 Degrees.

A G A I N.

As Sc. $\frac{1}{2}$ Z. cr^s. AE and AD, $55^{\circ} 00'$;

To Sc. $\frac{1}{2}$ X. cr^s. AE and AD, $15^{\circ} 00'$;

So is Tc. $\frac{1}{2}$ V. at A, $15^{\circ} 14'$,

To T. $\frac{1}{2}$ Z. VV. D and E, $80\ 48'\ 30''$.

This

Spherical Trigonometry.

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§ 5.

This Difference of the Angles unknown D and E $49^{\circ} 14' 30''$, being added unto the half Sum of the Angles $80^{\circ} 48' 30''$, (unknown) produceth the greater Angle D $130^{\circ} 03'$, and substracted from it, leaves the lesser Angle E, to wit, $31^{\circ} 34'$.

PROP. IV. Case 4.

Two Angles, and their Interjacent Side being known, to find the other Sides.

In the Triangle ADE, there is given the Angle at A $30^{\circ} 28'$, and the Angle at D $130^{\circ} 03'$, and their Interjacent-side AD $40^{\circ} 00'$, and the Sides DE and EA are required: Which is thus found.

Take the Sum and Difference of the two Angles, and work as followeth, saying,

 And here observe, That if the Sum of the two given Angles exceed a Semicircle, or 180° , substract them from a Semicircle, and proceed with the Residues, the Operation will produce each Side's Complement to a Semicircle, or 180 Degrees.

Fig. 39.

Angle. D $130^{\circ} 03'$.

Angle. A $30 28$.

Sum. $160 31$ } $\frac{1}{2}$ Z. is $80^{\circ} 15' 30''$.

Diff. $99 35$ } $\frac{1}{2}$ X. is $49 47 30$.

Side. AD $40 00$ } $\frac{1}{2}$ cr. is $20 00 00$.

As S. $\frac{1}{2}$ Z. of VV. A and D, $80^{\circ} 15' 30''$,

To S. $\frac{1}{2}$ X. of VV. A and D, $49 47 30$;

So is T. $\frac{1}{2}$ cr. AD, $20 00 00$,

To T. $\frac{1}{2}$ X crs. DE and EA, $15 45 00$.

H 2

AGAIN

§ 5.

A G A I N Say,

As Sc. $\frac{1}{2}$ Z. of *VV*. A and D, $80^{\circ} 15' 30''$,
 To Sc. $\frac{1}{2}$ X. of *VV*. A and D, $49 47 30$;
 So is T. $\frac{1}{2}$ cr. AD, $20 00 00$,
 To T. $\frac{1}{2}$ cr^s. Z. DE and AE. $54 15 00$.

Add the half Difference of the Sides DE and AE $15^{\circ} 45'$, unto half the Sum of the Sides DE and AE $54^{\circ} 15'$. It produceth the greater Side, the Side AE $70^{\circ} 00'$, but if deducted from it, leaves the lesser Side ED, which is $38^{\circ} 30'$, as was required.

P R O P. V. Case 5.

Two Sides and an Angle opposite to one of them given, to find the third Side.

Fig. 39. In the Triangle ADE, there is given the Side AE $70^{\circ} 00'$, the Side DE $38^{\circ} 30'$, and the Angle A $30^{\circ} 28'$, the Side AD is required.

First by Case 1. Prop. 1. I find the Angle at D to be $130^{\circ} 03'$, and then proceed thus:

First take the Sum and Difference of the two Angles; then also find the Difference of the two Sides given, and then work as followeth.

Angle D $130^{\circ} 03'$,

Angle A, $30 28$.

Sum.	160 31	}	$\frac{1}{2}$ Z. $80^{\circ} 15' 30''$.
Differ.	99 35	}	$\frac{1}{2}$ X. $49 47 30$.

Side.

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Side. AE 70 00.

Side. ED 38 30.

Diff. $\left. \begin{array}{r} \hline 31 \ 30. \\ \hline \end{array} \right\} \frac{1}{2} X. 15^{\circ} 45'.$

Now say,

As S. $\frac{1}{2} X.$ VV. D and A, $49^{\circ} 47' 30''$;

To S. $\frac{1}{2} Z.$ VV. D and A, $80 \ 15 \ 30$;

So is T. $\frac{1}{2} X.$ cr. AE and ED, $15 \ 45 \ 00$,

To T. $\frac{1}{2}$ cr. AD. $20^{\circ} 00' 00''$; which doubled giveth the Side AD, $40 \ 00 \ 00$, as was required.

PROP. VI. Case 6.

Two Angles and a Side opposite to one of them given, to find the third Angle.

In the Triangle ADE, there is given the Angle A $30^{\circ} 28'$, the Angle D $130^{\circ} 03'$, and his opposite Side AE $70^{\circ} 00'$, and 'tis required to find the Angle at E. Fig. 39.

First by Prop. 2. Case 2. I find the Side DE, opposed to the Angle A, to be $38^{\circ} 30'$, then proceed thus:

First find the Sum and Difference of the Sides. Then find the Difference of the Angles.

Side. DE $38^{\circ} 30'$,

Side. AE 70 00.

Sum. $\left. \begin{array}{r} \hline 108 \ 30. \\ \hline \end{array} \right\} \frac{1}{2} Z. 54^{\circ} 15'.$

Differ. $\left. \begin{array}{r} \hline 31 \ 30 \\ \hline \end{array} \right\} \frac{1}{2} X. 15 \ 45.$

H 3

Angle

§ 5.

Angle	D	130 03	
Angle	A	30 28	
Differ.		99 35	}

 $\frac{1}{2} X. 49^{\circ} 47' 30''.$

Now say,

As $S. \frac{1}{2} X.$ cr^s. DE and AE $15^{\circ} 45'$,To $S. \frac{1}{2} Z.$ cr^s. EA and DE $54 15$;So is $T. \frac{1}{2} X.$ VV. D and A $49 47 30''$,To Tc. $\frac{1}{2} V.$ at E $15^{\circ} 47' 00''$, which doubled giveth the Angle at E $31, 34$, as required.

P R O P. VII. Case 7.

Two Sides and an Angle opposite to one of them given, to find the included Angle.

Fig. 39. In the Triangle ADE, there is given the Side AE $70^{\circ} 00'$, the Side ED $38^{\circ} 30'$, and the Angle opposite thereunto at A $30^{\circ} 28'$, and the Angle E is required.

First by Prop. 1. Case 1. I find the Angle D opposite to AE, to be $130^{\circ} 03'$, then proceed thus;

First find the Difference of the Angles, then find the Sum and Difference of the Sides.

Angle D $130^{\circ} 03'$.Angle A $30 28$.

Differ.	99 35	}

 $\frac{1}{2} X. 49^{\circ} 47' 30''.$

Side.	AE	70° 00'	
Side.	ED	38 30.	
Sum.		180 30	} $\frac{1}{2}$ Z. 54° 15'.
Differ.		31 30	

§ 5.

Now say,

As S. $\frac{1}{2}$ X. cr^s. AE and ED 15° 45'.
 To S. $\frac{1}{2}$ Z. cr^s. AE and ED 54 15;
 So is T. $\frac{1}{2}$ X. of VV. D and A 49 47 30", Fig. 39.
 To Tc. $\frac{1}{2}$ V. at E 15 47': Which doubled is
 the Angle at E 31° 34', as was required.

P R O P. VIII. Case 8.

Two Angles and a Side opposite to one of them being known, to find the Interjacent Side.

In the Triangle ADE, there is given the Angle E 31° 34', the Angle D 130° 03', and his opposite Side AE 70° 00': Now the Side ED is required.

First by Prop. 2. Case 2. I find AD opposed to E, to be 40° 00', and then work thus.

Take the Sum and Difference of the Angles, then also find the Difference of the two Sides.

Angle	D	130° 03'	
Angle	E	31 34	
Sum		161 37	} $\frac{1}{2}$ Z. 80° 48' 30'.
Differ.		98 29	

Fig. 39.

H 4

Side

§ 5

Side AD $40^{\circ} 00'$ Side AE $70 00$

Differ.	30 00	}	: X. $15^{\circ} 00'$.
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Now say,

As S. $\frac{1}{2}$ X. VV. D and E $49^{\circ} 14' 30''$,To S. $\frac{1}{2}$ Z. VV. D and E $80 48 30$;So is T. $\frac{1}{2}$ X. cr^s. AD and AE $15 00 00''$,To T. $\frac{1}{2}$ cr^s. ED $19^{\circ} 15' 00''$, which being doubled, is the Side ED $38^{\circ} 30'$, as required.

P R O P. IX. Case 9.

*Two Sides and their included Angle being known,
to find the third Side.*

Fig. 40. In the Triangle APZ, there is given the Side ZP $38^{\circ} 30'$, the Side PA 70° , and the Angle P, let be $31^{\circ} 34$, and the Side AZ is required.

The Resolution of this Case depends on the *Catholick Proposition* of the Lord of Marchiston, by supposing the *Oblique-Triangle* to be divided (by a supposed *Perpendicular* falling either within or without the *Triangle*) into two *Rectangulars*.

Now in the Triangle AZP, let fall the *Perpendicular* ZR; so is the Triangle AZP divided into two *Rectangulars* ARZ and ZRP. Now the Side AZ may be found at two *Operations* thus: Say,

As the Radius or S. of	$90^{\circ} 00'$,
To Sc. of the included V, P.	$31 34$;
So is T. of the lesser Side PZ.	$38 30$,
To T. of a fourth Arch.	$34 08$.

If

Spherical Trigonometry.

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§ 5.

If the *contained Angle* be less than 90° , take this *fourth Arch* from the *greater Side*; but if it be greater than 90° , from its *Complement* unto 180° , the *Remainder* is the *Residual Arch*: Now again say,

As Sc. of the fourth Arch,	34°	$08'$,
To Sc. Residual Arch,	35	52 ;
So Sc. of the lesser Side PZ,	38	30 ,
To Sc. AZ the Side required,	40	00 .

 But note, that many times the Perpendicular will fall without the Triangle, as it doth now within; in such case the Sides of the Triangle must be continued, so will there be two Rectangulars, the one included within the other: As in the Triangle HIK, the Perpendicular let fall is KM, falling on the Side HE, and so the two Rectangulars found thereby will be IMK, and KMH, and so by the Directions in the former Proposition find out the Side IK, if required to be found.

Fig. 41.

 Note, That if the Angles at the Base be both of one kind, that then the Perpendicular falls within the Triangle: If of divers kinds, without the Triangle.

PROP. X. Case 10.

Two Angles and their Interjacent Side known, to find the third Angle.

In the Triangle AZP, there is given the Side Fig. 40. ZP $38^\circ 30'$, the Angle P $31^\circ 34'$, and the Angle Z $130^\circ 03'$, and the Angle at A is required.

First the Oblique-Triangle AZP, being reduced into two Rectangulars ARZ, and ZRP, by
Case

§ 5. *Case 9. foregoing, I find the Angle RZP, to be $64^{\circ} 19'$, (in the Triangle ZRP.) which taken out of Angle AZP $130^{\circ} 03'$, leaves the Angle AZR $65^{\circ} 44'$: Now the Angle A is found by this Analogy or Proportion.*

As S. V. PZR $64^{\circ} 19'$,

To S. V. AZR $65^{\circ} 44'$;

So is Sc. V. at P $31^{\circ} 34'$,

To Sc. V. at A $30^{\circ} 28'$: Which was required to be found out and known.

P R O P. XI. *Case 11.*

Three Sides given, to find an Angle.

In the Triangle APZ, the Side AZ is $40^{\circ} 00'$, the Side ZP is $38^{\circ} 30'$, the Side AP is $70^{\circ} 00'$, and the Angle Z is required. To find which do thus.

Fig. 40. Add the three Sides together, and from half their Sum, deduct the Side opposite to the required Angle; and then proceed as you see in the Operation following.

Side	AP	70° 00'	Co-Arith.	
Side	AZ	40 00		0, 191933
Side	ZP	38 30		0, 2 5850

Sum	148 30	Sines.
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$\frac{1}{2}$ Sum	74 15	9, 983380
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Side	AP	70 00
------	----	-------

Rem.	04 15	8, 869868
------	-------	-----------

Sum is		19, 251031 Fig. 40.
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$\frac{1}{2}$ Sum is $65^{\circ} 07' 30''$, the Sc. $\frac{1}{2} V.$ at Z, 9,625515 which doubled is $130^{\circ} 03' 12''$; the Angle at Z required.

This 11th Proposition or Case is of great Use in *Astronomy*, *Navigation*, and *Dyalling*.

For in *Astronomy* 'tis by this Proposition we find the time of Twilight's Beginning and End. In *Navigation* it finds the Sun's Azimuth, in order to find the Variation of the Needle; and in *Dyalling* it gets the Azimuth, by help of which, we find the Declination of any Wall or Plain.

P R O P. XII. Case 12.

Three Angles given, to find a Side.

In the Triangle AZP, the Angle A is $30^{\circ} 28' 11''$, the Angle Z $130^{\circ} 03' 12''$, the Angle P is $31^{\circ} 34' 26''$, and the Side AZ, opposite to P, is required.

This

§ 5.

This *Case* is likewise performed as the former *Case* or Proposition, the Angles being converted into Sides, and the Sides into Angles, by taking the *Complement* of the greatest Angle unto 180° : See the Work.

Angle E	31° 34' 26"	Co-Arith.
Angle A	30 28 11	0, 294920
Compl. V. Z	49 56 49	0, 116084

Sum is	111 59 26	Sines
--------	-----------	-------

$\frac{1}{2}$ Sum is	55 59 43	9, 918549
----------------------	----------	-----------

Fig. 40.

Angle P	31 34 26
---------	----------

Differ.	24 25 17	9, 616417
---------	----------	-----------

Sum total	19, 945970
-----------	------------

The $\frac{1}{2}$ Sum is Sc. of $20^\circ 00'$ half the

Side AZ,	9, 972985
----------	-----------

which being doubled, gives the Side AZ $40^\circ 00'$, required to be found out and known.

✍ But if the greater Side AP were required, the Operation would produce the *Complement* thereof unto a *Semicircle* or 180° ; therefore subtract it from 180° , it leaves the remaining required Side sought.

Thus have I laid down all the *Cases* of *Triangles*, both *Right-lined* and *Spherical*; either *Right*, or *Oblique-angled*; I might hereunto have annexed many *Varieties* unto each *Case*, and some fundamental *Axioms*, which somewhat more would have illustrated and demonstrated those *Cases* and *Proportions*; but because of the

the Smallness of this *Treatise*, which is intended more for *Practice* than *Theory*, I have for Brevity sake omitted them, and refer you for those things to larger Authors, who have largely discoursed thereon to good purpose.

§ 5.

CHAP. VI.
Of ASTRONOMY.

Defin. **A**STRONOMY is an Art Mathematical, which measureth the distinct Course of Times, Days, Years, &c. It sheweth the Distance, Magnitude, Natural Motions, Appearances and Passions, proper unto the Planets and fixed Stars, for any time past, present and to come. By this we are certified of the Distance of the starry Sky, and of each Planet, from the Center of the Earth, and the Magnitude of any fixed Star or Planet, in respect of the Earth's Magnitude; what is the Cause of Eclipses, and the Method of finding the Quantity of the Digits eclipsed.

SECT. I.

Of Astronomical Definitions.

§ 1.

Defin. I. **A** Sphere or Globe is a solid Body, contained under one *Superficies*, in whose middle there is a Point (called the *Center*), from which all right or streight Lines drawn

§ I. drawn unto the *Circumference* or *Superficies*, are Equal.

Fig. 43. 2. The *Poles* of the World are two fixed Points in the *Heavens* *Diametrically* opposite the one to the other; the one called the *Artick* or *North-Pole*, noted in the *Scheme* by P. The other is called the *Antartick* or *South-Pole*; as S. and is not to be seen of us, being in the lower *Hemisphere*.

Fig. 43. 3. The *Axis* of the World is an imaginary Line drawn from the *North-Pole*, through the *Center* of the *Earth*, unto the *South-Pole*, about which the *Diurnal Motion* is performed, from the *East* to the *West*; as the Line PS.

Fig. 43. 4. The *Meridians* are great *Circles*, concurring and intersecting one another in the *Poles* of the World; as PES and Pc. S.

Fig. 43. 5. The *Equinoctial* or *Equator* is a great *Circle*, 90 Deg. distant from the *Poles* of the World, cutting the *Meridians* at *Right-angles*, and divideth the World into two Equal Parts, called the *Northern* and *Southern Hemispheres*, as E = Q. in *Scheme* 42.

Fig. 43. 6. The *Ecliptick* is a great *Circle* crossing the *Equinoctial*, in the two opposite Points *Aries* and *Libra*, and maketh an Angle therewith (called, its *Obliquity*) of $23^{\circ} 30'$, represented by $\odot = \varpi$. This *Circle* is divided into 12 *Sines*, each containing $30^{\circ} 00'$: As *Aries* γ , *Taurus* δ , *Gemini* Π , *Cancer* $\odot$, *Leo* δ , *Virgo* xx , (which are called *Northern Sines*) *Libra* = , *Scorpio* m , *Sagittarius* f , *Capricornus* w , *Aquarius* = , and *Pisces* x ; these are called *Southern Sines*.

7. The

Astronomical Definitions.

III

7. The *Zodiack* is a *Zone* or *Girdle*, having § I.
8 Deg. of *Latitude* on either side the *Ecliptick*,
that is, the *Ecliptick* is just in the middle of the
Zodiack, in which space the *Planets* make their
Revolution. This *Circle* is a *Circle* which re- Fig. 43.
gulates the *Years*, *Months*, and
Seasons, * and is distinguished
in the *Scheme* by the 12 *Sines*.

* Which is by the Greeks cal-
led *Zōdiakos*, i. e. bring Life;
because the Life of all Creatures

depend on the Cause of that Circle; for the Sun ascending in it, and moving
towards us brings the Generation of things, and in descending the Cor-
ruption of all things sensible, and insensible, which are below the Concavity
of the Moon, &c.

8. The *Colures* are two *Meridians*, dividing
the *Ecliptick* and the *Equinoctial* into four equal
parts; one of which passeth by the *Equinoctial*
Points *Aries* and *Libra*, and is called the *Equi-* Fig. 43.
noctial Colure, as P = S. The other by the be-
ginning of *Cancer* and *Capricorn*, and is called
the *Solstitial Colure*, as P ⊗ S ∞.

9. The *Poles* of the *Ecliptick* are two Points,
23° 30' distant from the *Poles* of the World, as
I and K. Fig. 43.

10. The *Tropicks* are two small *Circles* Pa-
rallel unto the *Equinoctial*, and distant there-
from 23° 30', limiting the Sun's greatest Decli-
nation. The *Northern Tropick* passeth by the
beginning of *Cancer*, and is therefore called
the *Tropick of Cancer*, as ⊕ a D. The *Southern* Fig. 43.
Tropick passeth by the beginning of *Capricorn*,
and is therefore called the *Tropick of Capricorn*;
as Bb ∞.

11. The *Polar Circles* are two small *Circles*
parallel to the *Equinoctial*, and distant there-
from 66° 30'; and from the *Poles* of the World

23°

- §. 1. 23° 30'. That which is adjacent unto the North-Pole, is called the *Artick Circle*, as G d I.
- Fig. 43. and the other the *Antartick Circle*, as K d M.
12. The *Zenith* and the *Nadir* are two Points *Diametrically* opposite the one to the other: The *Zenith* is the *Vertical Point*, or the Point over our Heads, as Z. The *Nadir* is opposite thereto, as the Point N.
13. The *Azimuths* or *Vertical Circles* are great Circles of the *Sphere*, concurring and intersecting each other in the *Zenith* and *Nadir*; as Z f N.
- Fig. 43. 14. The *Horizon* is a great Circle, 90 Degrees distant from the *Zenith* and *Nadir*; cutting all the *Azimuths* at Right-angles, and dividing the World into two equal parts, the upper and visible *Hemisphere*, and the lower and invisible *Hemisphere*, represented by H a R.
- Fig. 43. 15. The *Meridian* of a Place is that *Meridian* which passeth by the *Zenith* and *Nadir* of the Place, as P Z S N.
16. The *Almicanthars*, or *Parallels of Altitude*, are small Circles parallel unto the *Horizon*, imagined to pass through every Degree and Minute of the *Meridian*, between the *Zenith* and *Horizon* B a F.
- Fig. 43. 17. *Parallels of Latitude*, or *Declination*, are small Circles parallel unto the *Equinoctial*; they are called *Parallels of Latitude*, in respect to any place on the *Earth*, and *Parallels of Declination*, in respect of the *Sun* or *Stars* in the *Heavens*. Hence the *Declination* of the *Sun*, *Moon*, or *Stars*, is no more than their Distance from the *Equator*.

18. The

Astronomical Definitions.

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18. The *Latitude* of a Place is the Height of the Pole above the *Horizon*, or the Distance between the *Zenith* and the *Equinoctial*; or 'tis the Distance of a Place from the *Equator*. § 1.

19. The *Latitude* of a Star is the *Arch* of a Circle, contained betwixt the *Center* of a Star, and the *Ecliptick Line*: This Circle making *Right-angles*, with the *Ecliptick*, is accounted either *Northward* or *Southward*, according to the Situation of the Star.

20. *Longitude* on Earth is measured by an *Arch* of the *Equinoctial*, contained between the *Primary Meridian*, (or *Meridian* of that place, where *Longitude* is assigned to begin) and the *Meridian* of any other place, counted always *Easterly*.

21. The *Longitude* of a Star is that part of the *Ecliptick*, which is contained between the Star's place in the *Ecliptick* and the beginning of *Aries*, counting them according unto the Succession of Signs.

22. The *Altitude* of the Sun or Stars is the *Arch* of an *Azimuth*, contained betwixt the *Center* of the Sun, or Star, and the *Horizon*.

23. *Ascension* is the rising of any Star, or part of the *Equinoctial*, to any Degree above the *Horizon*; and *Descension* is the setting of it.

24. *Right Ascension* is the Number of Degrees and Minutes of the *Equinoctial*; (i. e. from the beginning of *Aries*) which cometh unto the *Meridian*, with the Sun or Stars, or with any Portion of the *Ecliptick*.

25. *Oblique-Ascension* is an *Arch* of the *Equinoctial*, between the beginning of *Aries* and that part of the *Equinoctial*, which riseth with

§ 1. the *Center* of a *Star*; or with any *Portion* of the *Ecliptick* in an *Oblique Sphere*: And *Oblique Descension* is that part of the *Equinoctial*, that setteth therewith.

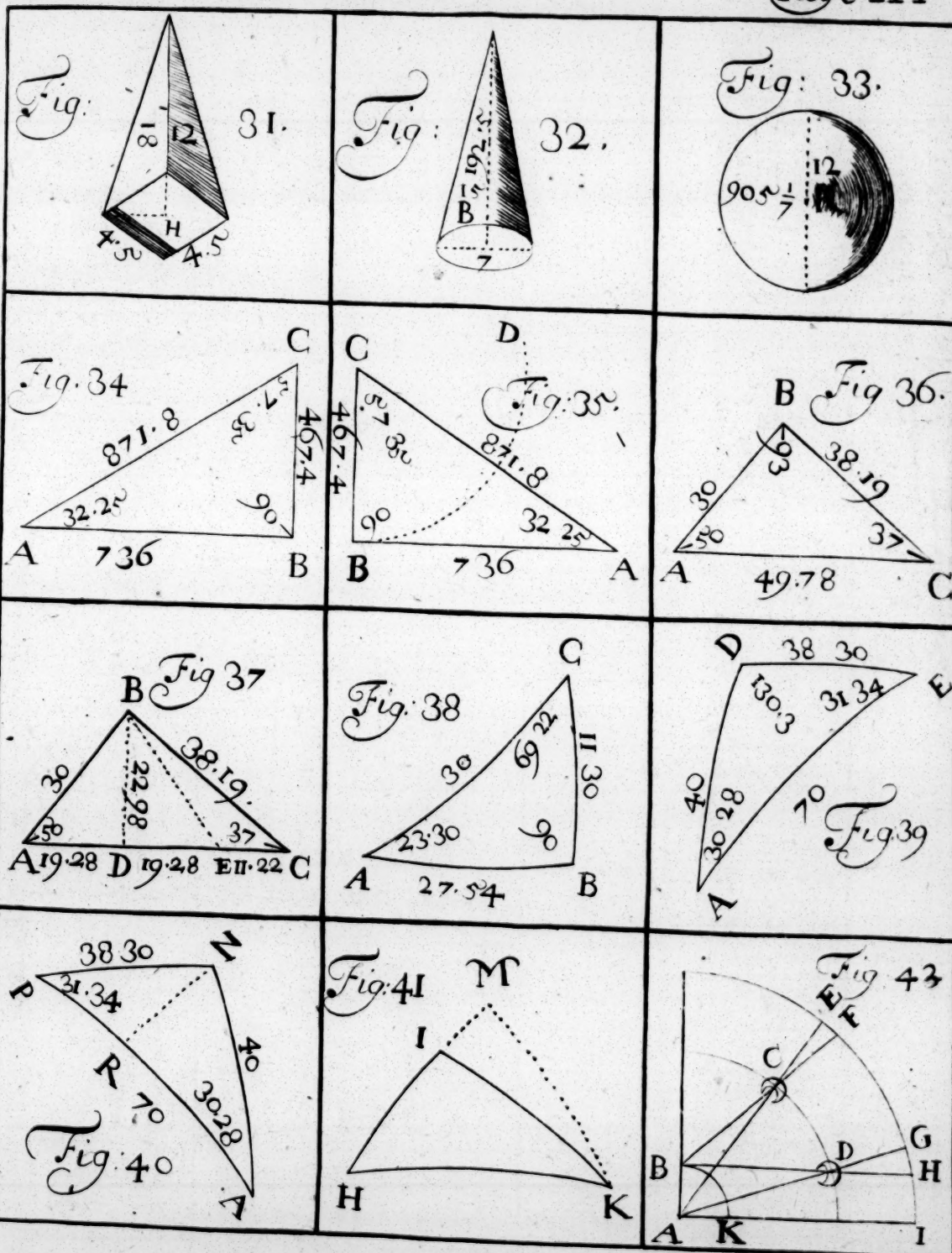
26. The *Ascensional Difference* is an *Arch* of the *Equinoctial*, being the *Difference* betwixt the *Right* and *Oblique-Ascension*.

27. The *Amplitude* of the *Sun* or *Stars* is the *Distance* of the rising or setting thereof, from the *East* or *West Point* of the *Horizon*.

28. The *Parallax* is the *Difference* between the *true* and *apparent Place* of the *Sun* or *Star*; so the *true Place* in respect of *Altitude*, is in the *Line ACE*, or *ADG*, the *Sun* or *Star* being at *C* or *D*, and the *apparent Place* in the *Line BCF* and *BDH*; so likewise the *Angles* of the *Parallax* are *ACB* or *ECF*, and *ADB* or *GDH*: Also in the said *Scheme* *ABK* representeth a *Quadrant* (of the *Globe* or *Earth*) on the *Earth's Superficies*: *A* the *Center* of the *Earth*, and *B* any *Point* of the *Earth's Surface*.

Fig. 42.

29. The *Refraction* of a *Star* is caused by the *Atmosphere*, or *Vaporous Thickness* of the *Air* near the *Earth's Superficies*, whereby the *Sun* and *Stars* seem always to rise sooner, and set later than really they do; that is, they appear above the *Horizon* when really they are under it.



S E C T. II.

Of Astronomical Propositions.

§ 2.

P R O P. I.

The Distance of the Sun from the next Equinoctial Point (either Aries or Libra) being known, to find his Declination.

THE *Analogy or Proportion.*

As Radius or S. 90° ,

To S. of the Sun's distance from the next Equinoctial Point ;

So is S. of the Sun's greatest Declination,

To the S. of the Sun's present Declination sought.

P R O P. II.

The Sun's Place given, to find his Right-Ascension.

This is the Analogy or Proportion.

As Radius or S. 90° ,

To T. of the Sun's Longitude from the next Equinoctial Point ;

So is the Sc. of his greatest Declination,

To T. of his Right-Ascension from the next Equinoctial Point.

§ 2.

P R O P. III.

To find the Sun's Place or Longitude from Aries,
his Declination being given.

This is the *Analogy* or *Proportion*.

As S. of the Sun's greatest Declination,

To Radius or S. $90^{\circ} 00'$;

So is S. of his present Declination,

To S. of the Sun's Place or

Longitude from Aries *.

* If the Sun's Declination be North, and increasing, this Proportion finds the Sun's Distance

from γ ; but if decreasing from ϵ in the Northern Sines. But if the Sun's Declination be South, and increasing from ϵ ; if decreasing from γ among the Southern Sines.

P R O P. IV.

By knowing the Sun's Declination, to find his Right Ascension.

This is the *Analogy* or *Proportion*.

As Radius or S. 90° ,

To Tc. of the Sun's greatest Declination;

So is T. of the Declination given,

To S. of the Sun's right Af-

† From the next Equinoctial Point, either γ or ϵ .

cension required †.

P R O P. V.

By knowing the Latitude of a Place, and the Sun's Declination, to find the Ascensional Difference.

This is the *Analogy* or *Proportion*.

As Radius or S. 90° ,

To

Astronomical Propositions.

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To Tc. of the Latitude given;

§ 2.

So is T. of the Sun's Declination given,

To the S. of the Ascensional Difference required.

 NOTE, That if you reduce the Degrees, &c. of the *Ascensional Difference* into Hours, it will shew you how much the *Sun* riseth or setteth before, or after six a Clock, in that Latitude.

P R O P. VI.

To find the *Sun's Oblique Ascension or Descension.*

First find the *Ascensional Difference* by the 5th Proposition, and his *Right Ascension* by the fourth: Now if the *Sun's Declination* be Northerly, deduct the *Ascensional Difference* out of his *Right Ascension*, from the beginning of γ , (for the six Northern Signs γ δ ϵ ζ η θ) it leaves the *Oblique Ascension*, and added unto the *Right Ascension*, giveth the *Oblique Descension*.

But if the *Sun's Declination* be Southerly, the *Ascensional Difference*, added to the *Right Ascension*, (for the six Southern Signs $\imath$ κ λ μ ν ξ) giveth the *Right Ascension*, and subtracted therefrom, leaves the *Oblique Descension*.

P R O P. VII.

By knowing the *Sun's Declination*, and the Latitude of a Place, to find the *Sun's Amplitude*.

This is the *Analogy or Proportion*.

As Sc. of the Latitude,

I 3

To

6 2.

*To the Radius or S. 90° ;
 So is the S. of the Sun's Declination,
 To the S. of the Amplitude from the East or
 West Points of the Horizon.*

*This Proposition is of great Use at Sea, in
 finding the Variation of the Needle: For the
 Summ of Difference of the Sun's Amplitude and
 Magnetick Amplitude, according as it is, is the
 Variation of the Needle.*

P R O P. VIII.

*By knowing the Sun's Declination and Amplitude,
 from the North part of the Horizon, to find
 the Latitude.*

*This is the Analogy or Proportion.
 As Sc. of the Amplitude from the North,
 To Radius or S. $90^{\circ} 00'$;
 So is S. of his Declination given,
 To Sc. of the required Latitude.*

P R O P. IX.

*By knowing the Latitude of a Place, and the Sun's
 Declination, to find at what time the Sun will
 be on the true East or West Points.*

*The Analogy or Proportion is,
 As T. of the given Latitude,
 To T. of the Sun's Declination propounded;
 So is Radius or S. $90^{\circ} 00'$,
 To Sc. of the Hour from Noon.*

P R O P.

P R O P. X.

By knowing the Sun's Declination, and Latitude of a Place, to find his Altitude at six a Clock.

This is the Analogy or Proportion.

As Radius or S. $90^{\circ} 00'$,

To S. of the Sun's Declination;

So is S. of the Latitude of the Place,

To S. of the Sun's Altitude at six a Clock.

P R O P. XI.

By knowing the Latitude of a Place, and the Sun's Declination, to find the Azimuth at six.

This is the Analogy or Proportion.

As Radius or S. $90^{\circ} 00'$,

To the T. of the Sun's Declination;

So is Sc. of the Latitude of the Place,

To the T. of the Azimuth sought.

P R O P. XII.

By knowing the Latitude of a Place, and the Sun's Declination, to find the Sun's Altitude when he is on the true East or West Points.

This is the Analogy or Proportion.

As S. of the Latitude,

To the Radius or S. $90^{\circ} 00'$;

So is the S. of the Declination,

To the S. of the Sun's Altitude being due East or West.

P R O P. XIII.

To find the Sun's Altitude at any time of the Day.

The *Analogy* or *Proportion* is,
As Radius or $S. 90^{\circ} 00'$,
To Tc. of the Poles Height ;
So is S. of the Sun's Distance,
From the Hour of Six,
To the T. of an Arch: Which being substra-
 ed from the *Sun's Distance* from the *Pole*, say,
As Sc. of the Arch found,
To Sc. of the remaining Arch of the *Sun's Di-*
stance from the *Pole* ;
So is S. of the Pole's Height,
To the S. of the Sun's Altitude at the Hour
 required.

P R O P. XIV.

By knowing the Latitude of a Place, with the
Sun's Declination, and Altitude, to find the
Hour of the Day.

To solve this Conclusion do thus: Get the
 Sum of the *Complements* of the *Latitude*, *Declination* and *Altitude* given*,
 Then find the Difference be-
 * *As in Case 11 of Oblique Spherical Triangles.* twixt their half Sum, and
 the *Complement* of the *Altitude*; then say,
As Radius or $S. 90^{\circ} 00'$,
To Sc. of the Sun's Altitude ;
So is Sc. of the Latitude of the *Place*,
To a fourth Sine: Then again say,
As the fourth S.
To the S. of $\frac{1}{2}$ Z. of the Lat. Declin. and Alt.

So

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§ 2.

So is the S. of X. of the Altitude to the $\frac{1}{2}$ Z,

To a fifth of S. unto which Sine, if you add the Radius or $90^{\circ} 00'$, half that Sum shall be the Sine of an Arch, whose double Complement is the Hour from the Meridian.

This is a very useful Proposition (tho' one of the most troublesome to calculate it being done by the 11th Case of Oblique Spheric Triangles:) For by it you find the Hour of the Day, having the Sun's Altitude given; and consequently may know how to set any Clock or Watch, or Sun-Dyal, if the Sun shines.

The Sun's Altitude, if he shines, is gotten thus: Set a strait Pin or Wyer exactly perpendicular on a true Horizontal Plain; then measure the Length of the Wyer, and the Length of the Shade of such Wyer, and say,

As the Length of the Shade to the Length of the Wyer, so is Radius to the Tangent of the Angle of the Sun's Height.

Also by having the same things, viz. Latitude, Declination, and Sun's Altitude given, 'tis easy to find the Sun's Azimuth from the North, and by Consequence from the South; which having gotten, is of great use in finding the Variation of the Needle.

This Azimuth is also always to be found, in order to find the Declination of any Wall or Plain on which any Dyal is to be delineated.

P R O P.

P R O P. XV.

To find the Time of the Sun's Rising or Setting, and consequently the Length of the Day or Night.

To resolve this Conclusion, first by Prop. the 5. find the *Ascensional Difference*, which reduced into *Hours* and *Minutes* of Time, by allowing for every 15 Deg. one Hour, and for every Deg. less than 15°, 4' of Time, and for every 15 Min. one Minute of Time.

Secondly, If the Sun's Declination be Northerly, the *Ascensional Difference* added unto 6 Hours, gives the Time of *Sun-setting*, and subtracted therefrom, leaves the Time of *Sun-Rising*: On the contrary, if the Sun's Declination be Southerly, the *Ascensional Difference* added unto 6 Hours, gives the Time of *Sun-Rising*, and deducted therefrom the Time of *Sun-setting*.

Thirdly, If you double the Time of *Sun-Rising*, it gives you the Length of the Night; and the Time of *Sun-setting*, the Length of the Day.

P R O P. XVI.

The Sun's Declination, Altitude and Azimuth known, to find the Hour of the Day.

The Analogy or Proportion is,
As the Sc. of the Sun's Declination,
To the S. of the Azimuth;
So is the Sc. of the Altitude,

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To the S. of the Hour from Noon; Which § 2.
converted into Time, will shew the Hour of
the Day.

P R O P. XVII.

By knowing the Sun's Declination, Altitude, and
Hour from Noon, to find the Azimuth.

The Analogy or Proportion is,
As Sc. of the Sun's Altitude,
To S. of the Hour from Noon;
So is Sc. of the Sun's Declination,
To the S. of the Azimuth required.

P R O P. XVIII.

By knowing the Latitude of a Place, the Altitude
of the Sun, and the Hour from Noon, to find
the Angle of the Sun's Position.

This is the Analogy or Proportion.
As Sc. of the Sun's Altitude,
To S. of the Hour from Noon;
So is Sc. of the Latitude,
To S. of the V. of the Sun's Position, at the
time of the Question.

P R O P. XIX.

By knowing the Sun's Altitude, Declination, and
Azimuth, to find the Latitude.

The Analogy or Proportion is,
As S. of the Sun's Azimuth,
To S. of his Distance from the North-pole;
So is S. of V. of the Sun's Position,
To Sc. of the Latitude required. PROP.

P R O P. XX.

To find the Length of the *Crepusculum*, or
Twilight.

The *Crepusculum* or *Twilight*, is nothing else but the *Refraction* of the *Sun's Beams* in the *Density* of the *Air*: Which the Learned *Pet. Nennius* found the Length of the *Crepusculum*

* Watched the Time after Sun-
setting, when the *Twilight* in
the West was shut in; so that
no more *Twilight* than in any
other part of the Sky near the
Horizon appeared there: Then
by one of the known fixed Stars,
having found the true Hour of
the Night, he found the Length
of the *Twilight*, to be as in the
Rule is mentioned.

(by his many strict Observa-
tions *) to continue from the
time of the *Sun's* passing be-
low the *Horizon* of a Place, un-
til the *Sun* had run below the
said *Horizon* $18^{\circ} 00'$, and then
followed the shutting in of
the *Twilight*, and until the
Sun was departed so low the
Twilight continued. — To find

which observe this *Analogy* or *Proportion*.

As *Radius* or $S. 90^{\circ}$,

To *Sc.* of the *Sun's Declination*;

So is *Sc.* of the *Pole's Height*,

To a fourth *Sine*, which keep.

Then out of the *Sun's Distance* from the
South Pole, subduct the *Complement* of the *Pole*;
and of that Remains and the Degrees 62, being
added to it, their *Sum* and *Difference* found, say
again;

As the fourth *Sine* found,

To $S. \frac{1}{2} Z.$ of the *Remainder* and $62^{\circ} 00'$;

So is $S. \frac{1}{2} X.$ of the *Remainder* and $62^{\circ} 00'$,

To a *Number*, which being multiplied by the
Radius is equal unto the *Quadrant* of the *Sine* of
the

the $\frac{1}{2}$ Angle of the Sun's Distance at the ending of the Twilight, from Noon next ensuing. § 2.

Then from the Sum of the whole Angle converted into Hours, subtract the Hour of the Sun's setting *, it gives you the Length of the Crepusculum * Or $\frac{1}{2}$ Diurnal Arch, or Twilight.

But the Sun being in the Winter Tropick, makes the Twilight longest of any Twilight, the whole Winter half Year: Now in a certain Parallel, betwixt that Tropick and the Equinoctial, is the shortest Crepusculum: The Declination of which Parallel is thus found.

To find the Length of the least Crepusculum or Twilight.

As the Tc. of the Pole,
To the S. of the Pole;
So is the T. of $09^{\circ} 00'$,
To S. of the Declination of the Parallel, in which the Sun maketh the shortest Crepusculum of the Year.

But before the Crepusculum come to be shortest, there is another Parallel, in which the Crepusculum is equal to that of the Equinoctial: the Declination of which is found thus.

As the Radius or S. $90^{\circ} 00'$,
To S. of the Pole's Elevation or Altitude;
So is S. of $18^{\circ} 00'$,
To S. of the Declination of the Parallel, in which the Sun maketh the Crepusculum equal to that in the Equinoctial.

PROP.

§ 2.

P R O P. XXI.

To find the Quantity of the Angles, which the Circles of the 12 Houses make with the Meridian.

This is the *Analogy* or *Proportion*.

As the Radius or S. 90° ,

To T. of 60° : for the 11th, 9th, 5th, and 3d House; or to the T. 30° for the 12th, 8th, 6th, and 2d House;

So is the Sc. of the Pole,

To the Tc. of any House, with the Meridian.

P R O P. XXII.

To find the Right Ascension of the Point in the Equinoctial: And also the Point in the Ecliptick, called *Medium Cæli*, or *Cor Cæli*.

First, To find the Right Ascension of the Point of the Equinoctial, called *Medium Cæli*, vel *Cor Cæli*, find out the Sun's Right Ascension, by Prop. 2. Then reduce the whole Time from Noon last past into Degrees, which add unto the Right Ascension of the Sun, so shall their Aggregate be the Right Ascension of the Point, which in the Equinoctial is called *Medium Cæli*, vel *Cor Cæli*, required to be found.

Secondly, By the 2 Propositions foregoing, you may find the Right Ascension of the Point in the Ecliptick Culminant in the Meridian, called *Medium Cæli*, vel *Cor Cæli*; which is the Cuspis of the Tenth House, and his Declination by Prop. the first.

P R O P.

P R O P. XXIII.

To find the Angle of the Ecliptick with the Meridian.

The *Analogy* or *Proportion* is,
As the *Radius* or $S. 90^{\circ}$,
To *S. of the Sun's Greatest Declination*;
So is *Sc. of the Sun's Right Ascension*, from
the next *Equinoctial Point*,
To *Sc. of the V. of the Ecliptick*, with the
Meridian.

P R O P. XXIV.

To find the Angle of the Ecliptick with the Horizon.

The *Analogy* or *Proportion* is,
As *Radius* or $S. 90^{\circ}$,
To *Sc. of the Altitude of Cor Cæli*;
So is *S. of the V. Ecliptick with the Meridian*,
To *Sc. of the V. of the Ecliptick and Horizon*
sought.

P R O P. XXV.

*To find the Amplitude Ortive of the Ascendant,
or Horoscopus.*

This is the *Analogy* or *Proportion*,
As *Radius* or $S. 90^{\circ}$,
To *S. of Altitude of Med. Cæli*;
So is *T. of V. Ecliptick with the Meridian*,
To *Tc. of the Amplitude Ortive of the Ascen-*
lant, or the Distance of the Azimuth from the
Meridian.

P R O P.

§ 2.

P R O P. XXVI.

To find the Ascendant Degree of the Ecliptick, or the Cuspis of the first House.

The *Amplitude Ortive* of the *Ascendant*, is equal to the *Distance* of the *Azimuth* of 90° , from the *Meridian*, wherefore the *Cuspis* of the first *House*, or *Ascendant Degree* of the *Ecliptick*, may be found thus :

As Radius or $S. 90^\circ$,

To Sc. of the *V. Ecliptick* with the *Meridian*;

So is Tc. of the *Altitude* of *Med. Cæli*,

To T. of the *Distance* of *Med. Cæli*, from the *Ascendant Degrees*.

P R O P. XXVII.

To find the Distance of the Cuspis of any House, from Med. Cæli.

This is the *Analogy* or *Proportion*,

As Sc. of the remaining part of *V.* of the *Ecliptick* with the *Meridian*, (found by Prop. 28.)

To Sc. of the former part of the *V*;

So is T. of the *Altitude* of *Med. Cæli*,

To T. of the *Distance* of the *Cuspis* of that *House* sought, from *Med. Cæli*.

P R O P. XXVIII.

To find the parts of the Angle of the Ecliptick with the Meridian, cut with an Arch perpendicular to the Circle of any of the Houses.

The *Analogy* or *Proportion* is,

As Radius or $S. 90^\circ$,

To

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§ 2.

To Sc. *Altitude of Med. Cæli,*

So is T. of the Circle of any House with the Meridian,

To Tc. of that part of that Angle which is next the Meridian :

Then substract that part found, out of the whole Angle, for the remaining or latter part.

P R O P. XXIX.

To find the Pole's Altitude above any of the Circles of the Houses.

The *Analogy or Proportion* is,

As the Radius or S. 90° ,

To S. of V. of the Circle of the House with the Meridian : (found by the 21 Prop.)

So is the S. of the Pole's Elevation, above the Horizon of the Place,

To S. of the Altitude of the Pole, above the Circle of Position.

P R O P. XXX.

By knowing the Latitude and Longitude of any fixed Star, to find his Right Ascension and Declination.

The *Analogy or Proportion* is,

1. As Radius or S. 90° ,

To S. of the Stars Longitude from the next Equinoctial Point;

So is Tc. of the Stars Latitude,

To T. of a fourth Arch.

Which compared with the Arch of Distance betwixt the Poles of the World and the Ecliptick

K

23° ,

- § 2. $23^{\circ}, 30'$: And if the *Latitude* and *Longitude* of the *Star* be both of one Dignity, *i. e.* when the *Star* hath *North Latitude* in the six *Northern Sines*, γ , δ , Π , S , Ω , M , or *South Latitude* in the six *Southern Sines*, m , f , v , w , x , z : Then shall the Difference between this *found Arch*, and the Distance of the *Poles* be your *fifth Arch*: But if the *Latitude* and *Longitude* of the *Star* be of contrary Qualities, *i. e.* the one *Northern* and the other *Southern*; then add this *fourth Arch* to the Distance of the *Poles* $23^{\circ} 30'$, and the Sum thereof shall be your *fifth Arch*; with which,

A G A I N say,

2. As S. of the *fourth Arch*,
To S. of the *fifth Arch*;
So is T. of the *Stars Longitude*,
To T. of the *Stars Right Ascension* from the
next *Equinoctial Point*.

3. As Sc. of the *fourth Arch*,
To Sc. of the *fifth Arch*;
So is S. of the *Stars Latitude*,
To S. of the *Stars Declination*.

 I might also shew how by having the *Latitude* and *Longitude* of any two *fixed Stars*, to find their Distance: But because 'tis the very same with finding the Distance of any two Places on Earth, I refer you to the Directions of *Prop. 1, 2, and 3, of Chap. 7. ensuing* where you will see the plain Demonstration thereof.

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§ 2.

P R O P. XXXI.

By knowing the Pole's Altitude, to find when any fixed Star shall be due East or West.

This is the *Analogy* or *Proportion*,
As *Radius* or S. 90° ,
To T. of the *Stars Declination*;
So is Tc. of the *Pole*,
To Sc. of the *Stars Horary Distance* from the
Meridian.

P R O P. XXXII.

By knowing the Pole's Altitude, to find the Elevation of any fixed Star above the Horizon, being due East or West.

This is the *Analogy* or *Proportion*,
As S. of the *Pole's Altitude*,
To *Radius* or S. 90° ;
So is S. of the *Stars Declination*,
To S. of the *Stars Elevation*, above the *Horizon*, at due East or West.

P R O P. XXXIII.

To find out the Horizontal Parallax of the Moon.

The *Analogy* or *Proportion*.
As the *Moon's Distance* from the *Center* of the
Earth,
To the *Earth's Semidiameter*;
So is *Radius* or S. 90° ,
To S. of the *Moon's Horizontal Parallax* in that
Distance.

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P R O P.

P R O P. XXXIV.

*The Horizontal Parallax of the Moon being known,
to find her Parallax in any apparent Latitude.*

This is the Analogy or Proportion.

As Radius or S. 90° ,

To S. of the Moon's Altitude;

So is S. of the Moon's Horizontal Parallax,

To S. of the Parallax in that Altitude.

P R O P. XXXV.

*By knowing the Moon's Place in the Ecliptick,
(having little or no Latitude) and her Parallax
of Altitude, to find the Parallaxes of her Lon-
gitude and Latitude.*

First, If the Moon be in the 90° of the Ecliptick, she hath then no Parallax of Longitude, and the Parallax of the Latitude is the very Parallax in that Altitude.

Secondly, But if the Moon be not in the 90^th Degree of the Ecliptick, to find the Parallaxes of the Latitude and Longitude, the Analogy or Proportion is,

1. As Radius or S. 90° ,

To T. of the V. of the Ecliptick and Horizon;

So is Sc. of the Moon's Distance from the Ascendant, or Descendant deg. of the Ecliptick,

To Tc. of the Ecliptick's V, with the Azimuth of the Moon.

AGAIN

A G A I N say,

2. *As the Radius* or $S. 90^{\circ}$,
To S. of that V. found;
So is the Parallax of the Moon's Altitude,
To the Parallax of her Latitude sought.

L A S T L Y say,

3. *As the Radius* or $S. 90^{\circ} 00'$,
To Sc. of the former V. found;
So is the Parallax of the Moon's Altitude,
To the Parallax of his Longitude sought, which
 being added to the true Motion of the Moon, if
 she be on the *East* part of the 90° of the *Ecliptick*.
 Or from it to be deducted, if she be on the *West*
 part of the 90° of the *Ecliptick*.

P R O P. XXXVI.

*How by knowing the Refraction of a Star, to find
 his true Altitude.*

He that hath any Knowledge in the *Diop-
 tricks* or the Doctrine of refracted Rays, must
 needs know that because of the *Atmosphere* or
 thick *vaporous Air* that surrounds the Earth, all
 the *Cœlestial Bodies*, but particularly the Stars,
 must needs appear higher than really they are:
 And therefore to find their true Height, I have
 annexed this Table of Refractions which serve
 for the Stars only, calculated by *Tycho Brahe*,
 a Nobleman of *Denmark*, and a most famous
 Astronomer.

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A Table

Astronomical Propositions.

A Table of the Refraction of the Stars observed by Tycho Brahe.

<i>Altitude.</i>	<i>Refraction.</i>	
0°	30'	00"
1	21	30
2	15	30
3	12	30
4	11	00
5	10	00
6	9	00
7	8	15
8	6	45
9	6	00
10	5	30
11	5	00
12	4	30
13	4	00
14	3	30
15	3	00
16	2	30
17	2	00
18	1	15
19	0	30
20	0	00

The USE of which TABLE is thus.

EXAMPLE:

Suppose the *Altitude* of a *Star* were found by Observation to be 13°; the correspondent *Refraction* is 4' 00", which subtracted from 13° leaves 12°, 56', which is the true *Altitude*.

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CHAP. VII.
OF COSMOGRAPHY.

SECT. I.

COSMOGRAPHY, according to its Etymology, is a Description of the visible World, being a Science that teaches the Knowledge of the Structure or Frame of the Universe; examining into its Figure, Magnitude and Disposition, with their respective Distances, Motions, and other Properties. It hath two Branches, Astronomical and Geographical.

The *Astronomical* part is that which informs us of the Distinction and Division of the Heavens, and of the Laws and Motions of the Stars and Planets, without some previous Knowledge of which, *Geography* cannot be well understood.

Now the best *Idea* we can have of the Figure of the World is, that it is a *Sphere* or *Globe* composed of the four Elements, included within the Heaven, as an Egg within its Shell; in which Sphere each Element hath a Situation most agreeable to its Nature.

The *Water* less solid, and so more liquid is above the Earth; but by the Decree of the Omnipotent, leaving certain Places open and undiscovered for the Habitation of *Man* and other *Animals*.

§ 1.

The *Air* more rare and clear is above both, and encompassing them on all parts.

The *Fire* more subtil, light and agil, is above the *Air*, and encompassing it every where, and is nearest to the Heaven.

The *Heaven* or *Firmament* environing the whole Body of Matter unknown to us, transparent, exquisite, incorruptible and unchangeable; which by reason of its Perfection, perpetual Motion, and by the Ornament of the Stars there placed, may be termed Divine Matter.

Now each particular Element hath a Quality predominant in it, and peculiarly natural to it; as the Earth is dry, Water-cold, Air humid, Fire-hot.

From all which, how ought we to admire the infinite Wisdom of the Great *God*, who to preserve a Concord among Contraries, hath placed a Moderator between two different Qualities, of whose Nature both the one and the other should participate: For that between the dry and the moist the Cold is placed, and between the Cold and the Hot the Humid?

So that each Element favours of the Quality of its Neighbouring Element; the Earth cold and dry, the Water cold and moist, the Air hot and moist, the Fire hot and dry.

But leaving this, which is more the Business of a *Philosopher*, I proceed more strictly into the Astronomical part; which informs us of the Description of the Heavens and Laws of the Stars, without some previous Knowledge of which (as I said before) the right Understanding of *Geography* cannot be attain'd: For we cannot comprehend why in one place the Days

are

are more long or short than in another, and in some Countries equal to the Night; and which is more to be admir'd, at certain Seasons equal throughout the World, and why the Qualities of the Winds are different according to the various Points of the Compass from whence they proceed.

And first of the Heaven: It is found by Experience there are many Heavens, one including another, and by Consequence one larger than another, in Number no less than *Eight*; of which, more in the following Discourse. Some are of Opinion there are *Nine*, others *Ten*, beginning at the lowest, according to the Order of the Seven Planets or wandring Stars.

The Heaven of the *Moon*, of *Mercury*, *Venus*, the *Sun*, of *Mars*, *Jupiter* and *Saturn*; the Heaven where the fixed Stars, the Twelve Signs, and the Poles are: And lastly, that which is called the *Primum Mobile* and *Emperique* Heaven, the Principal Habitation of God; for it is said in the *Scripture*, That *Heaven is my Seat*, &c.

The Heaven which environs the others beneath and within it, carried by the Motion of the *Primum Mobile*, moves according to the apparent Motion of the Sun, Moon and Stars, which is from East to West, as is very evident from a few Hours Observation in the Day and Night: But for the more plain Description and Division of this Heaven into certain parts, imagine a *Globe* or *Ball* between two Fingers, and that I cause it to turn without removing my Fingers, would you not allow that the two Minute Points under the Fingers are immoveable, that

§ 1. that is to say, they do not change their Place. And if you make a Line round this Ball equally distant from those two immoveable Points, will not the Ball then be divided into two equal Parts? Further, if when that is done you make two other Lines round the Ball, the one on the one side, the other on the other Side of the first Line equally distant, is it not reasonable to say, that the Space included between the two last Lines may be compared to a Girdle encompassing such Globe?

Just so is the Conception to be made of the Heavens: The two immoveable Points are the Poles, the one called the *Artick* or North, the other the *Antartick* or South; the Line dividing the Globe into two equal Parts, may be called the *Æquator*, and the two other Lines drawn on each side the *Æquator* may be termed the *Tropicks*.

And the Space on the Globe or Ball between those two last Lines, is what we call the *Torrid Zone*, in the Limits whereof the Sun is ever more confin'd, every Day making his *Tour* round the World from the East unto the West in the time of twenty four Hours; notwithstanding which Motion, it hath another Motion from one Tropick to the other, and is contrary to the former Motion, and therefore from West to East. By this Motion the Sun finisheth his Course in one Year; hence proceeds the Four Seasons, as shall be hereafter mentioned. In the whole Region of the Sun's Course rarely or never any Winds but the North and East blow, whereby frequent Showers and constant Breezes; and so renders the

the Air more temperate than its generally thought to be. This done, let there be drawn upon the Globe or Ball two Lines round, the one between one of the Poles and one of the *Tropicks*, and the other between the other Pole and the other *Tropick* of the like Distance as the first, which is 23 Deg. $\frac{1}{2}$ from each Pole; in such manner, that besides the *Torrid Zone*, the Globe shall be divided into four, which with the *Torrid Zone* makes up five *Zones*: Whereof the two next the Poles are *frozen* and extremely cold, the Sun never going beyond the *Tropicks*, that is to say, out of the *Torrid Zone* towards the Poles.

And those two *Zones* which are between the frozen, on the one side, and the other of the *Torrid*, are the *temperate Zones*; under which those parts of the *Earth* are most commodious for the Habitation of living Creatures: For the same Division is conceived to be made upon the Earth as was in the Heaven.

Therefore granting there are five like *Zones* upon the *Earth*, those which are under the Cold shall be cold, those under the Temperate temperate, and those under the *Torrid Zone*, extremely hot.

It is also observable, that the half of the Globe from the *Æquator* to the North Pole is called the North *Hemisphere*, and the other Moiety towards the South is called the Southern *Hemisphere*.

Having divided the *Sphere* into five Parts, as naturally it is, it remains that the other necessary Circles be described; and there is a principal one in the eighth *Sphere*, which is
more

§ 1.

more demonstrable upon a Cœlestial Globe, being somewhat difficult to be comprehended only by the Imagination. However, it is a Circle as large as one of the *Zones*, but obliquely set upon the Torrid Zone, cutting it in certain Places, exceeding the Limits of the Tropicks, and crossing in *Byas* (that is, obliquely) the *Æquator*: And this is what we call the *Zodiack*, where the Twelve Signs are; so called, by reason those Signs by the Position of the Stars make certain Figures of *Animals*, and likewise of the Temper of those *Animals* whose Names they bear. This Circle is divided into twelve Parts, the first whereof is *Aries* ♈, the next *Taurus* ♉, *Gemini* ♊, *Cancer* ♋, *Leo* ♌, *Virgo* ♍, *Libra* ♎, *Scorpio* ♏, *Sagittarius* ♐, *Capricornus* ♑, *Aquarius* ♒, *Pisces* ♓.

Now as to our Imagination, a Line passeth thro' the Torrid Zone called the *Æquator*, so through the middle of the *Zodiack* passeth a Line called the *Ecliptick*: For that when the Moon is in Opposition to the Sun in this Line, it causeth an *Eclipse*; of which, more in the following Discourse. And this Line never exceeds the Bounds of the *Tropicks*, there are other necessary Circles, as the *Horizon*, *Meridian*, &c. which shall be described before the end of this Treatise. But to proceed to the Planetary Orbs or Heavens.

The whole Heaven of the Planets turn under the *Zodiack* from the East to the West, making the *Tour* of the World in the Space of twenty four Hours; by reason of their being carried by the Motion of the *Primum Mobile*, and this Motion is called a *rapid Motion*: For they

They have a contrary Motion, which is natural to them, which is from the West to the East, which they continue notwithstanding the other Motion, as we have already said the Sun hath, and as Experience shews us; and as we shall make more clear by the Example of the two most apparent Planets, *viz.* the Sun and the Moon.

Manifest it is, when it is new Moon, she being then near the Sun, that she passeth with him towards the West, according to the rapid Motion, making her Tour in twenty four Hours; and proportionable to her Increase, removes from the Sun towards the East, and thus by little and little gets so far, that the fourteenth Day when it is full Moon, she is in the East at the time of Sun-set, which is made by the Motion proper and natural to her. By this Motion the Planets pass through all the Signs of the *Zodiack* one after another; and tho' by Experience we know this Motion to be contrary to the first, it is no wonder that for divers Causes they may at the same time have divers Motions, as on divers other Occasions: For if I am upon the Water in the Forecastle of a Ship sailing to the West, though I am carried with the Ship from the East, it does not hinder me to walk to the Stern towards the East: So that the Planets have a Motion proper, which is from the West to the East, while they are making the Tour of the World; and departing from a certain Point in the *Zodiack*, in a certain time they return to the same Point again: But the one sooner the other later, according to the Largeness of the Orbs they move in, or
by

§ 1.

by the Will of the God of Wonders, it is not therefore without Cause that *David* said, *The Heavens declare the Glory of God, and the Firmament sheweth his handy Work.*

It is also found by Experience, that the *Moon* which is in the lowest Sphere, and consequently hath a less Circle to make, finisheth her Course in twenty eight Days; *Mercury* which is above her in eighty eight Days, *Venus* in two hundred and twenty five Days, the *Sun* in three hundred sixty five Days and one Quarter, *Mars* in two Years, *Jupiter* in twelve, *Saturn* in thirty, and the *Eighth Heaven* in thirty six thousand Years; that is to say, every hundred Years one Degree.

And this Difference of Motion among the Planets, one advancing faster than another, is the Cause of the Conjunctions, Oppositions and Aspects of the Planets, which vary the Disposition of the Air: And from this Diversity of Motion, and from their Passage through the twelve Signs in the *Zodiack*, *Prognosticators* derive their *Predictions* of the Dispositions of Seasons of *Heat, Cold, Rain, Storms, Dearth, or Sicknes*; all which would be tolerable, did they not intermeddle with what depends not upon the Influences of the Stars. But as to that part which is called *Judiciary*, in regard it requires an exact Supputation of Degrees, Hours, and Minutes, being not necessary to such as are employ'd in Matters of great Moment, we leave it to the Learned, who profess it, and content our selves with their *Ephemerides*, a Work almost *Divine*: And it is sufficient Occasion for us to admire the wonderful Works of

of our *Creator*; how that by the Course of the Sun in the *Zodiack* is caused the four Seasons in the Year, viz. *Spring*, *Summer*, *Autumn*, *Winter*; and how that by the same Course the Moon increaseth and decreaseth, and how the *Eclipses* both of the one and the other doth happen. *Moses* in his Account of the Creation of the World, mentions only those two *Luminaries* which he calls the greater and the lesser Light.

So that the Sun by its proper Motion finisheth its Course in the *Zodiack* in one Year, or three hundred sixty five Days and one Quarter, and when he enters into the Sign *Aries*, which is part of the *Æquator*, then it is the beginning of *Spring*; and passing through *Taurus* and *Gemini*, draws near to the Point which is the highest, from us called *Cancer*, which is that Point or Part through which passeth the *Summer Tropick*: And being there the highest, and consequently the strongest, it causeth great Heat and long Days; of which more when I come to the Description of the *Horizon*. And continuing from *Cancer* through *Leo* and *Virgo*, he passeth into *Libra*, which is the other Sign answerable to *Aries* in the *Æquator*; and then begins *Autumn*, being near about the middle of *September*, and from *Libra* through *Scorpio* and *Sagittarius* enters into *Capricorn*, through the beginning whereof passeth the Northern *Tropick*: And then begins *Winter*, which continues till he be got through *Aquarius* and *Pisces*, and then he comes again into *Aries*; from whence he sets out, continuing constantly the same Course, which he finisheth, as hath been

§ 1. been already said, in one Year: And Astronomers have found by long Experience, that in one Year there are three hundred sixty five Revolutions, and one Quarter of the rapid Motion of the first Heaven, which are so many Days: For we call that Space of time a natural Day, which is between the rising of the Sun and the succeeding rising of the Sun, or from one Noon to the Noon following; and from this Quarter, which is six Hours, is made one Day every fourth Year, that the Account of the Year might agree with the Course of the Sun. This is attributed to *Cæsar* the Dictator; but in Reality, a Tropical Year is not three hundred sixty five Days and six Hours, but five Hours and forty nine Minutes: So that every Year there is wanting eleven Minutes to make up the six Hours.

NOTE, The Observation of *Cælestial Bodies* are the surest *Marks* and *Measures* of Time.

The Moon also goes on in the same Course through the twelve Signs in the Zodiack, without making Distinction of Seasons; but in making that Course, she shews her self to us in four different Shapes or Forms: For being under the Sun in the same Sign, which Sign ever it is, she is called New; and making her Progress towards the East in the Space of eight and twenty Days, in seven Days time she passeth through three Signs of the Zodiack, and therefore is removed from the Sun the fourth part of the Zodiack, and then it is increasing: And when removed from the Sun six Signs, that is to say half the Zodiack, she is in the full, and is in the East at the very Hour the Sun sets; and

and passing farther, when she hath gone through three parts of the *Zodiack*, which is Nine Signs, then she appears to be in her Decrease, or Last Quarter: And when she hath passed through the Twelve Signs, and comes again under the Sun, she is then again at New: And this is her perpetual and constant Course.

To the more plain understanding how that, in her Course, her Clearness Encreaseth and Decreaseth; first consider, That the *Moon* is a Body Round, Opaque and Globical; that is to say, not Transparent, borrowing her Light from the *Sun*, which, by reason of the Roundness and Opacity of the *Moon*, cannot Illuminate her on every Side; because all Rays of Light proceed in a direct Line: Insomuch, as hath been already shewn, when she is in Conjunction with the *Sun*, we see nothing of her Clearness; in regard the *Sun* being higher than her, and he Illuminating the Upper-part only: For her Body being Round, and not Transparent, the *Sun* can Illuminate no more than one half thereof, or something more; and therefore then we see not any thing of her Clearness: But removing from the *Sun*, that half which was illuminated begins to appear on the West-part thereof, the *Moon* tending towards the East: And this is what we call *Encreasing*, with regard to us, in proportion to her Distance from the *Sun*; and when removed the Distance of the half of Heaven, we see the whole part which the *Sun* illuminates; and thence approaching the *Sun* on the other side, she seems to Diminish, because the Shining-part seems to

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turn

§ I. turn its Countenance, and, in proportion to her drawing near to the *Sun*, we are deprived of the sight of that Clearness. And, as hath been already said, in divers times all the other Planets do the same; save only, that we cannot so sensibly observe either the *Diminution* or *Augmentation* of their Light: And, by means of the Diversity of their Motions, with the Sign wherein they are, it happens that their Influences, to which inferiour Bodies are subject, are diverse, and at the time of our Birth hath impressed in us various *Qualities* and *Temperaments*.

Now it sometimes happens that those two Great *Luminaries* are deprived of their Light; the *Moon* simply, the *Sun* only in appearance to us, which is call'd an *Eclipse*.

As to the *Sun*, his Clearness is always after the same manner, not subject to any Vicissitude, Alteration, or Change. The Eclipse of the Sun is when the Moon is Diametrically conjoin'd to the Sun, both being in the *Ecliptick-Line*, which cannot happen but when she is at New: And by reason of the Opacity and Obscurity of the Body of the *Moon*, which cannot be penetrated by the Light of the *Sun*, it obstructs our view of the Whole or Part of the Body of the *Sun*.

An *Eclipse* of the *Moon*, may be either Total or Universal, and that for divers Reasons: To the more easie understanding whereof, let us presuppose, that the *Sun* making its ordinary Tour with the Heavens round the World, drives the Shadow of the Earth into the contrary Part: So that the Night is nothing else but

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the Shadow of the Earth; and this Shadow thus turning, growing narrow towards the Heaven, not in the Form of a *Column*, but in a Round *Pyramid*, and rises up to a part of the Heavens, above the *Lunar-Heaven*: And, as hath already been affirm'd, that there is never any Eclipse of the *Sun*, but when the *Moon* is at New; so there can be no Eclipse of the *Moon*, but when she is in the Full. For while the *Moon* doth traverse the Point of the Shadow of the Earth, so us she is not visible during some space of time, sometimes more, sometimes less, according as by her Course she passeth through a broader or narrower quantity of the Shadow of the Earth; yet this doth not always happen, although every Fourteen Days she is at Full, and in Opposition to the *Sun*, being then half the Heavens distance from Him; yet she is not exactly Diametrically opposite, that being only when both are in Opposition to each other in the *Ecliptick-Line*, or very near: For she is not in the *Ecliptick* more than twice every Revolution, as the *Sun* is not in the *Æquator* more than twice in its *Annual* Course.

The Body of the *Sun* is much greater than the *Earth*, as is manifest from the following Reason. If an Opaque or Dark Body, opposite to an Illuminated Body, be greater than the Illuminated Body, it makes an infinite Shadow; that is to say, which terminates nowhere, but the more remote, the greater: As the Vulgar Experience of those who take Birds in the Night doth plainly shew, when they carry a Candle behind an Object much bigger than that Candle. And on the contrary, when

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an Illuminated Body is greater than the Dark Body which is opposite to it, the Shadow will terminate in a certain Point, and shall go on until such time as the Rays, which proceed from the Illuminated Body, assemble and set bounds to the Shadow. This is so agreeable to Reason, that it needs no other Demonstration.

It hath been here affirmed, That the Shadow of the Earth shuns the Sun, and extends it self on the side contrary to him: If the Earth were greater than the Body of the Sun, the Shadow of the Earth (which would be much larger) every Full-Moon, would *Eclipse* the Moon, which wou'd endure some Days both before and after; and this Shadow, by the same reason, wou'd darken all the Fixed and Wandring-Stars which shou'd be opposite to the Sun. And yet we see, that the Shadow does not obscure any of the Stars, and but sometimes causeth an *Eclipse* of the Moon. It may therefore be necessarily concluded, That although this Shadow reacheth the Heaven of the Moon, it reaches no farther, and moreover, that it terminates in a *Pyramid*; the Rays of the Sun, who is greater than the Earth, assembling round the Shadow.

Briefly to Conclude concerning the Magnitude of the Sun. Since the Shadow of the Earth terminates in some certain Place, and that it rises growing narrow at the end; it must be clear to a Rational Creature, That the Sun, which is an Illuminated Body, is larger than the Earth: For otherwise the Rays could not re-unite, and terminate the Shade of the Body of the Earth.

Hitherto

Hitherto we have only described the *Sphere*, that is to say, the Number of the Heavens, and shewn their Motions, and on what Side they turn, and in how long Time: We have likewise divided them into certain Parts, which are call'd *Zones*, speaking of their Qualities and Temperaments, and by which *Zones* the *Planets* move, and the reason of certain Accidents to them. And now that from thence some fruit may be drawn, to the better understanding of the Description of the *Earth*, for that all this Division of the Heaven should have place in the Description of the Earth, we must imagine, that the same Circles have place upon the *Terrestrial Globe*, that it may be better understood what Part of the Earth corresponds to each Zone. By which means, with the help of *Maps*, it may easily be known, who those are that inhabit the *Torrid*, *Temperate*, or *Frozen-Zones*, and the reason why it is more Cold or Hot in one Place than another, and what is nearer, or more remote, from the *East*, *West*, *South*, or *North*: Which could not be well understood, without a precedent Description of the *Heavens*.

To the better understanding why the Days in one Climate are Longer than in another, and why under the *Æquator* they are always Equal to the Nights, and that in *March* and *September*, they are Equal throughout the World, we must necessarily speak of the *Horizon*.

We must imagin the *Horizon*, or Limiter of our Sight, to be a Circle in the Sphere of the World: The *Latins* call it *Circulus Dividens*, because it Divides the Sphere into Two Equal

§ 1. Parts, but not like the others before-mention'd : And you are to presuppose, that in whatever Place you are, especially upon a Hill or Mountain, casting your Eye round, you see the Half of the Heaven. So that the *Horizon* is an Imaginary Circle, which Divides the Whole Heaven into Two Equal Parts ; that is to say, that which we do see, from what is not seen by us.

Considering then a *Right-Sphere* ; Suppose we are in some Part of the World under the *Torrid-Zone*, and directly under the *Æquator*, so that the *Æquator* being equally distant from the *Poles*, the South and North-Poles may be seen by us, and consequently within our *Horizon* ; that is to say, at that proper Place that separates that Part of the Heaven which we see, and is above us, from that Part of the Heavens which is below us, and which is not seen by us, in that manner which we behold the Sun, which in the Morning, at its Rising, enters into our *Horizon* : And the Line of Circumference which we may imagine to be in that Separation of the two Moieties of Heaven, is that we call the *Horizon*.

We may therefore say, That being under the *Æquator*, the Poles would be under our *Horizon*, and that the Sun continues there the space of Twelve Hours, for that it makes the whole Circle of the World in Twenty four Hours : And thence it is manifest, that the Days are Equal to the Night ; and that at Noon-tide, the Sun being in the *Æquator*, Bodies perpendicular to the *Horizon* have no Shadow : And so it is with those Persons inhabiting

§ 1.

biting that part of the Globe, having the Sun directly over their Heads, that is, in their Zenith.

Now removing out of the *Æquator*, this *Horizon* will shew us, by the Elevation of the Pole, how much our *Zenith* is distant from the *Æquator*: And, on the contrary, by the Distance from the *Æquator* to our *Zenith*, we shall know the Height of the Pole above our *Horizon*. And in regard more than Half the Heaven cannot be seen by us, the Bulk of the Earth obstructing our View, and that being under the *Æquator*, our *Zenith* is in the *Æquator*; and by this means we see the Poles on one side and the other of the *Horizon*; it cannot be otherwise, but that departing from the *Horizon* some remarkable space Northward, but that our *Zenith* changeth equally; and so much our Sight gaineth towards the North, and loseth towards the South; and hereby we lose the sight of the South-Pole, and elevate the North-Pole above our *Horizon*: And this is call'd an *Oblique-Sphere*, in regard it seems to lean on one side.

Put the case that we are in the Middle of the *Temperate-Zone*, we shall find, by the Way which the Sun shall make in our *Horizon*, that Matters will soon be changed: For the Days there will be Equal to the Nights but twice in the Year, viz. in *March* and *September*; and the remaining part they will be always Unequal, as in the beginning of *Winter* they are Short, in the beginning of *Summer* very long; and that Bodies at Mid-day shall always cast a Shadow. Which Things cannot be understood,

§ I. stood, without understanding the Nature of the *Horizon*; which cannot well be comprehended without some Instruction upon the Globe.

Take therefore a Terrestrial-Globe whereon there is a moveable Circle for the *Horizon*, and I will declare the Reason of what shall happen, placing the Circle where it ought to be for those who inhabit in the Middle of the *Northern-Temperate-Climate*, as those who inhabit in *France*, you will find, that as much as the Pole shall be elevated above the *Horizon*, so much their *Zenith* shall be distant from the *Æquator*. And that all the Lines which the Sun describes above the Horizon, by his Diurnal Motion, shall be Unequal, except it be when the Sun is in the *Æquator*, and that the Shortest Line of all is the *Southern-Tropick*, which we have already said to be the entrance into *Capricorn*, and the beginning of Winter; and the Longest Line of the *Northern-Tropick*, which is the entrance of the Sun into *Cancer*, and the beginning of Summer.

And when the Days are Equal to the Nights, in the Middle of the Temperate-Zone, the Sun is in the *Æquator*; which is wonderful: The reason is, for that although the *Horizon* rises or falls with regard to the Poles, it notwithstanding cuts the *Æquator* in Equal Parts; for all the Circles which divide a Globe into Equal Parts, whether they cut them Obliquely, or at Right-Angles, they can no other way intersect but in Equal Parts. Now the knowing how much our *Zenith* is distant from the *Æquator*, is the knowing how much our Sphere

is Oblique ; which the *Astrolabe* shews us in two manners, addressing our selves to the Pole elevated above the Horizon, or to the Sun at the Hour of Noon, adding or diminishing according to its Declination. To the understanding whereof, there will be some occasion of Experience ; which will not be difficult, after so short and easie a Declaration.

The Reason is, The Inhabitants under the Pole have the *Æquator* for their *Horizon* ; and hence it is that the half of the *Zodiack* always appears to them above the *Horizon* : And therefore such as inhabit under the *Artick-Pole* see the Sun from the Month of *March* to the Month of *September*, and the other Six Months it is out of their sight ; as more evidently may be demonstrated either by a *Cælestial* or *Terrestrial Globe*. For you will find, if the *Æquator* be the *Horizon*, we cannot see the Sun but when it is above the *Horizon*. The like in the other Hemisphere ; whence those that inhabit under either *Pole*, their Whole Year is but One Day and Night.

The *Antipodes* are those whose Feet (the Earth interposing) are opposite to ours : The Inhabitants under the *Antartick-Pole*, are the *Antipodes* to those under the *Artick*.

In Answer to a Vulgar Scruple. There is no more Danger of Falling Down, than that the Earth should be deprived of its Natural Situation : The Heavens encompass the Earth ; and our Bodies being Terrestrial, they adhere to the Earth ; for all things of Ponderosity tend downwards ; as a Bullet forc'd up perpendicularly, which, notwithstanding the Violence of
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§ 1. the Powder, at length tends to the Centre of the Earth. And though the Air, by reason of its Rarity, yields to any violent Motion, it is not a Consequence that it should give place to the Globe of the Earth ; for each Element preserves its Station entirely, and can be in no other Place ; so that it yields to no other Element, either in general or particular, without some Force ; as may be proved by the Air, which though it be moveable, doth always more or less resist any other Body : Experimented by any who run against the Wind ; and likewise by that of an empty Vessel's being plung'd in Water, which being full of Air, (for there is no Vacuity in Nature) and having a narrow Mouth, the Water, the Element of more gravity, tending downwards, to possess its natural station, the Air fallies out, and the Opposition between them, puts the Water into a violent Motion, with some Noise.

The Natural Cause of the Earth's continuing to be the Centre of the World, is its Gravity ; from which station it naturally cannot be removed ; for then, contrary to its Nature, it should ascend.

All Ponderous things tend to the Centre of the Earth : And it is demonstrable, that if a Hole were made in the Earth from one part to the other, and some weighty thing, as a Leaden-Bullet, should fall into that perforated part, it would have its Motion no farther than the Centre, not beyond it, for then it would ascend.

But to return to the *Antipodes*. The *Antipodes* are those who are diametrically opposite to

to us, and distant from us one half of the Terrestrial-Globe. Imagine then a Line passing from our *Zenith* through our Head, and through the Centre of the Earth to the other side: The Point where this Line goes out at is the *Antipodes*. So that to find this upon the Terrestrial-Globe, first observe in what *Parallel*, that is to say, what Distance from the *Æquator* we are, and then observing where the *Meridian* doth cross the *Parallel*, which answers to ours in the other *Hemisphere*, under our *Horizon*, and in that Part shall be the *Antipodes*. But this is common. And thus you will find that they are contrary to us; when it is Day with them, it will be Night with us; Winter with them, Summer with us; when the Days are Short with them, they are Long with us: And still they have their Seasons after the same manner as we have ours; because being in the same *Parallel* with us, they are as much distant from the *Æquator* as we are.

Thus much of COSMOGRAPHY, so necessary to the due understanding of GEOGRAPHY.

S E C T. II.

Of GEOGRAPHY, with its Definitions, &c.

GEOGRAPHY is an Art Mathematical, which sheweth how the Situation of Kingdoms, Provinces, Cities, Towns, Villages, Forts, Castles, Mountains, Woods, Havens, Rivers,

§ 2. Rivers, Creeks, &c. being on the Surface of the Terrestrial-Globe, may be Described, and Designed, in Commensuration analogical to Nature and Verity, and most aptly to our View may be Represented: That is, It is a Description of all the known Earth, imitated by Writing and Delineation; with all other things belonging thereunto.

I shall not here insist on the Names and Description of Kingdoms, Provinces, Cities, Villages, &c. nor of their Manners, Customs, Laws, Commodities, &c. but shall only give you some general Definitions of the Distinction and Denomination of several Parts of the Earth and Water, with some Problems for finding the Distance of any two Places on the Terrestrial-Globe.

DEFINITIONS.

1. The *Globe of the Earth*, is a Spherical Body, compos'd of Earth and Water, and is divided into *Continents, Islands* and *Seas*.

2. A *Continent*, is a great quantity of Land not separated, interlac'd or divided by the Sea, wherein are *Kingdoms, Principalities* and *Nations*; as EUROPE, ASIA and AFRICA, are one *Continent*; and AMERICA is another.

3. An *Island*, is such a Part of the Earth that is environ'd round with Water on every side; as the *Isle of Great-Britain, Java, Wight, &c.*

4. A *Peninsula*, is such a Tract of Land which being almost cut off from the Main-Land, and encompassed round with Water, yet nevertheless is joined unto the firm Land, by some little *Isthmus*; as *Peloponnesus, Peruviana, Taurica, Cymtrica, and Morea* in the *Levant*.

5. An

5. An *Isthmus*, is a little narrow Neck of Land which joineth the *Peninsula* unto the *Continent*.

§ 2.

6. A *Promontory*, is some high Mountain, which shooteth it self into the Sea, the utmost end of which is call'd a *Cape*; as *Cape-Boon*, *Esperance*, *Cape de Verde*, and *Cape de Coquibocao*.

7. The *Ocean*, is a general Collection of Waters, which environeth the World on every side, and produceth *Seas*, *Straits*, *Bays*, *Lakes* and *Rivers*: Of which, and other Waters, *Ovid* thus speaks in his *Metamorphosis*:

*Tum Freta diffudit, rapidisque tumescere ventis
Jussit, & ambita circumdare littora terra.*

" He spread the Seas, which then he did command
" To swell with Winds, and compass round the Land.

8. The *Sea*, is part of the *Ocean*, to which we cannot come but through some *Strait*; as the *Mediterranean*, or *Baltick-Sea*.

9. A *Strait*, is a part of the *Ocean* restrained within narrow bounds, yet openeth a way to the *Sea*; as, the *Straits* of *Gibraltar*, *Hellepont*, &c.

10. A *Creek*, is a crooked Shore, thrusting, as it were, two Arms forth to hold the *Sea*; as, the *Adriatick*, *Persian* and *Corinthian-Creeks*; from whence are produced *Rivers*, *Brooks* and *Fountains*; which are engender'd of *Congealed Air* in the *Earth's Concavity*, and seconded by *Sea-water* creeping through the hidden *Cranies* of the *Earth*.

11. A *Bay*, is a great In-let of Land; as, the *Bays* of *Mexico* and *Biscay*.

12. A *Gulph*, is a greater In-let of Land, and deeper than a *Bay*; as the *Gulphs* of *Venice* and *Florida*.

13. A

§ 2.

13. A *Climate*, is a certain space of Earth and Sea, included within the space of two *Parallels*; and there have been anciently accounted these seven, (*viz.*) 1. *Dia Meros*, 2. *Dia Syenes*, 3. *Dia Alexandria*, 4. *Dia Rhodes*, 5. *Dia Rhomes*, 6. *Dia Boristhenes*, and 7. *Dia Ripheos*.

14. A *Zone*, is a certain space of the Earth contained betwixt certain *Circles* of the *Sphere*, of which there are Five; (*viz.*) the *Torrid* or *Burning-Zone*, two *Temperate*, and two *Frigid* or *Frozen-Zones*.

The *Torrid-Zone*, is that which lieth on each side the *Equinoctial*, whose bounds are the two *Tropicks* of $\mathfrak{S}$ and $\mathfrak{V}$.

The two *Temperate-Zones*, are those which lie betwixt the two *Tropicks* of $\mathfrak{S}$ and $\mathfrak{V}$, and the *Polar-Circles*.

The two *Frigid-Zones* lie between the *Artick* and *Antartick-Circles*, and their respective *Poles*.

P R O P. I.

How to find the Distance of any two Cities or Places, which differ only in Latitude.

In this *Proposition* there are two *Varieties*, which are these :

1. If both the *Places* lie under one and the same *Meridian*, and on one and the same side the *Equinoctial*, either on the North or South-side thereof; then subtract the lesser *Latitude* from the greater, the *Difference* multiply by 70, (because 70 *English Miles* is a *Degree*) the *Product* is the *Distance* of the two *Places* in *English Miles*. If you multiply by 60, it gives

gives it in *Meridional Miles*, that is, such Miles § 2.
as are used in *Navigation*.

2. But if the two Places lie under one and the same *Meridian*, but the one on the South-side of the *Equinoctial*, and the other on the North-side; then add both their *Latitudes* together, and multiply their Sum, as before, it gives their Distance.

P R O P. II.

To find the Distance of any two Places, which differ only in Longitude.

There are also in this Proposition two Varieties.

1. The two Places may both lie under the *Equinoctial*, and so have no *Latitude*: And if so, subtract the lesser *Longitude* out of the greater, and convert the Remainder into Miles, by multiplying their Difference, as before; so have you the Distance of any two Places so posited. And here Note, That if the Difference of *Longitude* be above 180 deg. then you must use the Supplement of that to 360.

2. But if the two Places differ only in *Longitude*, and lie not under the *Equinoctial*, but under some other *Intermediate Parallel of Latitude*, between the *Equinoctial* and one of the Poles. Then to find their Distance, this is the Analogy or Proportion:

As Radius or S. 90°,

To Sc. of the Latitude;

So is S of $\frac{1}{2}$ X of Longitude,

To S. of $\frac{1}{2}$ their Distance: Which being Doubled, and converted into Miles, gives the required Distance.

NOTE,

- § 2. NOTE, We compare the *Sine* of half the *Difference of Longitude*, by reason the whole *Difference of Longitude* may be above 90 Deg. of which there is no *Sine*.

P R O P. III.

To find the Distance of any two Places, which differ both in Latitude and Longitude.

In this Proposition, three Varieties do present themselves to our View.

1. One of the Places may lie under the *Equinoctial* and have no *Latitude*, and the other under some *Parallel of Latitude* between the *Equinoctial* and one of the Poles. In such case, observe this *Analogy or Proportion* :

As *Radius* or *S. of 90°*,
To *Sc. of their X. of Longitude* ;
So is *Sc. of the given Latitude*,
To *Sc. of their Distance required*.

2. But if both the Places proposed shall be without the *Equinoctial*, but on the one side, either both towards the *North*, or both towards the *South*, then their Distance may be found, by this *Analogy or Proportion* :

As *Radius* or *S. 90°*,
To *Sc. of their X of Longitude* ;
So is *Tc. of the greater Latitude*,
To the *T. of a fourth Arch* : Which substracted from the *Complement* of the lesser *Latitude*, the Remainder must be the *fifth Arch*. Then say,
As *Sc. of the fourth Arch*,
To *Sc. of the fifth Arch* ;
So is *S. of the greater Latitudes*,
To *Sc. of the Distance of the two proposed Places*.

3. The

3. The two Places propounded may be so situated, that one of them may lie on the *North*, and the other on the *South*-side the *Equinoctial*; the Distance of Places so situated may be obtained by this *Analogy* or *Proportion* :

§ 2.

As Radius or S. 90°,
To Sc. of X. of Longitude ;
So is Tc. of the greater Latitude,
To T. of a fourth Arch : Which being subtracted out of the Sum of the other *Latitude*, and the *Radius* or 90°, the Remainder is a *fifth Arch*. Then say,

As Sc. of the fourth Arch,
To Sc. of the fifth Arch ;
So is S. of the Latitude first taken,
To Sc. of the Distance required.

These are all the Varieties of the Positions of Places on the *Terrestrial-Globe* : For if the Distance of any two Places be required, they must fall under one or other of these Varieties, and may be obtained by one or other of the *Proportions*, mention'd in the three foregoing *Propositions*.

Also, if you know the *Latitude* and *Longitude* of any two *Fixed-Stars*, or their *Right Ascension* and *Declination* ; then, by these Rules, their Distance may be found ; which is of good Use in *Astronomy*. It may also be applied to *Circular Sailing* ; of all other ways, the most perfect : Which is treated of in its due Place.

CHAP. VIII. Of NAVIGATION.

NAVIGATION, *so call'd from Navis a Ship, is an Art Mathematical; which sheweth how, by the shortest good Way, by the aptest Direction, and in the shortest Time, to conduct a Ship from any one Place unto any other Place assigned: It hath been highly esteemed by the Ancients: It is the Glory, Beauty, Bulwark, Wall and Wealth of the English Nation, and the Bridge that joins it to the Universe. Navigation is commonly divided into Three sorts of Sailing; viz. Plain-Sailing, Mercator's-Sailing, and Circular-Sailing: Of all which Three Parts, I shall treat in their Order.*

S E C T. I.

Of Plain-Sailing, or Sailing by the Plain-Chart.

§ I. **P**lain-Sailing, or Sailing by the Plain-Chart, is the plainest way, and the Foundation of all the rest: And although the Ground and Projection of the Plain-Chart is erroneous; yet is it more facile to the Learner, and may serve indifferently near the *Equinoctial*; because there the *Degrees of Longitude*, as well

Of Navigation.

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as the *Degrees* of *Latitude*, are Equal: Each Degree being divided into 60 *Minutes* or *Miles*, though they are somewhat more than *English Miles*, each *Minute* or *Mile* containing about 6000 Feet.

§ 1.

PROP. I.

The Rhumb, and Distance sailed thereon, being given; to find the Difference of Latitude, and the Departure from the Meridian.

The Doctrine of Right lined Triangles, both Right, and Oblique-Angled, applied to Propositions of Plain-Sailing.

Admit a Ship sails N. W. by N. 372' or Miles, or 124 Leagues; I demand her Difference of Latitude, and Departure from the Meridian?

In the Triangle ABC, the Hypotenuse AC representeth the Distance sailed, or Rhumb-line, BC the Departure from the Meridian, and AB the Difference of Latitude.

Fig. 44.

1. To find which, say,
As Radius or S. 90° ,
To Log. Distance sailed 372';
So is Sc. V. A. of the Course $56^{\circ} 15'$,
To Log. cr. AB $309\frac{3}{4}$ Minutes: Which being divided by 60', giveth $5^{\circ} 9' 18''$ for the Difference of Latitude.

2. To find the Departure from the Meridian, say, Fig 44.

As Radius or S 90° ,
To Log. Rhumb-line AC 372';
So is S. of V. of the Course A $33^{\circ} 45'$,
To Log. cr. BC $206\frac{6}{8}$ Minutes, the Departure from the Meridian: Which divided by 60, giveth $3^{\circ} 26' 36''$ for the Difference of Longitude.

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NOTE,

§ 1.

NOTE, That by this *Proposition*, you may keep an Account how much you have failed either *East* or *West*, *North* or *South*: The first of these is call'd *Northing* or *Southing*, the second *Easting* or *Westing*.

P R O P. II.

By the *Rhumb* and *Difference of Latitude* given; to find the *Distance*, and the *Departure* from the *Meridian*.

Admit a Ship sail N. W. by W. until her *Difference of Latitude* be $309\frac{3}{10}$ Minutes; I demand her *Distance* failed, and her *Departure* from the *Meridian*?

1. To find the *Distance*, say,
 Fig. 41. As Sc. of *V.* of the Course A $56^{\circ} 15'$,
 To Log. cr. AB the X. of Lat. $309\frac{3}{10}$ Minutes;
 So is the *Radius* or S 90° ,
 To Log. AC 372' the *Distance* failed.

2. For the *Departure*, say,
 Fig. 41. As Sc. of A *V.* of the Course $56^{\circ} 15'$,
 To Log. cr. AB X. of Lat. $309\frac{3}{10}$ Minutes;
 So is S. of *V.* of the Course A $33^{\circ} 45'$,
 To Log: cr. AB $206\frac{6}{10}$ Minutes, the *Departure* required.

By the help of this *Proposition*, when your *Latitude*, by *Observation*, doth not agree with your *Dead-Reckoning*, (kept by the former *Proposition*;) Then, according to this Rule, you may make your *Way* fail'd agree with your *observed Latitude*, and so correct your Account or *Dead-Reckoning*: For the *observed Latitude* is most to be rely'd on.

P R O P.

P R O P. III.

By knowing the Distance of the Meridians of two Places, and their Difference of Latitude; to find the Rhumb, and Distance.

Admit A to represent the Lizard, AB the Parallel thereof, C St. Mary's Island, (being one of the Azores) and CB the Meridian thereof.

In the Triangle ABC there is given the side AB 816 min. the Distance of the Lizard from the Meridian of St. Mary's, and the side CB Fig. 45. their Difference of Latitude 768 min. I demand the Rhumb, i. e. the Angle at C, and the Distance of the Lizard from St. Mary's?

1. For the Rhumb or Angle at C, say,

As Log. cr. CB 768',

To Radius or S. 90° ;

So is Log. cr. AB 816',

To T. of the Rhumb, or Angle at C $46^{\circ} 44'$, and is from the Lizard unto St. Mary's to the fourth Rhumb of the Meridian, and $1^{\circ} 44'$ more, viz. S. W. and $1^{\circ} 44'$ Westerly, or from Saint Mary's to the Lizard N. E. and $1^{\circ} 44'$ Easterly: And thus it shall be by the Plain-Chart.

2. For their Distance AC, say,

As S. Rhumb or V. at C $46^{\circ} 44'$,

To Log. cr. AB 816' ;

So is Radius or S. 90° ,

To Log. Hypoth. AC $1120' \frac{1}{16}$, which is the Fig. 45. Distance of the Lizard unto St. Mary's Island, and such should be the Distance by the Plain-Chart.

§ I.

P R O P. IV.

Admit two Ships to set sail from one Port, one Ship sails W. S. W. $40'$, the other W. by N. so far until she finds the first Ship to bear from her S. E. by E. I demand the second Ship's Distance from the Port, and their Distance asunder?

In the Triangle ADE, let A represent the Port, AD the W. S. W. Course, and AE the Course W. by N.

1. To find the second Ship's Distance from the Port, say,

Fig. 46. As S. of V. at E $22^\circ 30'$,
To Log. cr. AD $40'$;
So is S. of V. at D $123^\circ 45'$,
To Log. cr. AE $86\frac{28}{100}$ min. which is the Distance required.

2. To find the two Ships their Distance asunder, say,

Fig. 46. As S. of V. at E $22^\circ 30'$,
To Log. cr. AD $40'$;
So is S. of V. at A $33^\circ 45'$,
To Log. cr. DE $58\frac{12}{100}$ min. which is the Distance required.

P R O P. V.

Two Ships set sail from two Ports which lie N. and S. of each other, the one sails from the Northermost Port $72\frac{29}{100}$, and then meets the other Ship, which came from the Southermost Port, on a N. W. Course, and had sailed from thence $56\frac{80}{100}$; I demand the Rhumb on which the first Ship made her Way, and also the Distance between the two Ports?

In the *Triangle* ADE, let A be the Southernmost Port, AD the Course and Way of the second Ship N. W. $56^{\frac{8}{100}}$, let E be the Northernmost Port, ED the Course and Way of the other Ship $72^{\frac{29}{100}}$, and D the Place where they both meet. § 1.

1. To find the *Rhumb* on which the first Ship sailed, say,

As Log. cr. DE $72^{\frac{29}{100}}$,
To S. of V. at A $45^{\circ} 00'$;
So is Log. cr. DA $56^{\frac{8}{100}}$,
To S. of V. at E $33^{\circ} 45'$: Which sheweth the Course of the first Ship to be S. W. by S.

Fig. 47.

2. To find the Distance between the two Ports A and E, say,

As S. of V. at A. $45^{\circ} 00'$,
To Log. cr. DE $72^{\frac{29}{100}}$;
So is S. of V. at D $101^{\circ} 15'$,
To Log. cr. EA $100'$, which is the required Distance.

Fig. 47.

P R O P. VI.

Admit a Ship coming off the Main-Ocean, and I had sight of a Promontory or Cape, by which it is my desire to sail; I find it to bear from me S. S. E. and distant, by Estimation, 33' or Miles: But keeping still on my Course S. until the Evening, having sailed 36' or Miles; I wou'd then know how the Cape bears, and its Distance from the Ship?

In the *Triangle* ADE, Admit that at A, I do observe the Cape D to bear from me S. S. E. $33'$, and having sailed from A to E $36'$ South; I desire to know its Distance and Bearing? In

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the

§ 1. the Triangle there is therefore given AD 33',
AE 36', and the Angle at A $22^{\circ} 30'$.
Fig. 48.

1. To find the Angle at E, say,
As Z. cr^s AE, and AD 69',
To X. cr^s AD, and AE 36';
So is T. $\frac{1}{2}$ VV unknown D and E $78^{\circ} 45'$,
To the T. of $12^{\circ} 20'$: Which taken from $78^{\circ} 45'$,
leaves the Angle at E $66^{\circ} 25'$: So that the
Cape D then bears from me E.N.E. and $01^{\circ} 05'$
Northerly.

2. To find the Distance of the Cape ED from
the Ship, say,
As S V. at E $66^{\circ} 25'$,
To Log. cr. AD 33';
So is S. V. at A $22^{\circ} 30'$,
To Log. cr. ED $13\frac{78}{100}$ Miles distant: So that the
Cape is then distant from the Ship $13\frac{78}{100}$ Miles.

P R O P. VII.

Two Ports, both lying in one Latitude, distant 64'
or Miles, the Westermost of those Ports lieth op-
posite to an Island more Northerly distant there-
from 47' or Miles, which Island is also distant
from the Eastermost Port 34' or Miles; I de-
mand the Coast from the Westermost Port to
that Island?

In the Triangle ADE, let A be the Westermost
Port, and E the Eastermost Port, distant asun-
der 64'; and let D be the Island, distant from
A 47', and from E 34'; Then is the Angle at
A required, which is the Course or Rhumb from
the Westermost Port unto the Island: To find
which, say,
Fig. 49.

As Log. cr. AE 64',
 To Log. Z. cr^s AD, and ED 81';
 So is Log. X cr^s AD, and ED 13',
 To Log of a certain Line AO $16\frac{454}{1000}$;
 Which added to AE 64', is $80\frac{454}{1000}$;
 The $\frac{1}{2}$ whereof is AB $40\frac{227}{1000}$.

Then again, say,
 As Log. cr. AD 47',
 To Radius or S 90°;
 So is Log. AB $40\frac{227}{1000}$,
 To Sc. V. at A 58° 51', that is, N. E. by E.
 2° 36' Easterly; which is the Course from
 the Westermost Port A, unto the Island D.

S E C T. II.

*Of Sailing by the True Sea-Chart, commonly
 call'd MERCATOR'S Chart.*

THE True Sea-Chart, commonly call'd § 2.

MERCATOR'S Chart *, performs the
 same Conclusions as the Plain-Chart,
 and almost as speedily, but far more
 exactly: Because all Places may be
 laid down hereon, with the same
 Truth as on the Globe it self, both
 to their Latitudes, Longitudes, Bear-
 ing and Distance from each other.

* But is indeed the
 Invention of our Learned
 Countryman Mr. Edw.
 Wright, although this
 Stranger hath almost
 got the Name and Praise
 thereof.

And here it will be necessary to have a
 TABLE of Meridional Parts, which I have
 extracted out of Mr. Wright's Tables, to every
 Tenth Minute of Latitude, accounting it in
 single Miles, or Minutes of the Equinoctial; and
 have hereunto annexed the said Table,

A TA-

§ 2.

A TABLE of Meridional Miles.

		The Minutes of each Degree.						
		0	10	20	30	40	50	
		The Meridional Miles.						
The Deg. of Lat.		0	10	20	30	40	50	
0	0	60	70	80	90	100	110	10
1	120	130	140	150	160	170		10
2	180	190	200	210	220	230		10
3	240	250	260	270	280	290		10
4	300	310	320	330	340	350		10
5	360	370	380	390	400	410		10
6	421	431	441	451	461	471		10
7	481	491	501	511	521	532		10
8	542	552	562	572	582	592		10
9	603	613	623	633	643	653		10
10	664	674	684	694	704	715		10
11	725	735	745	755	766	776		10
12	786	797	807	817	827	838		10
13	848	858	869	879	889	900		10
14	910	920	931	941	951	962		10
15	972	983	993	1004	1014	1024		10
16	1035	1045	1056	1066	1077	1087		10
17	1098	1108	1119	1129	1140	1150		10
18	1161	1172	1182	1193	1203	1214		10
19	1225	1235	1246	1257	1267	1278		11
20	1289	1299	1310	1321	1332	1342		11
21	1353	1364	1375	1386	1396	1407		11
22	1418	1429	1440	1451	1462	1473		11
23	1484	1499	1505	1516	1527	1538		11
24	1549	1561	1572	1583	1594	1605		11
25	1616	1627	1638	1649	1661	1672		11
26	1683	1694	1705	1717	1728	1738		11
27	1751	1761	1773	1785	1796	1808		11
28	1819	1830	1842	1853	1865	1867		11

A Table of Meridional Miles.

§ 2.

<i>The Minutes of each Degree.</i>							
0	10	20	30	40	50		
<i>The Meridional Miles.</i>							
30	1888	1899	1911	1923	1934	1946	12
31	1958	1969	1981	1993	2004	2016	12
32	2028	2040	2052	2063	2075	2087	12
33	2099	2111	2123	2135	2147	2159	12
34	2171	2183	2195	2207	2219	2231	12
35	2244	2256	2268	2281	2293	2305	12
36	2318	2330	2342	2355	2367	2380	12
37	2392	2405	2417	2430	2442	2455	12
38	2468	2481	2493	2506	2519	2532	13
39	2544	2557	2570	2583	2596	2609	13
40	2622	2635	2648	2662	2675	2688	13
41	2701	2714	2718	2741	2754	2768	13
42	2781	2795	2808	2822	2835	2849	13
43	2863	2876	2890	2904	2918	2931	14
44	2945	2959	2973	2987	3001	3015	14
45	3030	3044	3050	3072	3086	3101	14
46	3115	3130	3144	3159	3173	3188	14
47	3202	3217	3232	3247	3261	3276	15
48	3291	3306	3321	3336	3351	3366	15
49	3382	3397	3412	3428	3443	3459	15
50	3474	3490	3505	3521	3537	3553	16
51	3568	3584	3600	3616	3632	3649	16
52	3665	3681	3697	3714	3730	3747	16
53	3763	3780	3797	3814	3830	3847	17
54	3864	3881	3899	3916	3933	3950	17
55	3968	3985	4003	4020	4038	4056	18
56	4074	4092	4110	4128	4146	4164	19
57	4182	4201	4219	4238	4257	4275	19
58	4294	4313	4331	4351	4370	4390	20
59	4409	4428	4448	4468	4487	4507	20

§ 2.

A Table of Meridional Miles.

The Deg. of Lat.	The Minutes of each Degree.						The Difference.
	0	10	20	30	40	50	
	The Meridional Miles.						
60	4527	4547	4567	4588	4608	4629	20
61	4643	4670	4691	4711	4733	4754	21
62	4775	4796	4818	4839	4861	4883	22
63	4905	4927	4949	4972	4994	5017	23
64	5039	5062	5085	5018	5132	5155	24
65	5179	5203	5226	5250	5275	5299	25
66	5324	5348	5373	5390	5423	5449	26
67	5474	5500	5520	5552	5678	5704	27
68	5631	5658	5685	5712	5739	5767	28
69	5795	5823	6021	5879	5908	5937	30
70	5966	5996	6125	6055	6085	6115	31
71	6146	6177	6208	6239	6271	6303	33
72	6335	6368	6401	6431	6468	6501	35
73	6535	6570	6605	6640	6675	6718	37
74	6747	6783	6820	6857	6895	6933	40
75	6972	7010	7050	7089	7130	7170	43
76	7211	7253	7295	7338	7381	7424	46
77	7469	7513	7559	7605	7651	7698	50
78	7746	7795	7844	7894	7944	7996	55
79	8048	8100	8154	8209	8264	8320	60
80	8377	8435	8495	8555	8616	8678	68
81	8742	8806	8872	8939	9007	9077	77
82	9148	9221	9295	9371	9449	9528	88
83	9609	9692	9778	9865	9954	10046	105
84	10141	10238	10338	10441	10547	10656	128
85	10770	10887	11007	11133	11263	11398	165
86	11539	11686	11839	11999	12168	12344	230
87	12521	12718	12927	13150	13388	13644	386
88	13920	14221	14550	14914	15321	15783	
89	16318	16950	17726	18729	20152	22623	

The

The Use of the TABLE is demonstrated by the several Examples following, after this manner :

P R O P. I.

To find by the Table; what Meridional Parts are contained in any Difference of Latitude.

In this Proposition, three Varieties present themselves unto our View.

1. When one Place is under the *Equinoctial*, the other having *North* or *South Latitude*, his Meridional Parts corresponding, is to be esteemed for the Meridional Difference of Latitude.

EXAMPLE : One Place upon the *Equator*, the other in 27° of *North Latitude* ; What's the Meridional Difference of Latitude ?

Look in the Table, under Title *The Deg. of Lat.* and against 27 you will find 1683, the Meridional Difference of Latitude required.

2. When both Places are towards one of the Poles, then the Meridional Parts of the lesser, taken from the Meridional Parts of the greater Latitude, the Remainder is the Meridional Difference required.

EXAMPLE : Suppose one Place in 14° of *North Latitude*, the other in 23° of the same Latitude ; What's the Meridional Difference of Latitude ?

The Meridional Parts answering to 14° of Lat. is 848, and the Meridional Parts answering

§ 2. ing to 23° of Lat. is 1418 ; the Difference of these, which is 570, is the *Meridional Difference of Latitude* required.

3. When one Place hath *North*, and the other *South Latitude*, their corresponding *Meridional Parts* added together gives the *Meridional Difference of Latitude* sought. Thus having found them out, they may thus be applied.

If the *Degrees of Latitude* had been the same as in the last Example, only one *North*, the other *South*, the *Sum* of the *Meridional Parts*, viz. 2266 would be the *Meridional Difference of Latitude*.

P R O P. II.

By knowing the *Latitudes*, and the *Difference of Longitude* of any two Places ; to find the *Rhumb*, and *Distance*.

Admit there be a Port in the *Latitude* of $50^{\circ} 00'$ North, and another in the *Latitude* of $13^{\circ} 12'$ North, and their *Difference of Longitude* is $52^{\circ} 57'$ West ; I demand the *Rhumb* and *Distance* ?

Fig. 50.

In the Triangle *Abc*, let *Ab* represent the proper *Difference of Latitude*, *bc* the *Departure*, *Ac* the *Distance* sailed, *A* the *Angle* of the *Course*, *c* the *Complement* of the *Course*.

In the Triangle *ABC*, *AB* is the *Meridional Difference of Latitude*, *BC* the *Difference of Longitude*, *A* the *Angle* of the *Rhumb*, *C* the *Compl.* of the *Angle* of the *Rhumb* : These things being understood, the Work evidently appears to be the same as in *Right-Angled Plain Triangles*.

There

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There is then required, first, the *Difference of Latitude*, and this falls under the second Variety.

§ 2.

One Port lieth in Lat. $50^{\circ} 00'$ N. 3475	} M. Parts {
The other lieth in — 13 12 N. 0799	
The Merid. Diff. of Lat. — 2676	

1. To find the *Rhumb* or *Course*, say,
 As *Merid. X. Lat.* 2676',
 To *Radius* or S. 90° ;
 So is *X. of Longitude* 3177',
 To *T. of the Rhumb* $49^{\circ} 53'$: The *Course* therefore is S. W. $\frac{1}{2}$ W. &c.

2. To find the *Distance*,

Fig. 50.

Lat. $50^{\circ} 00'$
 Lat. 13 12

Proper Diff. of Lat. — 36 48 which is 2208'.

As *Sc. Course* $40^{\circ} 7'$,
 To *proper X. of Lat.* 2208' ;
 So is *Radius* or S. 90° ,
 To the *Distance* 3426 *Minutes*, as required.

P R O P. III.

By knowing the *Latitudes*, and *Distance* of two Places ; to find the *Rhumb*, and *Difference of Longitude* ?

1. To find the *Rhumb* or *Course*, say,
 As the *Distance sailed*,
 To *Radius* or S. 90° ;

So

§ 2.

*So is the X. of Latitude,
To Sc. of the Rhumb required.*

2. To find the Difference of Longitude, say,
*As Radius or S. 90° ,
To the X. of Latitude in Merid. Parts ;
So is T. of the Rhumb,
To the X. of Longitude required.*

P R O P. IV.

*By knowing the Latitude and Rhumb of two Places;
to find their Distance, and Difference of Longitude.*

1. To find the Distance, say,
*As Sc. of the Rhumb,
To the X. of Latitude ;
So is Radius or S. 90° ,
To the Distance required.*

2. To find the Difference of Longitude, say,
*As Radius or S. 90° ,
To the X. of Latitude in Merid. Parts ;
So is T. of the Rhumb,
To the X. of Longitude required.*

P R O P. V.

*By knowing the Rhumb, Difference of Longitude,
and one Latitude ; To find the other Latitude,
and the Distance.*

1. To find the other Latitude, say,
*As T. of the Rhumb,
To the X. of Longitude in Parts ;*

So

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§ 2.

*So is Radius or S. 90° ,
To the Merid. X. of Latitude required.*

2. To find the Distance, say,
*As Sc. of the Rhumb,
To the X. of Latitude ;
So is Radius or S. 90° ,
To the required Distance.*

P R O P. VI.

*By knowing the Distance, one Latitude, and Rhumb ;
to find the other Latitude, and Difference of
Longitude.*

1. To find the Difference of Latitude, say,
*As Radius or S. 90° ,
To the Distance ;
So is Sc. of the Rhumb,
To the X. of Latitude required.*

2. To find the Difference of Longitude, say,
*As Radius or S. 90° ,
To the Merid. X. of Latitude ;
So is T. of the Rhumb,
To the X. of the Longitude required.*

P R O P. VII.

*By knowing the Latitudes and Departure of two
Places ; to find their Distance, Rhumb, and
Difference of Longitude.*

*Suppose one Place in the Latitude of $50^\circ 20'$
North, the other in $47^\circ 30'$ of the same Latitude,
and their Departure 130 Miles ; What's their*

N

true

§ 2. *true Distance, Course, and Difference of Longitude?*

First, find the *common Difference of Latitude*, (by taking the *lesser Latitude* from the *greater*) which is $2^{\circ} 50'$, or *190 Miles*; and also the *Meridional Difference of Latitude*, which is *258*. Then say,

As the common Dif. of Lat. 190 Miles,

To the Meridional Dif. of Lat. 258;

So is the Departure 130 Miles,

To the Difference of Longitude 177. Which 177 divided by 60, gives $2^{\circ} 57'$, the true Difference of Longitude.

Having thus gotten the *Difference of Longitude*; the *Distance* and *Rhumb* is gotten by PROP. II.

This *Case* is very useful, (*though omitted by the former Author*) for by it we sum up each *Day's Work* when taken from the *Log-board*, and enter it into the *Journal*, in order to find the *true Longitude* and *Latitude* the *Ship* is in at any time.

S E C T. III.

Of Circular-Sailing, or Sailing by the Arch of a great Circle.

§ 3. **T**HIS is, of all other, the most exact way of Sailing, and above all other most perfect, shewing the *Nearest Way*, and *Distance* between any two Places: And although it is hardly possible

fible to keep close unto the *Arch* of a *great Circle*, yet it is of great advantage to keep conveniently near it, especially in an *East* or *West Course*. In the former *Propositions* of Sailing, we used *Meridians*, *Parallels* and *Rhumbs*, as the *Sides* of every *Triangle*, whether by the *Plain* or *Mercator's Chart*: But in *Circular Sailing* the *Rhumbs* are not used so, because they are *Heli-spherical-lines*, and not *Circles*; nor the *Parallels*, because they are not *great Circles*: Whereas the *Sides* comprehending every *Spherical Triangle* are *Arches* of *great Circles*: Therefore here we use *Arches* of the *Meridians*, of the *Equinoctial*, and of other *great Circles* described, or so imagined to be described, from one Place unto another, on the *Spherical Superficies* of the *Earth* and *Sea*.

Therefore here ariseth Two things observable: And,

1. If the two Places lie under the *Equinoctial*, then is their Position *East* and *West*; and their Distance is their *Difference of Longitude*, converted into *Miles*. Or,

2. If the two Places proposed be in one and the same *Meridian*, then is their Position *North* and *South*, and their Distance is their *Difference of Latitude* converted into *Miles*.

And thus far doth *Circular Sailing* agree with the former; their *Difference* will evidently appear, by these following *Propositions*.

P R O P. I.

Two Places, the one under the *Equinoctial*, the other in any *Latitude* given; also, their *Difference of Longitude* given; to find,

N 2

1. Their

§ 3.

1. *Their Distance in the Arch of a great Circle.*
2. *The direct Position of the first Place from the second.*
3. *And of the second Place from the first.*

Here we call the *Angles* which the *Arch* makes with the *Meridians* of the *Places* propounded, the *Angles* of the *Direct Positions* of those *Places* one from the other; because the *Arch* of a *great Circle* drawn between two *Places* is the nearest *Distance*; and the most direct *Way* of the one, to the other *Place*. Now I shall not here demonstrate it by *Schemes*, as I have done in the other two *Sections*, but shall only lay down the *Proportions*, whereby the required *Parts* may be found; and so leave the ingenious *Seaman* to practise it with *Schemes* at his leisure: And,

1. To find the Nearest *Distance* from *Place* to *Place*, in the *Arch* of a *great Circle*, say, according to the 10th *Case* of *Rectangled Spherical Triangles*,

As the Radius,
To Sc. of X. of Longitude;
So is Sc. of X. of Latitude,
To Sc. of the Distance in the Arch required.

2. For the *Direct Position*, say, by the 11th *Case*, thus:

As the Radius,
To S. of X. of Latitude;
So is Tc. of X. of Longitude,
To Tc. of V. of Position required.

3. For

Of Navigation.

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3. For the *Direct Position* of the second Place § 3.
from the first, say, by the 11th Case, thus:

As the Radius,
To S. of the X. of Longitude;
So is Tc. of X. of Latitude,
To Tc. of V. of Position required.

P R O P. II.

Two Places proposed, the one lying under the Equinoctial, the other in any Latitude given, with their Distance in a great Circle of the same Places being also known; to find,

1. Their Difference of Longitude.
2. The *Direct Position* from the first to the second Place.
3. And from the second to the first Place.

1. For their Difference of Longitude, say, by Case 12.

As Sc. of the Latitude,
To Radius;
So is Sc. of their Distance in the Arch,
To Sc. of their Difference of Longitude required.

2. Now to find the *Direct Position* from the first Place to the second, say by the 13th Case:
And, 3dly, For the *Direct Position* from the second Place to the first, say by the 14th Case of Rectangulars.

P R O P. III.

Two Places lying in one Latitude given, their Difference of Longitude being also known; to find,

N 3

1. The

§ 3.

1. The Nearest Distance of those two Places.
2. The Direct Position of one Place from the other.

The Resolution of this Proposition depends on the 9th Case of Oblique Spherical Triangles; by supposing the Oblique Triangle to be transfigured or converted into two Rectangulars, by a supposed Perpendicular: And then,

1. To find the Nearest Distance in the Arch, say, by the 8th Case of Rectangulars,
As the Radius,
To Sc. of the Latitude;
So is S. of $\frac{1}{2}$ X. of Longitude,
To S. of half the required Distance: Which doubled, giveth the Distance of the two Places in the Arches, as sought.

2. For the Direct Position, say, by the 9th Case,
As the Radius,
To S. of the Latitude;
So is T of $\frac{1}{2}$ X. of Longitude,
To Tc. of V. of Position required.

P R O P. IV.

Two Places lying both in one Latitude given, and the Nearest Distance being also known; to find,

1. Their Difference of Longitude.
2. The Direct Position of one Place from the other.

The Resolution of this Proposition falls under the 11th Case of Oblique Spherical Triangles; for

for here you have the three Sides of the Triangle given, viz. the *Arch of Distance*, and the other two Sides (are both Equal) being the *Complements* of the Places *Latitude*: And here, seeing the two *Sides* are Equal, therefore the two *Angles of Position* are also Equal. Now there is required the *Three Angles*?

1. To find their *Difference of Longitude*, add the double of the *Complement of Latitude* to the *Arch of Distance*; then from half this Sum deduct the *Arch of Distance*, and then proceed in all points as you see in *Case 11th*: So shall their *Difference of Longitude* be obtained.

2. To find their *Direct Position*:

First, To the double *Complement of Latitude* add the *Arch of Distance*, then from half that aggregate deduct the *Complement of Latitude*, and then work as before; so shall the *Direct Position* be attained.

P R O P. V.

Two Places proposed lying in one *Latitude*, and the *Distance* of those Places in their *Parallel* given; to find,

1. Their *Difference of Longitude*.
2. Their *Distance in the Arch of a great Circle*.
3. The *Direct Position* of the one from the other.

Now you must understand, that as the *Semi-diameter* of a *Parallel*, is in *Proportion* to the *Semi-diameter* of the *Equinoctial*; so is any number of *Miles* in that *Parallel*, to the *Minutes* of *Longitude* answering to those *Miles*: So that if

§ 3. we suppose the *Semi-diameter* of the *Equinoctial* to be *Radius*, then the *Semi-diameter* of any *Parallel* is the *Sign* of that *Parallel's* *Distance* from the *Pole*, that is, the *Sc.* of the *Latitude* of that *Parallel*: Therefore,

1. To find the *Difference of Longitude*, say,
As Sc. of the Latitude,
To the Radius ;
So is the Distance in that Parallel,
To the Difference of Longitude required.

2. Now the *Difference of Latitude* being obtained the *nearest Distance* may be found, as in the 3d *Prop.* foregoing: 3. So likewise may the *Angles of Position* also.

P R O P. VI.

By knowing the Nearest Distance of two Places, their Difference of Longitude, and one of their Latitudes ; to find the Direct Position thereof from the other.

This *Proposition* falls under the 1st Case of *Oblique Spherical Triangles*, and is thus resolved: Therefore,

As S. of the Distance of the two Places,
To S. of their X. of Longitude ;
So is Sc. of the Latitude of the one Place given,
To S. of the Direct Position from the other as was so required,

P R O P. VII.

By knowing the Latitudes of two Places, and likewise their Difference of Longitude ; to find,

1. The

1. The Distance in the Arch.
2. The direct Position from the first to the second Place.
3. The direct Position from the second to the first Place.
4. The Latitudes and Longitudes by which the Arch passeth.
5. The Course and Distance from Place to Place through those Latitudes and Longitudes, according to Mercator.

I shall here make use of Mr. Norwood's Example of a Voyage from the *Summer-Islands*, unto the *Lizard*: Now because the Work is various, I have therefore illustrated it with a Scheme, and shall be as brief and facile as possible. Therefore,

In the Triangle ADE, let A be the *Summer-Islands*, whose Latitude is $32^{\circ} 25'$, AD the Complement thereof $57^{\circ} 35'$, let E represent the *Lizard* whose Latitude is $50^{\circ} 00'$, and ED the Complement thereof $40^{\circ} 00'$, and let their Difference of Longitude, namely, the Angle ADE, be $70^{\circ} 00'$, now D representeth the *North-Pole*, and AE an Arch of a great Circle passing by these two Places: Now see the Operation:

Fig. 51.

1. By having the Complements of the Latitudes of the two Places, viz. AD $57^{\circ} 35'$, and ED $40^{\circ} 00'$, and their Difference of Longitude, namely, the Angle EDA, $70^{\circ} 00'$: You may find the Nearest Distance EA to be $53^{\circ} 24'$, by Case 9. § 5. Chap. 5.

2. Then having found the Nearest Distance in the Arch EA to be $53^{\circ} 24'$ (or 3204 Miles,) the Angle of Position from the *Summer-Islands* to the

Fig 51.

§ 3. the *Lizard*, namely, the *Angle* DAE, may be found by *Case* 1. § 5. *Chap.* 5. to be $48^{\circ} 48'$, that is N. E. and $03^{\circ} 48'$ Easterly.

3. And also, by the same *Case*, may the *Direct Position* from the *Lizard* to the *Summer-Islands*, namely, the *Angle* AED, be found to be $81^{\circ} 10'$, that is W. by N. and $2^{\circ} 25'$ Westerly.

Fig. 51. 4. In order to the finding the *Latitudes* and *Longitudes* by which the *Arch* passeth, first let fall the *Perpendicular* DB, so is the *Oblique Triangle* ADE converted into two *Rectangulars*, viz. ABD, and DBE: Secondly, by *Case* 8. § 4. *Chap.* 5. you may find the *Length* of the *Perpendicular* DB to be $39^{\circ} 26'$, whose *Complement* is $50^{\circ} 34'$ which is the greatest *Latitude*

Fig. 51.

BDc	48°	$31'$
BDf	38	31
BDg	28	31
BDh	18	31
BDj	08	31

by which the *Arch* ABE passeth; so the greatest *Obliquity* of the *Equinoctial* from that *Circle* is $50^{\circ} 34'$. — Thirdly, by *Case* 9. § 4. *Chap.* 5. you must find the *Vertical Angles*, viz. ADB,

and BDE, which will appear, the *Angle* ADB to be $58^{\circ} 31'$, and EDB to be $11^{\circ} 29'$: Now these things being obtained, the *Latitudes* by which the *Arch* passeth at every 10th *deg.* of *Longitude* from A, may be found, by resolving the several *Right-Angled Triangles*, viz. BDc, Bdf, &c. subtracting 10° from ADB $58^{\circ} 31'$, there remains BDc $48^{\circ} 31'$; and so for the rest, as in the *Table*. Now by knowing these *Angles* last found, and the *Perpendicular* BD before found to be $39^{\circ} 26'$, you may, by *Case* 3. § 4. *Chap.* 5. find the *Latitudes* of the several *Points* A, c, f, g, h, i, B and E, to be as in the subsequent *Table*.

5. Thus

5. Thus having found the *Latitudes* and *Longitudes* of the *Arch*, and the other required *Parts* before-mention'd; we

now come to shew, how the *Course*, and the *Distance* from Place to Place, according to *Mercator*, may be found. So to find, first the *Course* and *Distance* *Ac*, now there is given the *Latitude* of *A* $32^{\circ} 25'$,

	<i>Latitude.</i>	<i>Longitude.</i>
A	$32^{\circ} 25'$	$00^{\circ} 00'$
c	$38^{\circ} 51'$	$10^{\circ} 00'$
f	$43^{\circ} 34'$	$20^{\circ} 00'$
g	$46^{\circ} 54'$	$30^{\circ} 00'$
h	$49^{\circ} 04'$	$40^{\circ} 00'$
i	$50^{\circ} 15'$	$50^{\circ} 00'$
B	$50^{\circ} 34'$	$60^{\circ} 00'$
E	$50^{\circ} 00'$	$70^{\circ} 00'$

Fig. 51.

and of *c* $38^{\circ} 51'$, and their *Difference of Longitude* is $10^{\circ} 00'$, now the *proper Difference of Latitude* is $6^{\circ} 26'$, or $386'$, and *Meridional Difference of Latitude* is $475'$. Now knowing these things by *Prop. 2. § 2. Chap. 8.* you may find the *Course* from *A* to *c* to be *N. E. $51^{\circ} 38'$* , and the *Distance* *Ac* to be $622'$; and so those *Rules* prosecuted, will shew the *Course* and *Distance* from *c* to *f*, from *f* to *g*, from *g* to *h*, &c. So of the rest, which for brevity sake I shall omit, and leave the *Ingenious Seaman* to calculate at his *Pleasure*.

Fig. 51.

I might hereunto annex many more *Propositions of Circular-Sailing*; but because of the smallness of this *Treatise*, and that those *Propositions* already handled, being by the *Ingenious Seaman* well understood, will be sufficient to enable him to perform any other *Conclusion* in *Circular-Sailing* whatsoever, I therefore here omit, and hasten forwards unto the other *Parts* of this *Mathematical Treasury*.

§ 3.

A TABLE of Angles, which every Rhumb maketh with the Meridian.

<i>These on this side the WEST, encline towards the North-end of the Meridian.</i>	<i>Angles of Inclination with the Meridian.</i>		<i>These on this side the EAST, encline unto the North-end of the Meridian.</i>
Rhumbs.	NORTH		Rhumbs.
N. & by W.	11°	15'	N. & by E.
N. N. W.	22	30	N. N. E.
N. W. & by N.	33	45	N. E. & by N.
North-West	45	00	North-East
N. W. & by W.	56	15	N. E. & by E.
W. N. W.	67	30	E. N. E.
W. & by N.	78	45	E. & by N.
WEST	90	00	EAST
W. & by S.	78	45	E. & by S.
W. S. W.	67	30	E. S. E.
S. W. & by W.	56	15	S. E. & by E.
South-West	45	00	South-East
S. W. & by S.	33	45	S. E. & by S.
S. S. W.	22	30	S. S. E.
S. & by W.	11	15	S. & by E.
Rhumbs.	SOUTH		Rhumbs.
<i>These on this side the WEST, encline unto the South-end of the Meridian.</i>			<i>These on this side the EAST, encline towards the South-end of the Meridian.</i>

Note, That if you account in Quarter of Points, add for One Quarter 2° 48', for One Half 5° 31', for Three Quarters 8° 26', (not regarding the Seconds in Navigation.)

Fig 43.

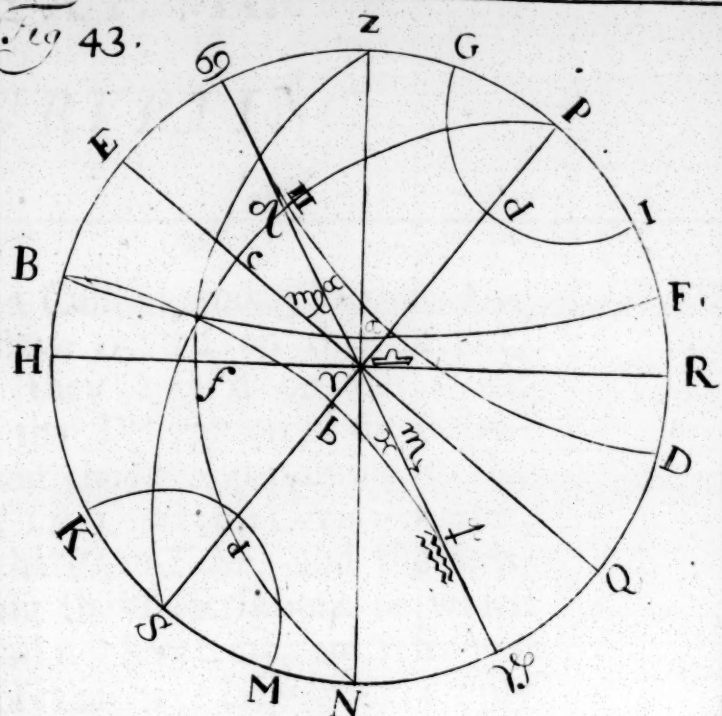


Fig: 44

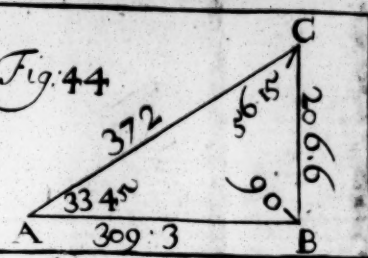
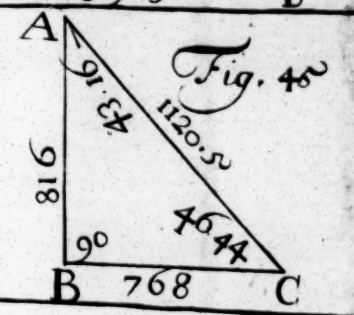


Fig. 45



D Fig: 46

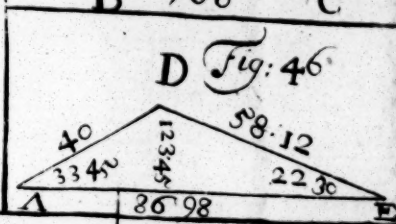


Fig: 47

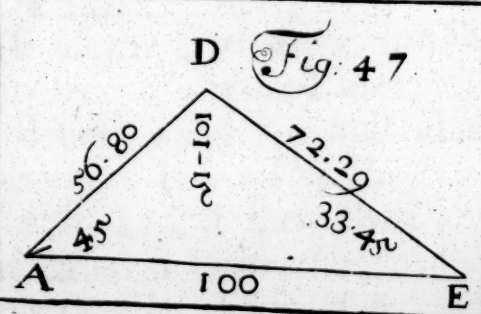


Fig: 48

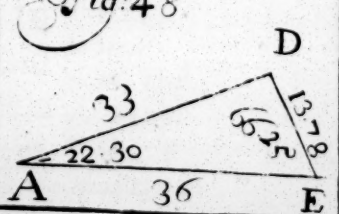


Fig: 49

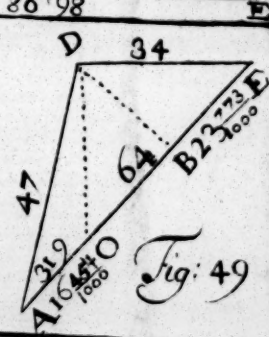


Fig: 50

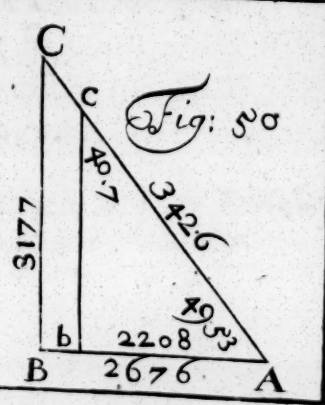


Fig: 51

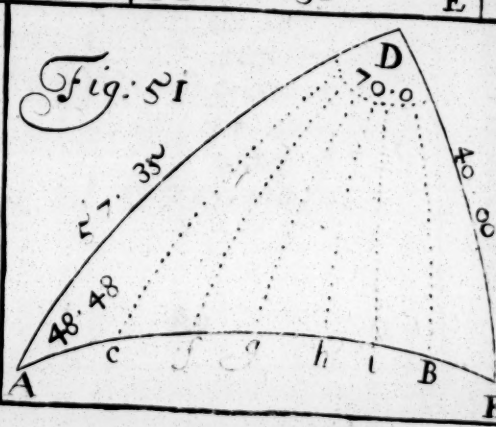


Fig: 52



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CHAP. IX.

OF SURVEYING.

IT hath been a Custom among Modern Authors that have treated on this Subject, that before they enter'd on the Work itself, to give the Description of the Instruments used in, and chiefly appertaining to the Art of SURVEYING, viz. the *Circumferentor*, the *Theodolite*, the *Plain-Table*, and the *Semi-circle*; concerning the Descriptions of which Instruments I shall not here treat, but refer you unto those Authors that have largely and amply described them. I shall in this place only demonstrate the Use of the *Semi-circle*, in taking the *Plots of Enclosures, Champain-Plains, Woods and Mountains*, divers ways^{*}; and also, in taking of *Accessible and Inaccessible Heights and Distances*: And also, I shall shew the Use of a little Instrument call'd a *Protractor*, in the delineating on Paper the *Plot of a Field, &c.* which Instrument being so commonly known, and so generally used, makes me omit the Description thereof as superfluous.

^{*} Which Instrument, and the *Plain-Table*, I esteem as the two aptest Instruments for Surveying of

Land; i. e. the *Plain-Table* for small *Enclosures*, and the *Semi-circle* for *Champain Plains, Woods and Mountains*.

As

As for your *Chain*, I wou'd have you have it made of good round *Wyre*; to contain in length four *Poles* or *Perch*, to be divided into an hundred equal Parts call'd *Links*.

And here, before we enter on the Work itself, it will be necessary to understand how, by the *Protractor*, to lay down an *Angle* of any quantity of degrees propounded, or to find the quantity of an *Angle* given.

S E C T. I.

Of the Use of the Protractor.

P R O P. I.

By the Protractor, to Protract an Angle of any quantity of degrees propounded.

- § 1. **A**N *Angle* may be laid down easily, according to the Directions of *Prop. 5. § 1. Ch. 4.* but because this is more useful in *Surveying*,
 Fig. 52. Know, that if it be required to Protract an *Angle* of 50 deg. having drawn the line *AB* at pleasure, place the Centre of your *Protractor* on *C*, and moving it by your *Protracting-Pin*, until the *Meridional-line* thereof be directly on the line *AB*, then make a Mark by the division of 50° on the *limb* of the *Protractor*, as at *D*, and draw the line *CD*, so shall the *Angle DCB* be an *Angle* of 50 degrees.

P R O P.

PROP. II.

By the Protractor given, to Measure an Angle given.

This is performed by the line of *Chords* also, according to *Prop. 6. § 1. Chap. 4.* and by the *Protractor* is found thus: Suppose *DCB* were *Fig. 52.*
an *Angle* whose quantity were desired, to find which, first the *Centre* of the *Protractor* applied into the *Angular Point C*, and its *Meridional-line* lying justly with *CB*, you shall perceive the *Point D* to touch the *limb* of the *Circle* at 50 deg. Therefore I conclude the Measure of the *Angle DCB* to be 50 degrees.

SECT. II.

Of the manifold Use of the Semi-circle, in taking the Plots of small Enclosures, Plains, Woods or Mountains, divers Ways.

PROP. I.

How to take the Plot of a Field, by the Semi-circle, at one Station taken in any part thereof, from whence all the Angles may be seen, and measuring from the Station unto every Angle thereof.

Suppose *ABCDEF* were a Field, and 'tis § 2.
required to take the Plot thereof: Having placed marks at all the Angles thereof, and Fig. 53.
made choice of your Station, which let be *K*;
at which place your Instrument, and turning it

§ 2. it about until the *Needle* hang over the *Meridian* or *North* and *South-line* of the *Chart*, then screw it fast : Then directing your sight to A you'll find the Degree out, by the *Index*, to be $40^{\circ} 15'$: Then measuring KA with your *Chain* it appears to be 5 *Chains*, and 20 *Links*, which note down in your *Field-book* : And so do by all the rest, untill you have found all the *Angles* and *Distances* from your Station K, to each respective Angle ; which finished, your *Work* will stand thus :

<i>Angles.</i>	D	M	C	L
A	40	15	5	20
B	88	00	6	10
C	130	00	5	50
D	200	00	7	00
E	250	00	5	00
F	310	00	5	20

P R O P. II.

How to delineate on Paper any Observation taken according to the Doctrine of the last Proposition

Fig. 53.

Upon your Paper draw a Line to represent the *Meridian-line*, as M, H ; then placing the Centre of your *Protractor* on the Point K, laying the *Meridian-line* of the *Protractor* on the *Meridian-line* M, H ; then seeing the Angle at A was $40^{\circ} 15'$, make a mark against $40^{\circ} 15'$ of the *Protractor*, as at A, and so do with all the other Angles, as you find them in your *Table* : Then

Then remove your *Protractor*, and draw the Lines *KA*, *KB*, &c. This done, lay down on each line his respective Measure, as it appeareth in the *Table*. Lastly, Draw the Lines *AB*, *BC*, &c. So have you on the Paper the exact Figure of the *Field*.

This Way will serve very well for any such Field, Close or Common, from one Point in which you may see all the Angles; Provided that the Hedge, Bank, or whatever Fence it be, run straight between every Angle, and not waving in and out: In such Case, this Way or Method is as good as any, if there be not too many Angles.

P R O P. III.

How, by the Semi-circle, to take the Plot of a Field, at one Station, in any Angle thereof, from whence you may view all the other Angles, by measuring from the Stationary-Angle, unto all the other Angles.

Admit *A, B, C, D, E, F, G*, to be a Field, Fig. 45.
whose Plot is required: Place your *Semi-circle* at *G*, and turning it about until the *Needle* hang over the *Meridian* or *North* and *South-line* of the *Chart*, and there screw it fast: Then direct your *Sights* to the several Angles, viz. *B, C, D*, &c. in order one after the other, and so shall each respective Angle be found, as in the subsequent *Table*: Then with your *Chain* measure from your *Stationary-Angle G*, to all the other respective Angles; which done, you have finished, and the Work standeth as in the following *Table*.

O

Angles.

<i>Angles.</i>	D	M	C	L
B	40	00	5	00
C	88	00	6	00
D	120	15	6	40
E	165	00	6	30
F	193	00	3	40
A	348	07	4	00

P R O P. IV.

How to delineate any Observation taken according to the Doctrine of the last Proposition.

Fig. 54.

Upon your Paper draw a straight line as M.N, then take a Point therein as G, to represent the *Stationary-Angle*, to which Point apply the *Centre* of your *Protractor*, (in all respects as is before taught;) then, according to the Notes in the *Table*, prick off all the *Angles*, viz. B, C, &c. according to their due quantity; then draw all the lines, viz. GB, GC, GD, &c. and on them place their respective measure, (as appeareth in your Notes;) lastly, draw the lines AB, BC, CD, &c. So is there on the Paper the exact *Figure* of the *Field*, as was required.

This Method is seldom used in Surveying any Field; because we cannot stand just in the Corner, by reason of the branching out of the Hedges: but where 'tis bounded with Banks and Ditches, this Way is very commodious and short.

P R O P.

P R O P. V.

How, by the Semi-circle, to take the Plot of a Field, at two Stations, by measuring from each Station to the visible Angles; the Field being so Irregular, that from no one Place thereof all the Angles can be seen.

Admit A, B, C, D, E, F, G, H, I, K, to be Fig. 55. the Figure of a Field, whose Plot is required: Having made choice of your two Stations, viz. Q and P, and placed Marks in all the Angles; then place your *Semi-circle* at Q, and there fix it, with the *Needle* hanging over the *Meridian* of the *Chart*, represented by R, Q, X, and direct your Sights unto all the visible Angles, viz. A, B, C, D, E, and F, and note down the quantity of each Angle in your *Field-book*: Then measure with your *Chain* from your Station Q, to the Angles A, B, C, D, E and F; and their Length so found, note down in your *Field-Book* also.

This done, direct your Sight unto your *second* Fig. 55. Station P, and note down in your *Field-book* the Degree of *Declination*, of your *second* Station P, from the *Meridian*: Then measure the *Stationary-Distance* PQ with your *Chain*, and note it down in your *Field-book* also.

Then remove the *Instrument* unto P, your *second* Station, and there fix it, with the *Needle* hanging over the *Meridian-line* of the *Chart*, represented by TPB; then direct your Sights to the several visible Angles at this *second* Station, viz. F, G, H, I and K, in order one after another, and note down the quantity of each Angle

§ 2. Angle in your *Field-book* ; then with your *Chain* measure from your *Station P*, to these several Angles G, H, I and K, (in all respects as at the first *Station Q*;) and their Length so found, note down in your *Field-Book* likewise: So have you finished your Observation, and your Work standeth thus :

The Observation taken at the First Station Q.

Fig. 55.

<i>Angles.</i>	D	M	C	L
A	50	0	6	60
B	80	0	7	65
C	140	12	12	0
D	220	7	11	10
E	270	5	12	60
F	330	0	6	0

The *Declination* of the *Station P*, from the *Meridian R Q X*, is $30^{\circ} 00'$, and the *Stationary Distance Q P* is 9 Chains.

The Observation taken at the Second Station P.

Fig. 55.

<i>Angles.</i>	D	M	C	L
F	227	11	0	0
G	297	0	12	0
H	347	16	9	90
I	60	0	6	0
K	90	0	6	26

NOTE,

NOTE, That the manner of taking the lot of a large *Champaign Field*, at many Stations, is almost the same with this *Proposition*; for he that can do the one, can also perform the other: Therefore, for brevity-sake, I here omit it, as superfluous.

§ 2.

P R O P. VI.

How to delineate any Observation taken according to the Doctrine of the last Proposition.

Upon your Paper draw the *Meridian-line* Fig. 55.

Q X, then place the Centre of your *Protractor* in Q (representing your *first Station*,) and the *Meridional-line* lay equal to R Q X, then prick off the *Angles* visible at your *first Station* Q, viz. A, B C, D, E and F, of their due Quantity; then draw Q A, Q B, &c. laying on them their corresponding Measure, noted in your *Field-book*. Now because your *second Station* P doth decline $30^{\circ} 00'$ from the *Meridian* R Q X, prick off $30^{\circ} 00'$, and draw P Q, making it 9 *Chains*, as in your *Field-book* appeareth, so doth P represent your *second Station*: Then in all respects as before, place your *Protractor* at P your *second Station*, and draw the *Meridian* T P B parallel to R Q X; then prick off the several *Angles*, viz. F, G, H, I and K, of their due Quantity, and then draw P F, P H, P I, &c. of their due Length; lastly, draw the lines A B, B C, C D, &c. and so shall you have on your Paper the exact Figure of the *Field*, as required.

This Method is very proper to be used in Champaign-Fields, or in large Heaths or Commons.

P R O P. VII.

How by the Semi-circle to take the Plot of a Field at two Stations, which lieth remote from you, when either by Opposition of Enemies you may not, or by some other Impediment you cannot come into the same.

Fig. 56.

Admit the Figure A, B, C, D, E, F, to be a Field into which by no means you can possibly enter, and yet of necessity the Plot thereof must be had, for the obtaining of which, choose any *two Stations*, it mattereth not whether near at hand, or far off, so that all the *Angles* may be seen: Let your *two Stations* be H and L (the full length of the Field, if possible) then place your *Instrument* at H, and fixing it, as is afore-shewed, direct your *Sights* to the several *Angles* of the Field, viz. A, B, C, &c. orderly one after another, observing their *Degrees* as is afore-taught, noting it down in your *Field-book*; then take up your *Instrument*, leaving a mark in its room at H, and measure with your *Chain* from H unto L your *second Station*, which note down in your *Field-book*; then placing your *Instrument* at L your *second Station*, and (as is before taught) fixing it there, make the like *Observation* to the several *Angles*, viz. A, B, C, D, &c. as at the *first Station* H, and note it down in your *Field-book* also: And having so done, you have finished, and your *Work* standeth as in the following *Table*.

Observation

Observations at the First Station H, are,

The Angle from H the first Station, unto the second Station, is $80^{\circ} 00'$, the Stationary-Distance H L is 60 Chains.

Observations at the Second Station L, are,

1. Angles.	D	M
A	104	0
B	88	7
C	59	0
D	48	0
E	26	0
F	21	30
2. Angles.	D	M
A	16	0
B	39	0
C	50	9
D	74	0
E	100	0
F	29	15

Fig. 56.

P R O P. VIII.

How to delineate any Observation taken according to the Doctrine of the last Proposition.

Upon your Paper draw a Line as H L, which make equal to 60 Chains; then placing the Centre of your Protractor on H your first Station, prick off all the Angles A, B, C, &c. as you find them in your Field-book, and draw H A, H B, H C, &c. at pleasure; then remove your Protractor unto your second Station L, placing it as before, and prick off all the Angles A, B, C, D, &c. as you find them in your Field-notes, and draw the Lines L A, L B, L C, &c. at length, until they intersect the former Lines

O 4

H A,

§ 2. H A, H B, &c. in the Points A, B, C, &c. which *Points of Intersection* are the *Angles* of the *Field*; lastly, draw A B, B C, C D, &c. So shall you have on your Paper the Figure of your *Field* required.

P R O P. IX.

How by the Semi-circle, to take the Plot of a great Champain-Plain, Wood, or other over-grown Ground, by measuring round about the same, and making Observation at every Angle thereof.

Fig. 57. Admit A, B, C, D, be the Figure of a large overgrown Champain-Field, whose Plot is required: First place your *Instrument* at A, laying the *Index* on the *Diameter*, and turn it about until you espy the *Angle* at D, and there fix it fast, and direct your *Sights* to B, and note the *Degree* cut by your *Index*, in your *Field-book*, (as afore is taught) then remove your *Instrument* to B, and there make the like *Observation*, and so to C, and D, noting it down in your *Field-book*, as afore; Then with your *Chain* measure the *Sides* A B, B C, C D and D A, whose *Length* note down in your *Field-book*; and so you have finished, and your *Work* standeth thus:

<i>Angles.</i>	D	M	C	L
DAB	100	0	12	20
ABC	117	15	10	0
BCD	71	30	19	20
CDA	71	15	12	20

P R O P.

P R O P. X.

How to delineate any Observation taken according to the Doctrine of the last Proposition.

Upon your Paper draw the Line A B, at pleasure, and placing the Centre of your Protractor on the Point A, prick off an Angle of 100° , and draw A D, setting on it, and also on A B, their corresponding Measure, in your Notes; then on B protract an Angle of $117^{\circ} 15'$, draw B C of its due length; then draw the Line C D: So have you the exact Figure of the Field on your Paper. Fig. 57.

This is the general, best, and surest Way of Surveying, especially of large Quantities of Land, which consist of divers Parcels; also, when we take the Plot of a Lordship or Manour, we begin to take the Boundary or Out-line after this Method, observing (as we go round) the Buttings of the Hedges and Banks, also the Crossings of Roads and Ditches, noting them down in the Field-Book, with their Distances from their last proper Angle: From which Notes and Observations, the Principal Line, with the Buttings of the Fences or Divisionary Lines of such Lordship or Manour, are first delineated, in order to the cutting the same into its true and proper Divisions.

P R O P. XI.

How to take the Plot of any Field, by the help of the Chain only.

Admit the Figure A, B, C, D, E, to represent Fig. 58.
a Field whose Plot is required; To obtain which,

§ 2. which, first measure the Sides CD, CB, and BD, and note their due Length down in your *Field-Book*, and then measure the Sides CA, and BA, and then note down their Length in your *Field-Book*: Then measure the Sides BE, and ED, (for the Sides BC and BD were before known) which note down in your *Field-Book*: So is your *Field* A, B, C, D, E, reduced into three *Triangles*, viz. CBD, CAB, and BED, the Length of whose Sides are all known. Thus you have finished, and the Work stands as you see:

		Sides.	Ch.	Lin.
Fig. 58.	In the Triangle CBD	CD	5	97
		CB	8	28
		BD	8	25
	In the Triangle CAB	CA	4	24
		CB	8	28
		BA	6	51
	In the Triangle BED	BE	5	28
		ED	5	25
		BD	8	25

P R O P. XII.

How to delineate any Observation taken according to the Doctrine of the last Proposition.

Fig. 58. Upon your Paper draw a straight Line, as CD, make it 5 Chains, $\frac{97}{100}$; take CB in your *Compasses*, and strike an *obscure Arch*; then take BD,

BD, and with that extent in D cross the former *Arch* in B, and draw BC, and BD: Then take in your *Compasses* BE, and on B strike an *Arch*; then take DE, and also cross the former *Arch* in E, and draw BE, and ED: Lastly, take the Line CA, and on C strike an *Arch*; then take AB, and on B intersect the former *Arch* in A, then draw CA, and AB; so have you on your Paper the exact Figure of the Field A, B, C, D, E, as was required. § 2. Fig. 58.

The best way of doing it, (as I find by Practice) is thus: Draw an *Eye-draught*, as near the shape as you can, suppose ABEDC; then measure all the Sides, setting down each Measure upon its correspondent Side: This done from any one *Angle*, as B, (from whence you may see all the rest) measure *Diagonals*, setting down upon them their proper Measure; from which rough-Draught, as well as from the Field-Book-Notes, the Field, Court, Yard, &c. may be Plotted.

This Method is very exact and true, and very proper for a small Enclosure, or for a Court, Yard, Garden, or the like, where there are not many *Angles*.

S E C T. III.

Of finding the Area or superficial Content of any Field lying in any Regular or Irregular Form; by reducing the Irregular Fields into Regular Forms.

HAVING already shewed how to take the Plot of any Field divers Ways, by the § 3.
Semi-

§ 3. *Semi-circle and Chain*, and also by the *Protractor* how to delineate the Draught thereof on Paper, &c. I come now to shew how the *Area* or *superficial Content* of a Field may be attained, i.e. how many *Acres*, *Roods* and *Perches* are therein contained. To which end, Know, That a *Statute Pole* or *Perch* contains $16\frac{1}{2}$ Feet; That 40 of those *Perches* in length, and 4 in breadth, makes an *Acre*: So that an *Acre* contains 160 *Perches*, and a *Rood* 40 *Perches*, according to the Statute of 33d of Edw. I.

Now the Original of the *Menfuration* of *Land*, and all other *Superficies*, depends on the *Menfuration* of certain *Geometrical Figures*; as a *Triangle*, *Square*, &c. which may be measured according to the *Directions* of § 2. Chap. 4. of *Geometry*: It would therefore here be superfluous to make a repetition of things already handled: I shall therefore omit it, and come to shew how any *Field* lying in any *Irregular Form*, may be Measured by converting it into *Regular Figures*; for it seldom happeneth, but that the Plot of a Field is an *Irregular Figure*, consisting of three, four, five, or more *Unequal Sides*.

If it be a *Triangle*, it is Measur'd by Chap. 4. § 2. Prop. 3.

If it be a *Four-sided Irregular Figure*, 'tis divided into two *Triangles*, as in Plate 4. Fig. 59. The *Four-sided Irregular Figure* ABDC is divided by the *Diagonal* AD, drawn from opposite Corners; then let fall two *Perpendiculars* from B and C to the *Base* AD: This done, find the *Area* of the two *Triangles* ABD, ACD; which may be done at once, (*viz.*) by *Multiplying*

the Sum of both the Perpendiculars by Half the Base, or the Whole Base by Half the Sum of the Perpendiculars. By which Rule, the Content of the Four-sided Figure or Trapezium BCD is found to be 630 Perches. If it be an Irregular Figure, 'tis Measured by Chap. 4. § 2. Prop. 7. § 3.

P R O P. I.

How to reduce any Number of Perches into Acres, and, on the contrary, Acres into Perches.

To find how many Acres are contained in any Number of Perches given, you must consider that 160 Perches do make a Statute Acre; therefore if you Divide the Number of Perches propounded, by 160, the Quotient is the Number of Acres contained therein; and if there be a Remainder which exceeds 40, then Divide it by 40, the Quotient shall be Roods, and the Remainder Perches.

But, on the contrary, if it were required to find how many Perches are contained in a certain Number of Acres propounded; you must Multiply the Number of Acres by 160; the Product shall be the Perches contained therein.

P R O P. II.

To reduce Dimensions taken in Chains and Links, into Acres, Roods and Perches.

In regard we generally take our Dimensions in Chains and Links, that is, by Gunter's Chain, (which contains 4 Pole, or 100 Links, as before has

§ 3. has been noted) it will be necessary to shew you how to Cast up the *Area* of a Piece of Land, from Dimensions taken in those Denominations.

Suppose the *Four-sided Figure* A B C D, in Plate 4. Fig. 60. was Plotted, and the Length of the *Base* DB, and two *Perpendiculars* dropt from A and C to be as in the said *Figure*, as followeth, viz. the *Base* 12 Chains, 48 Links, or 1248 Links; the *Long Perpendicular* 8 Chains, 46 Links, or 846 Links; and the *Less Perpendicular* 3 Chains, 94 Links, or 394 Links: Then I take them out, and place them as followeth.

846 *Great Perpendicular.*

394 *Less Perpendicular.*

1240 *Sum of them.*

620 *Half Sum.*

1248 *Base.*

.4960

2480

1240

620

7.73760 *Acres and Parts.*

4

2.95040 *Roods and Parts.*

40

38.01600 *Perches and Parts.*

And

And here NOTE, That *Gunter's Chain* (as before was hinted) contains 4 *Statute Pole*, or 100 *Links*; so that any Number of *Chain* is so many 100 *Links*; as, 5 *Chain* is 500 *Links*, 7 *Chain* is 700 *Links*, &c. And further, you are to Observe, That 160 *Square Statute Pole* is an *Acre*, each *Pole* being 16 *Foot* and $\frac{1}{2}$. Now 'tis evident, In a *Square Chain* there is 16 *Square Pole*: Divide therefore 160 the *Square Pole* in an *Acre*, by 16 the *Square Pole* in a *Square Chain*, the Quote is 10 the *Square Chain* in an *Acre*. But every *Square Chain* contains 10000 *Square Links*, (for 'tis 100 Multiply'd by 100) and consequently an *Acre* 100000 *Square Links*.

Hence the Reason of the foregoing Operation is evident; for the *Half Sum* of the *Perpendicular* is 620 *Links*, and the *Base* 1248 *Links*, consequently the Product 773760 *Square Links*. Now because 100000 *Square Links* is an *Acre*, therefore I Divide 773760 by 100000 *Links*: But to Divide by 100000, is no more than to cut five figures to the Right; so will the remaining figures on the Left be *Acres*, and those cut off to the Right 73760 *Hundred thousandth Parts* of an *Acre*. If these *Parts* be Multiply'd by 4, and from the Product five figures be again cut off to the Right-hand, those on the Left shall be *Roods*, and those on the Right *Parts* of a *Rood*. And if those so cut off be Multiply'd by 40, and from the Product five figures be cut off to the Right-hand, as before, those on the Left are *Poles*, and those on the Right *Parts* of a *Pole*. In the preceeding Operation, there is 7 *Acres*, 2 *Roods*, 38 *Poles*, and 1600 *Hundred thousandths* of a *Pole*.

P R O P.

To reduce Statute Measure to Customary;
& *contrá.*

The Statute Pole or Perch is $16\frac{1}{2}$ Foot long: Now divers Parts of England uses a Pole of 18 Foot long; others, one of 21 Foot; and some of 24 Foot long. And therefore, to turn one sort of Measure into another, suppose Statute into Customary, do thus: Multiply any Number of Acres, Roods or Perches Statute Measure by the Square $\frac{1}{2}$ Yards or Square $\frac{1}{2}$ Feet in a Square Pole of the Statute Measure; the Product Divide by the Square $\frac{1}{2}$ Yards, Square $\frac{1}{2}$ Feet in a Pole of the Customary Measure you would bring it into; the Quotient gives you the Acres, Roods or Perches of that Customary Measure.

EXAMP. I. In 786 Acres Statute Measure, How many Acres of 18 Foot to the Perch?

NOTE, In a Statute Pole there is $5\frac{1}{2}$ Yards, that is 11 Half-Yards; consequently, in a Square Statute Pole there is 121 Square Half-Yards; and in a Square Pole of 18 Foot there is 144 Square Half-Yards.

$$\begin{array}{r}
 786 \text{ Acres Statute Measure.} \\
 121 \text{ Square } \frac{1}{2} \text{ Yards in a Stat. Pole.} \\
 \hline
 786 \\
 1572 \\
 786 \\
 \hline
 144 \left) 95106 \right. (660 \text{ Acres Customary.} \\
 \underline{870} \\
 66
 \end{array}$$

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§ 3.

So that the *Customary Acres* (at 18 Foot to the *Perch*) in 786 *Statute Acres*, is 660, and $\frac{66}{144}$ of an *Acre*. This *Overplus* Multiply'd by 4, and the *Product* Divided by 144, gives *Roods*. And if the *Remainder*, after this Division for *Roods* be Multiply'd by 40, and the *Product* Divided by 144, will give *Perches*; see the following Operation of the precedeing *Remainder* :

$$\begin{array}{r}
 66 \\
 4 \\
 \hline
 144 \overline{) 264} \left(1 \text{ Rood.} \right. \\
 \underline{120} \\
 40 \\
 \hline
 144 \overline{) 4800} \left(33 \text{ Perches.} \right. \\
 \underline{480} \\
 48 \text{ Remainder.}
 \end{array}$$

So that 786 *Acres Statute Measure*, is 660 *Acres*, 1 *Rood*, 33 *Perches Customary*, with some small *Remainder* over.

NOTE, If there had been any *Roods* or *Perches* with the given *Acres*, then you must have turn'd all into *Perches*; and having Multiply'd and Divided as before directed, Divide the *Quote* by 160; or else by 40, and that *Quote* again by 4.

P

EXAMP.

§ 3.

EXAMP. II. In 649 Acres Customary, at 18 Foot to the Perch; How many Acres Statute Measure?

649 Acres Customary.
144 Sq. $\frac{1}{2}$ Yards Custom. Pole.

$$\begin{array}{r}
 2596 \\
 2596 \\
 649 \\
 \hline
 121 \overline{) 93456} \left(772 \text{ Acres Stat.} \right. \\
 \underline{875} \\
 286 \\
 \hline
 44 \text{ Remainder.} \\
 4 \\
 121 \overline{) 176} \left(1 \text{ Rood.} \right. \\
 \underline{55} \\
 40 \\
 121 \overline{) 2200} \left(18 \text{ Perches.} \right. \\
 \underline{990} \\
 22 \text{ Remainder.}
 \end{array}$$

So that in 649 Acres Customary, at 18 Foot to the Perch, there are 772 Acres, 1 Rood, 18 Perches Statute Measure.

The

§ 3.

The Reason of this Operation would be more evident, did we Multiply the *Acres* by 160 to bring them into *Perches*, and those *Perches* by the *Square* $\frac{1}{2}$ *Yards* in a *Perch*: For by this Way, we should get the *Square* $\frac{1}{2}$ *Yards* in the given Number of *Statute Acres*; and then it would only be Dividing by the *Square* $\frac{1}{2}$ *Yards* in a *Customary Perch*, and so get the Number of *Customary Perches* in the given *Statute Acres*; which being Divided by 160, will give the *Customary Acres* required. But Multiplying by 160, and then Dividing again by the same, is needless; because the Quotient would be the same, were those Operations omitted; therefore we only Multiply the *Statute Acres* by the *Square* $\frac{1}{2}$ *Yards* in a *Statute Perch*, and Divide the *Product* by the *Square* $\frac{1}{2}$ *Yards* in a *Customary Perch*, the *Quotient* gives the *Customary Acres* in those *Statute Acres*.

NOTE, That *Customary*, as well as *Statute Acres*, contain 160 *Square Perch*; so that the Difference is only in the Bigness of the *Perch*.

S E C T. IV.

Of the Use of the Semi-circle, in taking Altitudes, Distances, &c.

P R O P. I.

How by the Semi-circle to take an Accessible Altitude.

§ 4.

Admit AB be the Height of a Tower, which is required to be known. First, placing

P 2
your

§ 4. your *Semi-circle* at D, (with the *Arch* downwards, and the two *Sights* fixed,) place it *Horizontal* *, and screw it fast:

* Which to do, is no more than this; with a *Thread* and *Plummet* fastened at the *Centre* of the *Semi-circle*, so that it hath liberty to play, move the *Semi-circle* till the *Thread* playeth against 90 deg. then screw it fast, and it is *Horizontal*.

Then move your *Index*, till through the *Sights* thereof you espy the top of the *Tower* at B, and observe what *Degree* the lower part of the *Index* cutteth, and that will be equal unto the *Angle* at D 50 deg. Then Measure the Distance DA, which let be 299 Feet. Now the Height of the *Tower* AB, is found, according to Ch. 5. § 2. Prop. 1. thus:

Fig. 61. As Sc. V. at B $50^{\circ} 00'$,
To Log cr. DA 299 Feet;
So is S. V. at D $50^{\circ} 00'$,
To Log. AB $356 \frac{3}{10}$ Feet, the Height of the
Tower required.

P R O P. II.

How by the *Semi-circle* to take an *Inaccessible* Altitude, at two Stations.

Let AB be a *Tower* whose Height is required; Having placed your *Instrument* at E, as before, direct your *Sights* unto the Top of the *Tower* at B, and finding the *Degree* cut by the *Index* to be $23^{\circ} 43'$, I say, it is the Quantity of the *Angle* at E. Now by reason of Water or such-like Impediment, you can approach no nearer the *Base* of the *Tower* than D; therefore Measure ED, which is found to be 512 Feet: Then at D make the like Observation, and the *Angle* at D appeareth to be $50^{\circ} 00'$, whose

Com-

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Complement is the *Angle* DBA $40^{\circ} 00'$, and the *Complement* of the *Angle* E $23^{\circ} 43'$, is the *Angle* EBA $66^{\circ} 17'$: Now if the *lesser Angle* at B be taken out of the *greater*, the *Remainder* is $26^{\circ} 17'$, the *Angle* EBD. Now, first, to find the *Side* BD of the *Triangle* EBD, say, according to Chap. 5. § 3. Prop. 1. thus:

§ 4.

As S. of V. EBD $26^{\circ} 17'$,
To Log. cr. ED 512 Feet;
So is S. of V. at E $23^{\circ} 43'$,
To Log. cr. BD $465 \frac{2}{10}$ Feet required.

Fig. 61.

Now to find the *Height* of the *Tower* AB, say, according to Chap. 5. § 2. Prop. 2. thus:

As *Radius* or S 90° ,
To Log. cr. DB $465 \frac{2}{10}$ Feet found;
So is S. of V. BDA $50^{\circ} 00'$,
To Log. cr. BA $356 \frac{3}{10}$ Feet; which is the *Height* of the *Tower* required.

NOTE, That in taking any manner of *Altitude*, the *Height* of your *Instrument* must be added unto the *Height* found, and that will give you the *true Altitude* required.

P R O P. III.

How by the *Semi-circle* to take an *Inaccessible Distance*, at two *Stations*.

Admit A and B be the two *Stations*, from either of which it is required to find the *Distance* unto the *Church* at C: Placing your *Instrument* at B, the *Index* lying on the *Diameter*, and direct your *Sights* unto the *Church* at C; fasten your *Instrument*, and turn your *Sights* about, until you see through your *Sights* your *second*

P 3

Station

§ 4. Station at A, so will you find your *Index* to cut $30^{\circ} 00'$, which is the Quantity of the *Angle* ABC: Then Measure the *Distance* AB, which is found to be 250 Yards; then with your *Instrument* at A, make the like Observation as before, and you will find the *Angle* BAC to contain $50^{\circ} 00'$. Now by the Third Maxim of *Plain Triangles*, Chap. 5. § 1. you find also the *Angle* ACB to be $100^{\circ} 00'$. Now to find the *Distance* AC and BC, you may, by their opposite Proportion, according to Chap. 5. § 3. Prop. 1. find the *Distance* of AC, thus:

Fig. 62. As S. of V. at C $100^{\circ} 00'$,
To Log. cr. AB 250 Yards;
So is S. of V. B $30^{\circ} 00'$,
To Log. cr. AC 127 Yards: Which is the
Distance of the Church from A.

Now to find the *Distance* BC, say,
As S. of V. at A $100^{\circ} 00'$,
To Log. cr. AB 250 Yards;
So is S. of V. at A $50^{\circ} 00'$,
To Log. cr. BC $194\frac{4}{16}$ Yards: Which is the
Distance of the Station B from the Church at C.

P R O P. IV.

How to find the Horizontal-Line of any Hill or Mountain, by the Semi-circle.

Let Fig. 63 be a Mountain, whose Horizontal-Line AB is required to be found: To find which, place your *Instrument* at A; and having caused a Mark to be placed on the Top of the Mountain at C, (of the just Height of your *Instrument*) then move your *Index* until
through

th
a
A
th
u
v
to
fi

o
m
E
5
M
P
g

F
w
M

m
t
I
d
t

through the Sights thereof you espy the Mark at C, so will you find the Quantity of the *Angle CAD* to be $50^{\circ} 00'$, and by consequence the *Angle ACD* to be $40^{\circ} 00'$; then Measure up the *Hill AC*, which is 346 Yards. Now having obtained these several things, 'tis required to find the Length of AD part of AB: To find which, say,

As Radius or S. 90° ,
To Log. cr. AC 346 Feet;
So is Sc. of V. at A $50^{\circ} 00'$,
To Log. cr. AD $222 \frac{4}{16}$ Feet;

Fig. 63.

Now seeing the *Hill* or *Mountain* descendeth on the other side, you must place your *Instrument* at C, and direct your *Sights* unto the *Bottom* at B, and the *Angle DCB* will be found $50^{\circ} 00'$, and the *Angle CBD* $40^{\circ} 00'$: Then Measuring down the *Mountain* as CB, it appeareth to be 415 Feet; then have you the *Angles DCB*, and *CBD*.

To find DB, part of AB, say,

As Radius or S. 90° ,
To Log. cr. CB 415 Feet;
So is S. of V. BCD $50^{\circ} 00'$,

Fig 63.

To Log. cr. DB 318 Feet. Now AD $222 \frac{4}{16}$ Feet added thereunto, produceth AB $540 \frac{4}{16}$ Feet, which is the *Horizontal-Line* required of the *Mountain ACBD*.

 NOTE, That when you come to delineate a Field wherein are *Hills*, you must protract the Line AB, instead of the *Hypothenusal-Lines AC*, and *CB*; and 'twill be necessary to distinguish those kind of Fields, by Shadowing them off with *Hills* and *Dales*.

S E C T. V.

How to find whether Water may be conveyed from a Spring-Head, unto any appointed Place.

§ 5. **T**HE Art of conveying of *Water* from a *Spring-Head* unto any appointed Place, hath a special respect unto *Measuring*; and therefore I think it not amiss to insert it in this Place, and enroll it under the Title of **SURVEYING**.

In the Performance of which, we make Use of a *Water-Level*; the Construction and Making whereof is sufficiently known to those who make *Mathematical Instruments*. Now if it were required to find whether Water may be conveyed in *Pipes*, &c. to any Place assigned: To perform which, observe these Rules.

First, At some 10, 20, 30, 40, 60, or 100 Yards distance from the *Spring-Head*, in a right-line towards the Place unto which your Water is to be conveyed, place your *Water-Level*, being prepared of two *Station-Staffs* with moveable *Vanes* on each of them, graduated also after the usual manner: Cause your *first Assistant* to set up one of them at the *Spring-Head*, Perpendicular unto the *Horizon*; and your *second Assistant* to erect another, as far from your *Water-Level* towards the Place to which the Water is to be conveyed, as your *Water-Level* is distant from the *Spring-Head*. Now the *Station-Staffs* in this order erected, and your *Water-Level* placed precisely *Horizontal*, go unto the end

nd of the *Level*, and looking through the *Sights*, cause your *first Assistant* to move a Leaf of Paper up or down your *Station-Staff*, until through the *Sights* you espy the very edge thereof, and then by some known *sign* or *sound*, intimate to your *Assistant* that the Paper is then in its true Position; then let the *first Assistant* note against what Number of *Feet*, *Inches*, and *Parts* of an *Inch* the edge of the Paper resteth; which he must note down in a Paper. Then your *Water-Level* remaining immoveable, go to the other end thereof, and looking through the *Sights* towards your other *Station-Staff*, cause your *second Assistant* to move a Leaf of Paper along the *Staff*, till you see the very edge thereof through the *Sights*, and then cause him, by some known *sign* or *sound*, to take notice what number of *Feet*, &c. are cut by the said Paper; which let him keep, as your *first Assistant* did.

This done, let your *first Assistant* bring his *Station-Staff* from the *Spring-Head*, and cause your *second Assistant* to take that *Staff*, and carry it forwards towards the Place unto which the Water is to be conveyed, some 30, 40, 60, or 100 Yards, and there to erect it *Perpendicular*, as before, letting your *second Assistant's Staff* stand immoveable, and your *first Assistant* to stand by it: Then in the Midway between your two *Assistants* place your *Water-Level* exactly *Horizontal*, and looking through the *Sights* thereof, cause your *first Assistant*, and after that your *second*, to make their several Observations in all respects as before.

§ 5.

In this manner you must go along from the *Spring-Head*, to the Place unto which you would have the *Water* conveyed; and if there be never so many several *Stations*, you must in all of them observe this manner of Work precisely: So that by comparing the *Notes* of your two *Assistants* together, you may easily know whether the *Water* may be conveyed from the *Spring-Head*, or not, by calling your two *Assistants* together, and causing them to give in their *Notes* of Observation at each *Station*; which add together severally: Then if the *Notes* of the *second Assistant* exceed the *Notes* of the *first Assistant*, take the *lesser* out of the *greater*, and the *Remainder* will shew you how much the appointed *Place*, to which the *Water* is to be conveyed, is Lower than the *Spring-Head*.

The *first Assistant's*
NOTES.

Station.	Feet.	Inch	Parts.
1	15	3	. 50
2	2	1	. 25
3	1	6	. 0
Sum.	18	10	. 75

The *second Assistant's*
NOTES.

Station.	Feet.	Inch.	Parts.
1	3	2	. 75
2	14	0	. 25
3	3	11	. 0
Sum	21	2	. 0

By these two *Tables* you may perceive, that the *Notes* of the *first Assistant*, collected at his several *Stations*, being added together, amount unto 18 *Feet*, 10 *Inches*, and $\frac{75}{100}$ or $\frac{3}{4}$ of an *Inch*; and the *Notes* of your *second Assistant*, collected

at his several *Stations*, amounts unto 21 Feet, 2 Inches : So that the Number of the *first Assistant's Observations* being taken from the *second's*, there will remain 2 Feet, 3 Inches, and $\frac{25}{100}$ or $\frac{1}{4}$ of an Inch : And so much is the Place unto which the *Water* is to be brought, Lower than the *Spring-Head*, (according to the sleight *Water-Level*) and therefore the *Water* may easily be conveyed thither. And here Observe these *Notes* :

 1. In your Passage between the *Spring-Head* and the appointed Place, from *Station* to *Station*, you must observe this Order ; That your *first Assistant*, at every *Station*, must stand between the *Spring-Head* and your *Water-Level*, otherwise great Errors will ensue.

 2. That if the *Notes* of your *first Assistant* exceed the *Notes* of the *second Assistant*, then 'tis impossible to bring the *Water* from that *Spring-Head* unto the appointed Place : But if their *Notes* are equal, it may be done, if the Distance be but short.

 3. That the most approved Authors concerning this Particular, do averr, That at every *Mile's* end there ought to be allowed $4\frac{1}{2}$ Inches more than the *Straight-Level*, for the Current of the *Water*.

 4. That if there be any *Mountains* lying in the Way betwixt the *Spring-Head* and the Place to which the *Water* is to be conveyed, you must then cut a *Trench* by the side of the *Mountain*, in which you must lay your *Pipes* equal with the *straight Water-Level*, with the former Allowance : And in case there be a *Valley*, you must then make a *Trunk* of strong Wood, well under-
propt

§ 5. propt with strong Pieces of Timber, well Pitcht or Leaded, (as is done in divers Places between *Ware* and *London*.)

 5. That when the *Spring* will have too violent a *Current*, you must then convey your *Water* to the Place assign'd, by a *Crooked* or *Winding-Line*; and you also ought to lay the *Pipes*, the one up, and the other down, that thereby the *Violence* of the *Current* may be stopt.

CHAP. X.

Of MEASURING,

Of Board, Glass, Tiling, Paving, Timber, Stone, and Irregular Solids, such as Geometry can give no Rule for the Measuring thereof.

SECT. I.

Of the Measuring of Board, Glass, Paving, Tiling, &c.

§ 1. **I** Have already, in the 4th Chapter of this Book, and the 2d Section thereof, apply'd Geometry to the finding out of the *Superficial Content* of all *Regular Superficies*. I have also in the 9th Chapter, and the 3d Section thereof,

Of Measuring.

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hereof, shewed how the *Superficial Content* of any *Irregular Superficies* may be found, by reducing them into *Regular Forms*; which I have explained amply in that *Section*, I shall therefore here be as plain and brief as is possible. § 1.

P R O P. I.

To Measure a Piece of Board, Plank, Glass, &c.

In Measuring of Board, Glass, &c. Carpenters and other *Mechanicks* Measure by the Foot, 12 Inches unto the Foot; so that a Foot of Board, or Glass, contains 144 Square Inches.

Now if a Piece of Board, Plank, or Glass, be required to be Measured, let it be either a *Parallelogram*, or *Tapering Piece*: First, by the Rules aforegoing, find the *Content* thereof in Inches, and that *Product* Divide by 144; the *Quotient* is the *Content* of that *Superficies* in Feet.

P R O P. II.

To Measure Tiling, Flooring, Roofing, and Partitioning-works.

In Tiling, Flooring, Roofing, and Partitioning-work, Carpenters and other *Workmen* reckon by the Square, which is 10 Feet every way; so that a Square containeth 100 Feet.

EXAMPLE. There is a Roof 14 Feet Broad; What Length thereof shall make a Square?

Divide 100 by 14, it yields $7\frac{2}{7}$ Feet.

Now if you have any Number of Feet given, and the Number of Squares therein contain'd are

§ 1. are required ; Divide that Number by 100
the Product is Squares.

P R O P. III.

To Measure Paving, Plastering, Wainscoting
and Painting-work.

In Paving, Plastering, Wainscoting and
Painting-work, Mechanicks reckon by the Yard
Square ; so each Yard is equal unto 9 Square
Feet.

By the Rules foregoing, find the *Superficie*
Content of the Court, Alley, &c. in Feet ; which
Divide by 9, the *Quotient* is the Number of
Yards in that Work contained.

P R O P. IV.

To find how many Inches in Length will make
Square Foot.

EXAMP. Suppose a Piece of Board or Plank
be 3 Inches Broad ; How much in Length will
make a Foot Square ?

Consider that in a Square Foot there are 144
Square Inches : Divide therefore 144 by 3, the
Quote is 48 ; which shews, that a Piece of Board
3 Inches Broad, requires 48 Inches, or 4 Foot in
Length, to make a Foot Square.

Whence 'tis manifest, that if you Divide
144 by any given Breadth, you get the Length
that will make a Foot Square of that Breadth.
The reason is clear ; for if you Multiply the
Divisor and Quote, that is, the Length and
Breadth, the Product will be 144 the Square
Inches in a Square Foot.

And

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§ 1.

And after this manner is the TABLE upon the Common Two-Foot Rules made, call'd the TABLE of Board Measure ; which shews, upon any Breadth, how much in Length will make a Foot Square.

A TABLE of Board Measure.

Inch.	1	2	3	4	5	6	7	8
Feet	12	6	4	3	2	2	1	1
Inch.	0	0	0	0	4 $\frac{4}{5}$	0	8 $\frac{4}{7}$	6

From this TABLE it appears, that if a Piece of Board be an Inch Broad, 'twill require 12 Foot in Length to make a Foot Square : If 2 Inches Broad, 6 Foot in Length will make a Foot Square : If 8 Inches Broad, 1 Foot 6 Inches will make a Foot Square.

S E C T. II.

Of the Measuring of Timber, Stone, and Irregular Solids.

§ 2.

IN Superficial Measure, a Superficial Foot contains 144 Square Inches ; but in Solid Measure, a Foot contains 1728 Cubick Inches. Now having already, in the 4th Chapter of this Book, and the 3d Section thereof, largely apply'd Geometry unto the Measuring of all Regular Solids ; I shall therefore in this Place be as brief as possible, only I shall be somewhat larger

§ 2. larger in the Mensuration of *Irregular Solids* which is of special Moment in sundry Parts of the *Mathematical Practices*.

P R O P. I.

How to Measure any kind of Timber, or Stone whether Three-square, Four-square, Many-square, Round, or of any other fashion, provided it be straight and equal all along.

To perform which, first, by the Rules aforegoing in *Chap. 4. § 2.* get the *Superficial Content* at the End, and then say,

As 144, the Inches of the Superficial Content of the End of a Cubick Foot,

To a Cubick Foot containing 1000 Parts ;

So is the Superficial Content of the End of any Piece of Timber,

To the Solid Content of One Foot Length of the said Piece of Timber.

According to which, Mr. Philips calculated the ensuing TABLE, which I have thought fit hereunto to annex.

CASE II. *Or the Solid Content in Feet, &c. may be found otherwise, thus :*

By the Rules aforegoing, find the *Content* of the End of the Piece of Timber in *Inches*, which *Content* Multiply by the Length of the said Piece of Timber or Stone in *Inches*, and that *Product* Divide by 1728, it produceth the *Solid Content* of that Piece of Timber or Stone, in Feet, and Parts of a Foot.

A TA-

Of Measuring.

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§ 2.

A TABLE, shewing the Solid Content of One Foot-length of any Piece of Timber, according to the Superficial Content at the End thereof.

	Feet.			Feet.	
	Parts.			Parts.	
<i>The Inches of the Content at the End.</i>	0	7	200	1	398
	0	14	300	2	83
	0	21	400	2	778
	0	28	500	3	472
	0	35	600	4	167
	0	42	700	4	861
	0	49	800	5	356
	0	56	900	6	250
	0	62	1000	6	944
	0	69	2000	13	888
	0	139	3000	20	833
	0	208	4000	27	778
	0	278	5000	34	722
	0	347	6000	41	666
	0	417	7000	48	711
	0	485	8000	55	555
<i>The Inches of the Content at the End.</i>	0	556	9000	62	500
	0	625	10000	69	444
	0	694	20000	138	888

PROP. II.

To Measure Round Timber which is Hollow, or any other Hollow Body.

If Hollow Timber be to be Measured, first Measure the Stick as though it were not Hollow, then

Q

- § 2. then find the *Solidity* of the *Concavity* as though it were *Massie Timber*; then Subtract this last found *Content* out of the whole *Content* before found, the *Remainder* is the *Content* of that *Hollow Body*.

P R O P. III.

To Measure Tapering Timber, or Stone.

Those *Tapering Bodies* are either *Segments* of *Cones*, or *Pyramids*: Now the Way to Measure such *Bodies*, is demonstrated in *Chap. 4. § 3. Prop. 4, and 5.* But now to find the *Content* of these *Segments*, do thus: Measure the *Solidity* of the whole *Cone*, or *Pyramid*, and then find the *Content* of the *Top-part* thereof cut off, (as if it were a *Cone* or *Pyramid* of it self) and the *Content* thereof Deduct from the *Content* of the whole *Cone*, or *Pyramid*; so shall the *Remainder* be the *Content* of the *Segment* required: which reduced into *Feet*, gives the *Solid Content* of that *Piece of Timber* in *Feet*. Now to find the *Length* of the *Top-part* cut off, from the *Cone*, or *Pyramid*, say,

*As the Difference of the Breadth of the two Ends,
To the Length between them;
So is the Breadth of the greater End,
To the whole Length of the Cone, or Pyramid.*

P R O P. IV.

How to find the Solid Content of any Solid Body, in any strange form, such as Geometry can give no Rule for the Measuring thereof.

These strange Forms, are either *Branches* in *Metal*, *Crowns*, *Cups*, *Bowles*, *Pots*, *Screws*, or
Twisted

Of Measuring.

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§ 2.

Twisted Ballisters *, or any other *Irregular Solid*, that keep not in thickness one quantity, but are thicker in one place than in another, so that no Man, by *Geometry*, can possibly Measure their *Solidity*.

* *Whose Surface is bounded by a Line call'd (by Proclus) a Helicoides; but it may also be call'd a Helix, a Twist or Wreath, &c.*

Now for the finding the *Content* of any such-like *Irregular Body* in *Inches* or *Feet*, do thus : Cause to be made a *Hollow Cube*, or *Parallelopipedon*, so that you may Measure it with an *Inch-Rule* without difficulty ; and so to know the true *Content* of the Whole, or any Part thereof, at pleasure, within the *Concavity* : Then take some other convenient Vessel, and put pure *Spring-Water* therein ; then having fill'd the Vessel to a known Measure, make a Mark precisely round the very Edge of the Water, then take the *Solid Body* and put it therein ; then take out as much of the Water as (by means of the *Body* put therein) is arisen above the Mark, until the Water do justly touch at the Mark again : Then put the Water taken forth into the *Hollow Cube*, and find the *Solid Content* thereof (being transform'd into a *Cubick Body*) in *Feet*, *Inches*, and *Parts* of an *Inch* : Which *Content* is the just *Solidity* of the *Body* put into the Water. (*Archimedes*, by this Proposition, found the Deceit of the *Crown of Gold*, which Gelo the Son of *Hiero* had vowed unto his *Gods* ; now the Workmen had mixed *Silver* with the *Gold* ; which Theft was discovered by the great Skill of *Archimedes* *.) And herein you must be very curious not to spill any of the Water, or take out of the Vessel, or put into the

* See Proclus, l. 2. c. 3. & Vitruvius, l. 9. c. 3.

Q 2

Cube,

§ 2. *Cube*, any more than the just quantity arisen above the Mark ; for if you do, it will produce infinite Errors : And thus may the *Solidity* of any *Irregular Body* be found.

P R O P. V.

To find how much in Length of any Square Piece will make a Foot Solid.

EXAMP. I. A Piece of Timber is 8 Inches Square, (consequently the Area is 64 Inches) How much in Length will make a Foot Solid ?

Remember, in a Solid Foot are 1728 Solid Inches ; Divide therefore 1728 by 64, the Quote is 27 Inches, that is 2 Foot, 3 Inches, the Length that will make a Foot Solid upon 8 Inches Square.

If the Piece be not exactly Square, the thing differs not ; for the Area of the Oblong must be found.

EXAMP. II. A Piece of Timber is 9 Inches one way, and 4 Inches the other, (consequently the Area is 36) How much in Length will make a Foot Solid.

Here, as before, I Divide 1728 by 36, the Quote is 48 Inches, that is 4 Foot, the Length that will make a Foot Solid.

The Reason is plain ; for the Breadth Multiplied into the Thickness, is the Divisor ; and that Multiplied by the Quote, which is the Length, will make 1728 the Inches in One Solid Foot.

And thus is the Table of Timber Measure, put on the Common Two-foot Rules, made ; which

which shews how much in Length any exact Square Piece will make a Foot Solid. § 2.

A TABLE of Timber Measure.

In. Sq.	1	2	3	4	5	6	7	8	9	10
Feet.	144	36	16	9	5	4	2	2	1	1
Inch.	0	0	0	0	9 $\frac{3}{4}$	0	11 $\frac{1}{8}$	3	9 $\frac{1}{4}$	5 $\frac{7}{8}$

The Use of this TABLE is thus: If a Piece of Timber be 8 Inches Square, it shews you, that 2 Foot 3 Inches in Length will make Foot Solid: If it be 9 Inches, then you will find that 1 Foot 9 Inches $\frac{1}{3}$ of an Inch in Length will make a Solid Foot.

CHAP. XI. Of GAUGING.

IN GAUGING, there are Two things chiefly necessary to be noted, yet both controverted. 1st. That seeing all manner of Casks made to hold Liquor in, are for the most part the Trunk of a Spheroid, cut off with two Circles, at Rightangles with the Base, and therefore Irregular; Therefore they must be reduced into a Regular Proportion.

And the 2d thing necessary to be noted, is to find the true Quantity of an *Ale*, or *Wine* Gallon, in *Cubick-Inches* or *Parts* of a Foot that thereby the *Content* of the *Vessel* or *Cask* in *Gallons*, may be known.

S E C T. I.

Of Gauging any *Beer*, *Ale*, or *Wine* Cask also any manner of *Brewers Tunns*.

P R O P. I.

To find the *Solid Content* in *Inches*, of any *Cask*.

§ 1.

I Shall follow *Mr. Oughtred's Method*, which is ; Take the *Diameter* of the *Cask* both at *Head* and *Bung*, by which find the *Area's* of their *Circles* ; which done, then take Two thirds of the *Area* of the *Bung*, and One-third of the *Area* at the *Head* ; which added together shall be the *Mean Area* of the *Cask* ; which Multiply'd into the *Length* of the *Vessel*, it will shew how many *Solid Inches* are contained therein.

EXAMP. Suppose the *Diameter* at the *Head* of a *Vessel* be 18, and at the *Bung* 32, and *Length* is 40 *Inches*.

Now I find the *Aggregate* of the two *Circles* to be 620 and 989 *Cubick Inches* ; which Multiply'd by 40 the *Length*, produceth 24839 *Cubick Inches*, for the whole *Content* of that *Cask* in *Cubick Inches*.

P R O P.

Of Gauging.

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§ I.

PROP. II.

To find the Content of a Vessel, in Wine or Ale-gallons.

The Wine-Gallon is establish'd by the Consent of Artists, in these and other Nations, to contain 231 Cubick Inches *. Yet

Dr. Wybard affirms it to be somewhat less, to wit, 225 at most.

The Ale-Gallon contains 282 Cubick Inches, according to the Establishment of Excise. Herein

Artists differ somewhat in their Experiments.

* See Mr. Oughtred in his Book of the Circles of Proportion, pag. 57 and Mr. Edm. Gunter in his Book of the Cross-Staff, Part 21. Chap. 4.

Now having already shewed how to find the Content in Inches of any Cask, I now come to shew how to find the Content in Gallons, of any Beer, Ale, or Wine-Cask, which is thus: Divide the Number of Inches given by 231 for Wine Measure, and 282 for Ale Measure. In the former Example I find the said Cask to contain 107, 53 Wine-Gallons, and 88, 8, &c. Gallons in Ale Measure.

PROP. III.

How to Gauge or Measure Brewers Tuns, &c.

Those Tuns are most commonly Segments of Cones or Pyramid, whose Basis is either a Square Parallelogram, Circle, or Oval; To Measure which, let their Form be what it will, you must do thus: By the former Rules of Measuring such Segments or Bodies, you must find their Solid Content in Cubick Inches, (as in Chap. 10. § 2. Prop. 3.) which Content Divide by 282 Inches

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(the

- § 1. (the *Inches* in One Gallon) it sheweth the Content in Gallons; and Dividing the Gallons by 36 (the Number of Gallons in a Barrel) it shews the Content in Barrels.

S E C T. II.

Of Gauging or Measuring, and the Moulding of Ships.

P R O P. I.

To Gauge a Ship, thereby to find how many Tuns her Burthen is.

- § 2. **I**N the Gauging or Measuring of Ships, *Naupegers*. (or *Ship-wrights*) observe these Three particular Rules: 1st. That if you Measure the Ship *within*, you shall find the Content, or the Burthen the Ship will hold or take in. 2^{dly}. If the Ship be Measured on the *outside*, to her *light mark* as she swims being *unladen*, you shall have the Content of the *Empty Ship*. 3^{dly}. But if you Measure from the *light mark*, to her *full draught* of Water being *laden*, you shall have the true Burthen of the Ship.

Now to find the Content of the *Queen's Royal Ships*; Measure the *Length* of the *Keel*, the *Breadth* of the *Mid-ship Beam*, and the *Depth* of the *Hold*: Which three Multiply into one another, and Divide their *Product* by 100; so shall you find how many Tuns her Burthen is.

But for *Merchants Ships*, which give no Allowance for *Ordnance*, *Masts*, *Sails*, *Cables*, *Anchors*,

Of Gauging.

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Anchors, &c. which are all a Burthen, but no Tonnage, you must Divide the Product by 95, so shall their true Burthen be found. § 2.

P R O P. II.

By knowing the Measure of a Ship of one Burthen, to make another Ship of the same Mould, which shall be double, or triple, or in any Proportion, either more or less than the said Ship.

First, you shall Multiply the Keel Cubically, and in like manner every Beam; the Mid-ship Beams, Multiply them Cubically; and also the Reaking of the Ship, both at Stem and Stem-Post, Multiply them Cubically; likewise the principal Timbers, that doth mould the Ship, Multiply them Cubically; and the Depth of the Hold, Multiply it Cubically; and so consequently every Place, or Places, which doth lead any Work, Multiply them Cubically: Then if it be required to have a Ship as big again, or thrice as big, double or triple each respective Cubical Number: Then by Chap. 1. § 1. Prop. 9. or by Chap. 2. § 2. Prop. 4. find the Cube-Roots thereunto belonging: Then according unto these respective Numbers make your Keel, your Timbers, Beams, &c. Which being done, you shall make a Ship of the Mould and Proportion desir'd.

The Demonstration of this last Proposition depends upon the 33d Prop. of 11 of Euclid, where 'tis prov'd, That all Similar Solids are in Triplicate Ratio to that of their Homologous Sides; that is, they are to each other as the Cubes of their Diameters.

C H A P.

C H A P. XII.

Of D I A L L I N G.

HOROLOGIOGRAPHIA, or the Art of DIALLING, is an Art Mathematical, which demonstrateth the precise Distinction of Times, by the Sun, Moon and Stars

* Which doth appear to have been in Use above 2400 Years, for King Achaz had a Dial: This Art requireth good skill in Geometry and Astronomy. Now Ctesibius that famous Philosopher measured the Hours and Times by the orderly running of Water. Then by Sand was the Hours measured. After that, by Trochilike with Weights: And of late, with Trochilike with Springs.

whereby the Time of the Day or Night may be known *. Now the Demonstrative Delineation of Dials, consisteth chiefly in the finding out the Hour-Lines, and their true Distance one from the other; which Lines are great Circles of a Sphere, which being projected on a plain Superficies, become straight Lines; which Lines do continually vary, according as the Planes on which they are de-

scribed, or projected, do lie situated in respect of the Horizon of the Place. Now a Dial may be made on any Plain Superficies; for all Plain Superficies are posited, either Perpendicular, Parallel, or Oblique, to the Horizon of the Place in which the Plane is seated.

In the Delineation of all which Dials in this Chapter described, (which are the most Eminent and Useful Dials now used) I have used this Method: 1st. I have shewed how to Delineate

lineate them by Geometrical Projection, by Scale and Compass only : And, 2dly. how they may be Described by Arithmetical Calculation : Of both which I have been very plain and large.

S E C T. I.

Of the Delineation and Projection of sundry most Useful Dials.

P R O P. I.

How to draw the Hour-Lines on an Equinoctial Plane.

AN Equinoctial Plane, is such which lieth Parallel unto the Equinoctial, and is an Horizontal Plane, under the Pole. This is the first and plainest kind of Dials, and is made after this manner : First describe the Circle ÆW E R for your Plane, then cross it with the two Diameters Æ W and Æ R : Then divide the Semi-circle E W R into 12 equal Parts, in the Points $\odot, \odot, \odot, \&c.$ Then from the Centre Q , and through the said Points, draw straight Lines, which shall be the true Hour-Lines belonging unto this Equinoctial Plane. Now because these Planes are capable of receiving all the Hour-lines from Sun-rising unto the Sun-setting, in Summer ; therefore the Hour-lines of 4 and 5 in the Morning, and 7 and 8 in the Evening, must be delineated as you see done in the Figure : These Hours may be subdivided

§ I.

Fig. 64.

§ 1. divided into *Half-Hours*, and *Quarters*: The *Stile* of this *Dial* must be a straight *Pin*, or
 Fig. 64. *Wyre*, set *Perpendicular* to the *Plane* on the *Centre Q*, and of any convenient *Length*. This *Dial* may be made for any *Latitude*, and is of good Use for *Seamen* and others.

If this *Dial* be put up in the *Latitude* of *London*, it must make an *Angle* of $38^{\circ} 28'$; the *Under Face* next the *Horizon* serving for the *Winter Half-Year*, and the *Upper Face* the *Summer Half-Year*.

P R O P. II.

How to draw the Hour-lines on a Polar Plane.

A *Polar Plane* is one that lies *Parallel* unto the *Pole*, and under the *Equinoctial* is an *Horizontal Dial*. The Way to make this *Dial* is thus:
 Fig. 65. First draw the Line *AB*, for the *Horizontal-line* of the *Plane*; and cross it at the *Middle* at *Right-angles* with the Line *12 Q 12*, which is the *Meridian* or *Hour-line* of *12*: Then upon the Line *12 Q 12*, either above or below the Point *Q*, assume any Point, as *S*; then setting one foot of your *Compasses* in *S*, describe the *Semi-circle* *CED*, which Divide into 12 Equal Parts, in the Points $\odot, \odot, \odot, \&c.$ Then lay a *Ruler* unto *S*, and unto the several Points $\odot, \odot, \odot, \&c.$ and it will cross the Line *AB*, in the Points, *x, x, x, &c.* Then through those Points draw (by *Chap. 4. § 1. Prop. 4.*) *Right-Lines* all *Parallel* unto *12 Q 12*, and so is your *Dial* finished. Then, according unto the *Breadth* of the *Plane*,
 you

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you may proportion your *Stile*; * whose Height § 1.
must be Equal to the Distance between the two

Hour-lines 12 and 9, or 12
and 3, and then will the *Shadow*
of the upper edge thereof shew
the Hour of the Day. The
Height of the *Stile* is also found
thus :

Fig. 56.

* Which may be either a Pin
of the length of QS, placed on
Q, and Perpendicular unto the
Plane ; or it may be a piece of
Brass or else what of the breadth
of 12, to 3, or 9.

*As the Tangent of the Hour-line 4 or 5,
To the Distance thereof from the Meridian ;
So is the Radius,
To the Height of the Stile.*

Then for the other *Hour-line*, say,
*As the Radius,
To the Height of the Stile ;
So is the Tangent of any other Hour-line,
To the Distance thereof from the Meridian-line.*

P R O P. III.

*How to draw the Hour-lines on a Meridian-Plane,
which is an East or West Dial.*

A *Meridian Plane* stands upright directly in
the *Meridian*, and hath two Faces, one to-
wards the *East*, and the other towards the
West.

Now admit it be required to make a Direct
East Dial, in the Latitude of $51^{\circ} 32'$: Let ABCD Fig. 66.
be a *Dial-Plane* on which you would describe
a Direct *East Dial* ; on the Point D describe
an obscure Arch HG, with the Radius of your
Line of Chords ; then take $38^{\circ} 28'$, the Com-
plement of your Latitude, place it from G to L ;
then

- § 1. then draw DL quite through the *Plane*: Then, to proportion your *Stile* unto your *Plane*,
 Fig. 66. so that all the *Hours* may be placed thereon, from *Sun-rising* to 11 a *Clock*, assume two Points in the Line LD, as K, for 11, and I, for the 6 a *Clock* Hour-lines; then draw 6, I 6, and 11, K 11, *Perpendicular* to LD. This done, with the *Radius* of your Line of *Chords* on L, strike the *Arch* OP, and from P to O place $15^{\circ} 00'$; and draw OK, to cut 6 I 6 in M, so shall IM be the Height of the *Stile* proportion'd unto this *Plane*; which may be a Plate of *Brass*, whose Beadth must be equal to the Distance between the *Hour-lines* of 6 and 9, which must be placed *Perpendicular* to the *Plane*, on the Line 6 I 6, whose *Shadow* of the upper edge shall shew the Hour of the Day. Now to draw the *Hour-lines*, with the *Radius* of your Line of *Chords*, on M strike the *Arch* QN, which divide into five equal Parts, in the Points ●, ●, ●, &c. Then lay a Ruler from M unto each of those Points, and it will cut the Line IK in the Points *, *, *, &c. through which Points (by Chap. 4. § 1. Prop. 4.) draw *Parallels* to 6 I 6, as the Lines 77, 88, &c. which shall be the true *Hour-lines* of an *East Plane*, from 6 in the *Morning*, till 11 before *Noon*. Then for the *Hour-lines* of 4 and 5, you must prick off 5 as far from 6, as 6 is from 7; and 4, as far as 6 is from 8; and draw the *Hour-lines* 55, and 44, as before. Thus is your *Dial* compleated, and in the forming of which, you have made both an *East* and a *West Dial*; which is the same in all respects, only whereas the *Arch* HG, through which the *Equinoctial* passed in the

the *East Dial* was described on the Right-hand of the *Plane*, in the *West* it must be drawn on the Left-hand; and the *Hour-lines* 4, 5, 6, 7, 8, 9, 10, and 11, in the *Fore-noon* in the *East Dial*, must be 8, 7, 6, 5, 4, 3, 2, and 1, in the *West* in the *Afternoon*; as in the *Figure* plainly appear- Fig. 67.
eth. Or if you have drawn an *East Dial*, 'tis made a *West* by pricking it through, and drawing the Lines on the backside of the Paper: But Note also, That the *Hour-line* of 11 in the *East Dial* will be the *Hour-line* of 1 in the *West Dial*, and the *Hour-line* of 10 in the *East* the *Hour-line* of 2 in the *West*, and so on. Now you may find the Distance of the *Hour-lines* from the *Substile*, by this *Analogy* or *Proportion*:

As the *Radius*,
To the *Height of the Stile*;
So is the *Tangent of any Hour's Distance from 6*,
To the *Distance thereof from the Substile*.

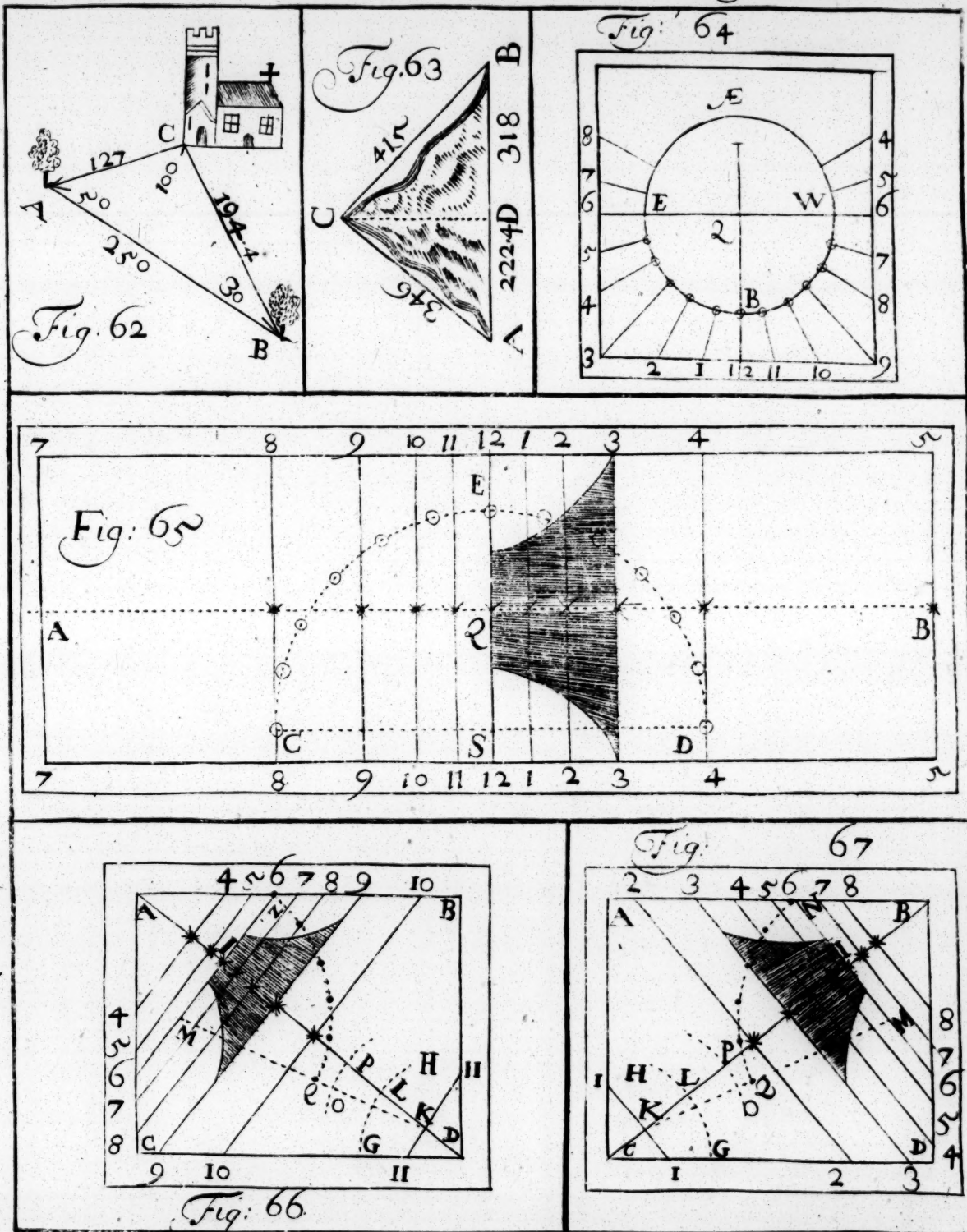
P R O P. IV.

How to draw the *Hour-lines* on a *Direct South* and *North Plane*.

This *Plane* or *Dial* must stand upright, having his *Face* or *Plane*, if it be a *South Dial*, directly opposite unto the *South*; but if a *North Plane*, directly opposite to the *North*. Now admit it be required to make a *Direct South Dial*, for the *Latitude* of $51^{\circ} 32'$: To make which, first describe the *Circle* ABCD, to represent an Fig. 68.
Erect Direct South Plane; cross it with the *Diameters* CB and AD; then out of your *Line of Chords* take $38^{\circ} 28'$, the *Complement* of the *Latitude*,

§ 1. *titude*, and set it from A unto a, and from B unto b: Then lay a Ruler from c unto a, and it will cut the Meridian ARD in P, the Poles of the World; a Ruler also laid from C to b, will cut the Meridian ARD in Æ, which is the Point through which the Equinoctial must pass; for the drawing of which, you have three Points given, viz. C, Æ, and B, and the Centre will always be in the Meridian-line ARD. To find the Centre thereof, do thus: Draw B Æ, and divide it equally in K, erect KM Perpendicular to ÆB (by Chap. 4. § 1. Prop. 1.) and continue it until it cutteth the Line MAR in M, so is M the Centre of the Equinoctial-Circle C Æ B, which describe, then divide it into 12 equal Parts, in the Points ●, ●, ●, &c. the Semi-circle CDB; then lay a Ruler unto R, and to each of those Points, and it will cross the Equinoctial-Circle C Æ B in the Points *, *, *, &c. dividing that into 12 unequal Parts; then lay a Ruler from P, and to every of those Points, and it will cross the Circle of the Plane CDB in the Points I, I, I, &c. Lastly, draw from the Centre R, and through those Points the Right Lines, which shall be the true Hour-lines of an Erect Direct South Dial. Now for the Stile, take $33^{\circ} 28'$, the Complement of the Latitude, and place it from D unto e, and draw R e for the Axis of the Stile, which must hang directly over the Hour-line of 12, or the Meridian, and must point downward, towards the South Pole, because the Plane beholdeth the South-part of the Meridian.

Now in making this Erect Direct South Dial, you have also made an Erect Direct North Dial, which is but the backside of the South Dial, lying



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§ 1.

lying in the same *prime Vertical*; only as this hath the *South Pole* Elevated above it, and beholdeth the *South-part* of the *Meridian*, so the other hath the *North Pole* of the *World* Elevated above it, and beholdeth the *North-part* of the *Meridian*. The *Hour-lines* of 9, 10, 11, or 1, 2, and 3, is not express'd on this *Plane*, because 12, representeth 12 at *Midnight*; neither are the other said *Hours* express'd, because the *Sun* is never above the *Horizon*, at those *Hours*: Therefore the *North Dial* is capable only to receive these *Hours*, namely, 4, 5, 6, 7 and 8 in the *Morning*, and 4, 5, 6, 7 and 8 at *Night*; as doth plainly appear in the *Figure*. Now the Distance of the *Hour-lines* from the *Meridian* may be found, by this *Analogy* or *Proportion*:

As *Radius* or $S\ 90^\circ$,
To *Sc. of the Latitude*;
So is *T. of the Hour from Noon*,
To *T. of the Hour-line from the Meridian*.

Fig. 69.

P R O P. V.

How to draw the Hour-lines on a Horizontal Plane.

This *Horizontal Plane*, or *Dial*, is one of the best and most useful *Dials* in our *Oblique Hemisphere*. Admit it be required to make an *Horizontal Dial* for the *Latitude* of $51^\circ\ 32'$: To make which, first describe the *Circle ABCD*, Fig. 70. which representeth your *Horizontal Plane*; then cross it with the two *Diameters ARC*, and *BRD*; then take $51^\circ\ 32'$ out of your *Line of Chords*, and set it from *B* to *a*, and from *C*

R

to

§ I. to b: Then lay a Ruler from A unto a, and it will cut the *Meridian* BD, in P the *Pole* of the *World*; then lay a Ruler from A unto b, and it will cut ABD the *Meridian*, in the Point Æ, where the *Equinoctial* cutteth the *Meridian*; then through the three Points A, Æ, and C, draw the *Equinoctial Circle*, whose *Centre* is at H, (and found as in the former *Proposition*.) Then divide the *Semi-circle* ADC into 12 equal Parts, in the Points ●, ●, ●, &c. Then lay a Ruler to R the *Centre* of the *Plane*, and on those Points, so shall the *Equinoctial Circle* AÆC be by it divided into 12 Unequal Parts in the

Fig. 70. Points *, *, *, *, &c. Then a Ruler laid unto P the *Pole* of the *World*, and those Points, shall cut the *Semi-circle* CDA in those Points I, I, I, &c. Lastly, from the *Centre* R, and through those Points, let there be drawn *Right Lines*, which shall be the true *Hour-lines* of such an *Horizontal Plane*, from 6 in the *Morning* until 6 at *Night*; but for the Hours of 4 and 5 in the *Morning*, and 7 and 8 in the *Evening*, they are delineated by producing 4 and 5 in the *Evening*, through the *Centre* R, and 7 and 8 in the *Morning*, extending them out unto the other side of the *Plane*, so shall you have those *Hour-lines* also on your *Plane* delineated as you

Fig. 70. see in the *Figure*. The *Stile* of this *Plane* may be a thin *Plate* of *Brass*, cut exactly unto the quantity of an *Angle* of $51^{\circ} 32'$, and set *Perpendicular* on the *Meridian-line*, for the forming of this *Stile*, take out of your *Line of Chords* $51^{\circ} 32'$, and set it from D unto e, and draw Re, which shall be the *Axis* of the *Stile*. You may also prefix the *Halfs* and *Quarters* of

Hours,

Of Dialling.

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Hours, in the very same manner as the *Hours* themselves were drawn.

§ 1.

Now to find out the Distance of the *Hour-lines* from the *Meridian*, say,

Fig. 70.

As the *Radius* or $S. 90^{\circ}$,
To the *S. of the Latitude* ;
So is the *T. of the Hour from Noon*,
To the *T. of the Hour-line from the Meridian-line*.

These kinds of *Dials* being so frequently used with us, in this *Oblique Sphere* ; for the help of the speedy Delineating of them, I have annexed hereunto the TABLE of *Longomontanus*, wherein the *Hour-lines*, for many *Latitudes*, are calculated.

The Use of the following TABLE is thus : The First Column next the Left-hand is the *Latitude*, beginning at 30 deg. and ending at 60 deg. The Second Column is the *Distances* on an *Horizontal Plane* from xii. to xi. and i. in any of those *Latitudes*. The Third is the *Horizontal Distance* betwixt xii. and x. and xii. and ii. and so on. The Last Column, next the Right-hand, is the *Complement* of the former *Latitudes* ; and the Use of it is this : The *Distances* for an *Horizontal Dial* in the *Latitude* of 30 deg. will serve for an *Erect Direct South Dial* in the *Latitude* of 60 deg. So that the *Distances* here calculated serve for two *Dials* ; as, if it be an *Horizontal* in the *Latitude* of 51 deg. 'twill be an *Erect Direct South* in the *Latitude* of 39 deg. Understand the like of others.

R 2

A TA-

§ 1.

A TABLE, shewing the Distance of the Hour-lines from the Meridian, in these Degrees of Latitude.													
The Hour-lines from the Meridian.													
xi.		i. x.		ii. ix.		iii. viii.		iv. vii.		v.		vi.	
D		MD		MD		MD		MD		MD		M	
30	7	38	16	6	26	34	40	54	61	49	90	0	60
31	7	51	16	34	27	14	41	42	62	28	90	0	59
32	8	4	17	1	27	53	42	30	63	6	90	0	58
33	8	17	17	27	28	34	43	17	63	45	90	0	57
34	8	30	17	54	29	13	44	5	64	42	90	0	56
35	8	43	18	20	29	49	44	46	64	56	90	0	55
36	8	56	18	45	30	25	45	28	65	27	90	0	54
37	9	9	19	9	31	1	46	9	65	58	90	0	53
38	9	21	19	34	31	37	46	50	66	29	90	0	52
39	9	33	19	57	32	9	47	26	66	55	90	0	51
40	9	46	20	20	32	40	48	1	67	20	90	0	50
41	9	58	20	43	33	14	48	37	67	45	90	0	49
42	10	10	21	7	33	47	49	13	68	11	90	0	48
43	10	22	21	29	34	17	49	44	68	32	90	0	47
44	10	32	21	50	34	46	50	14	68	52	90	0	46
45	10	43	22	12	35	15	50	45	69	14	90	0	45
46	10	54	22	33	35	44	51	16	69	37	90	0	44
47	11	5	22	53	36	10	51	43	69	53	90	0	43
48	11	16	23	12	36	35	52	9	70	10	90	0	42
49	11	26	23	32	37	1	52	35	70	28	90	0	41
50	11	36	23	51	37	27	53	1	70	43	90	0	40
51	11	46	24	9	37	50	53	24	70	58	90	0	39
52	11	56	24	26	38	13	53	46	71	12	90	0	38
53	12	5	24	44	38	36	54	8	71	27	90	0	37
54	12	14	25	2	38	59	54	30	71	41	90	0	36
55	12	23	25	18	39	18	54	50	71	53	90	0	35
56	12	32	25	33	39	38	55	9	72	4	90	0	34
57	12	46	25	49	39	58	55	28	72	16	90	0	33
58	12	48	26	5	40	18	55	46	72	27	90	0	32
59	13	56	26	19	40	36	56	1	72	38	90	0	31
60	13	58	26	30	40	53	56	15	72	47	90	0	30

PROP.

P R O P. VI.

How to draw the Hour-lines on an Erect Declining Plane.

These Planes are made to set on the Sides of Houses, wherein the Meridian is always a Perpendicular, drawn on the Plane, on whose top is the Centre, where the Substile and the Hour-lines all meet. Fig 71.

Now before we can delineate the Hour-lines on any such Planes, Two things must be given; as, the Latitude of the Place, and the Plane's Declination: By having which, we must find these Three things; viz. the Pole's Height above the Plane; the Distance of the Substile from the Meridian; and the Plane's Difference of Longitude.

For the finding of which Requisites, by Geometrical Projection, we describe on the Dial Plane these Circles of the Sphere, viz. the Horizon, Meridian, and Equinoctial; which being described in their true Position on the Plane, we proceed thus:

Admit it be required to make a Direct South Dial, on an Erect Direct South Plane, Declining Westward $24^{\circ} 20'$, in the Latitude of $51^{\circ} 32'$. Fig. 71.

Now in order to find the Requisites before-mention'd, describe the Circle ZHNO, and cross it with the two Diameters ZQN, and HQO: Now Z is the Zenith, N the Nadir, ZQN the Hour-line of 12, HQO the Horizon. Now seeing the Plane declines S. W. $34^{\circ} 20'$; make Na, and Ob, each equal to $34^{\circ} 20'$: Then a Ruler laid from Z to a, will cut the Horizon in S the South Point of the Horizon, through which draw

§ 1. the *Meridian* ZSN, whose *Centre* is at Y, found as in the 4th *Proposition* aforegoing: Then a Ruler laid from Z to b, will cut the *Horizon* in W, the *West* Point thereof. Now the *Horizon*, and the *Meridian* being projected on the *Plane*, take out of your *Line of Chords* $51^{\circ} 32'$, which place from H unto c, and from N unto d; then lay a Ruler from W unto c, and it cutteth the *Meridian* in P, the *Pole* of the *World*. Then through P and Q draw the Line PQD, which representeth the *Axis* of the *World*, and the *Substilar-line* of the *Dial*; then lay a Ruler from W to d, it cutteth the *Meridian* in Æ, so is W Æ two Points through which the *Equinoctial* must pass, whose *Centre* is found, as afore to be at M, (being always in the *Axis*) of the *World*,) so have you on your *Plane* the *Horizon* HQO, the *Meridian* ZPS Æ N, and the *Equinoctial* LÆKWG, described on the *Plane*, as required.

Now, *First*, To find the *Pole's Height* above the *Plane*, which in this *Scheme* is represented by BP, lay a Ruler from G unto P, and it shall cut the *Plane* in V; then measure the Distance BV on your *Line of Chords*, and you will find it to contain $34^{\circ} 33'$, which is the *Pole's Height* above the *Plane*.

Fig. 71. *Secondly*, To find the *Distance* of the *Substile* from the *Meridian*, represented in the *Scheme* by the *Arch* ZB, or ND; which measured as afore, will appear to be $18^{\circ} 08'$, the *Distance* of the *Substile* from the *Meridian*.

Thirdly, To find the *Plane's Difference* of *Longitude*, which in the *Scheme* is represented by the *Angle* ÆPK, lay a Ruler from P unto Æ, and

and it cutteth the Plane in X, then measure the Arch DX, as afore, and so will you find the Plane's Difference of Longitude, to be $30^{\circ} 00'$: Thus by Geometrical Projection have we found all the three Requisites: Now to find them by Arithmetical Calculation observe these Analogies or Proportions. § 1.

1. For the Poles height above the Plane, say:
 As Radius or S. 90° ,
 To Sc of the Latitude $38^{\circ} 28'$;
 So is Sc. of the Declination $65^{\circ} 40'$;
 To S. of the Poles height above the Plane $34^{\circ} 33'$.

2. For the Distance of the Substile, from the Meridian, say:
 As the Radius or S. $90^{\circ} 00'$,
 To the S. of the Plane's Declination $24^{\circ} 20'$;
 So is Tc of the Latitude $38^{\circ} 28'$,
 To the T. of the Substilar Distance from the Meridian $18^{\circ} 10'$.

3. For the Plane's Difference of Longitude, say: Fig. 71.
 As the Sc. of the Latitude $38^{\circ} 28'$,
 To the Radius or S. $90^{\circ} 00'$;
 So is S. of the Substilar Distance $18^{\circ} 10'$,
 To the S. of the Difference of Longitude 30 Deg.

Or it may be found thus:
 As the S. of the Latitude,
 To the Radius;
 So is the T. of the Declination,
 To the T. of the Difference of Longitude required.

These things found, we come now to shew how the Hour-lines may be projected. To project which observe, First, to lay a Ruler from

R 4

P the

§ 1. P the *Pole* of the *World*, to Æ the *Interfection* of the *Equinoctial* with the *Meridian*, and it will cut the *Plane* in x , where begin to divide the *Semicircle* LxG , into 12 Equal Parts in the Points $\bullet, \bullet, \bullet, \bullet, \text{\&c.}$ Then lay a Ruler from Q , to every of those Parts, and it shall cut the *Equinoctial*; and divide it into 12 unequal Parts, in the Points $*, *, *, *, \text{\&c.}$ Then a Ruler laid from P the *Pole* of the *World* unto each of these Points, it will divide the *Plane* into 12 unequal Parts in the Points $I, I, I, I, \text{\&c.}$ Then by a Ruler laid from the *Center* Q , to those Points, draw right Lines, which shall be the *true Hour-lines* proper unto such a *Declining Plane*, as you see plainly demonstrated by the *Scheme*.

Now the *Substilar Line* falleth in this *Dial*, just on the *Hour-line* of 2, in the *Afternoon*, because the *Plane* declineth *Westerly*. The *Angle* of the *Stile* is $DQR\ 34^\circ\ 33'$. which may be either a *Plate* or *Wyre*, brought into such an *Angle*, which must be placed *Perpendicular* to the *Plane*, and directly over the *Substilar Line* $QD\ 2$.

Now the distance of the *Hour-lines*, from the *Substilar Line*, may also be found by this *Analogy* or *Proportion*.

Fig. 71. As the *Radius*,
To the *S. height* of the *Pole* above the *Plane*;
So is the *T* of the *Hour-line* from the *Meridian* of the *Plane*,
To the *T. of the Hour-line* from the *Substile*.

Thus have you compleated your *Dial*; as you see in the *Scheme*, and here you may take notice that having finished a *West Decliner*, you

you have also made an *East Decliner*; if you only convert the *Hour-lines* of the *West Decliner*, in such manner as you see in *Fig. 72.* on the *East Decliner*, and compleat all as you see in that *Scheme.* § 1.

Thus I have explained the making and delineating of the best and most useful *Dials*, both by *Geometrical Projection*, and also by *Arithmetical Calculations*, in as brief and compendious a manner as possible. There are sundry other kind of *Dials*, as *Incliners*, *Decliners*, and *Recliners*, which being not so useful, for brevity sake, they are here omitted: As for *Instrumental Dials*, as *Quadrants*, *Rings*, *Cylinders*, &c. Which depend on the Sun's height, I refer you to Mr. *Edm. Gunter's* Book, wherein they are largely described.

As for the Beautifying and Adorning of those *Dials*, &c. by describing on them the *Equinoctial*, *Tropicks*, *Parallels of Declination*, *Parallels of the Sun's Place*, *Length of Days*, the *Sun's Rising and Setting*, *Jewish*, *Italian* and *Babylonish Hours*, *Almicantbars*, *Azimuths*, *Circles of Position*, the *Signs Right Ascending*, *Descending*, *Culminating*, &c. I do advise you to consult Mr. *Gunter*, Mr. *Foster*, Mr. *Wells*, and Mr. *Holwel's Works*, all which Authors have very learnedly shewed the describing of them, by several large *Schemes* and *Figures*, for the plainer Illustration thereof.

Now seeing the *Latitude* of a Place must be first known, before a *Dial* can be made to it, I have therefore hereunto annexed a Table of the *Latitudes* of all the principal *Cities*, *Towns*, and *Islands*, in and about *Great Britain* and *Ireland*;

§ I. Ireland; so that if you are to make a *Dial*, for any of those Parts, you may have recourse to this Table, and make your *Dial* to the Latitude of that Place, which you find to be the nearest to the Place, for which you are to make your *Dial*.

A TABLE of the Names and Latitudes of all the Principal Cities, Towns and Islands, in and about Great Britain and Ireland.

ENGLAND.		D. M.
A	Rundel	51 00
	Bedford	52 15
	Barwick	55 54
	Bristol	51 35
	Buckingham	52 10
	Cambridge	52 20
	Canterbury	51 25
	Carlisle	55 20
	Chichester	50 48
	Chester	53 18
	Colchester	52 08
	Dover	51 20
	Derby	53 00
	Dorchester	50 50
	Durham	54 56
	Exeter	50 48
	Falmouth	55 22
	Glocester	51 57
	Guilford	51 12
	Hartford	51 54
	Hereford	52 17
	Huntington	52 30
	Ipswich	52 20
	London	51 30
	Lincoln	53 20
	Leicester	52 45
	Lancaster	54 15
	Northampton	52 24
	Norwich	52 45
	Nottingham	53 00
	Newcastle	55 12
	Oxford	51 50
	Portsmouth	51 08
	Plimouth	50 36
	Reading	51 40
	Salisbury	51 12
	Stafford	52 50
	Stanford	54 44
	Shrewsbury	52 50
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Fig: 68

Fig: 69

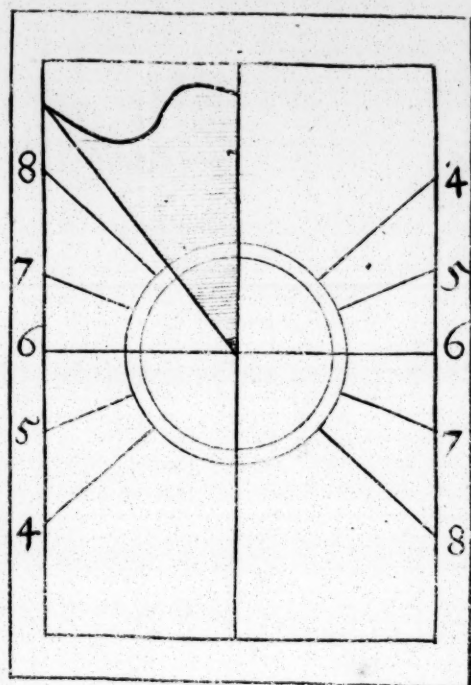
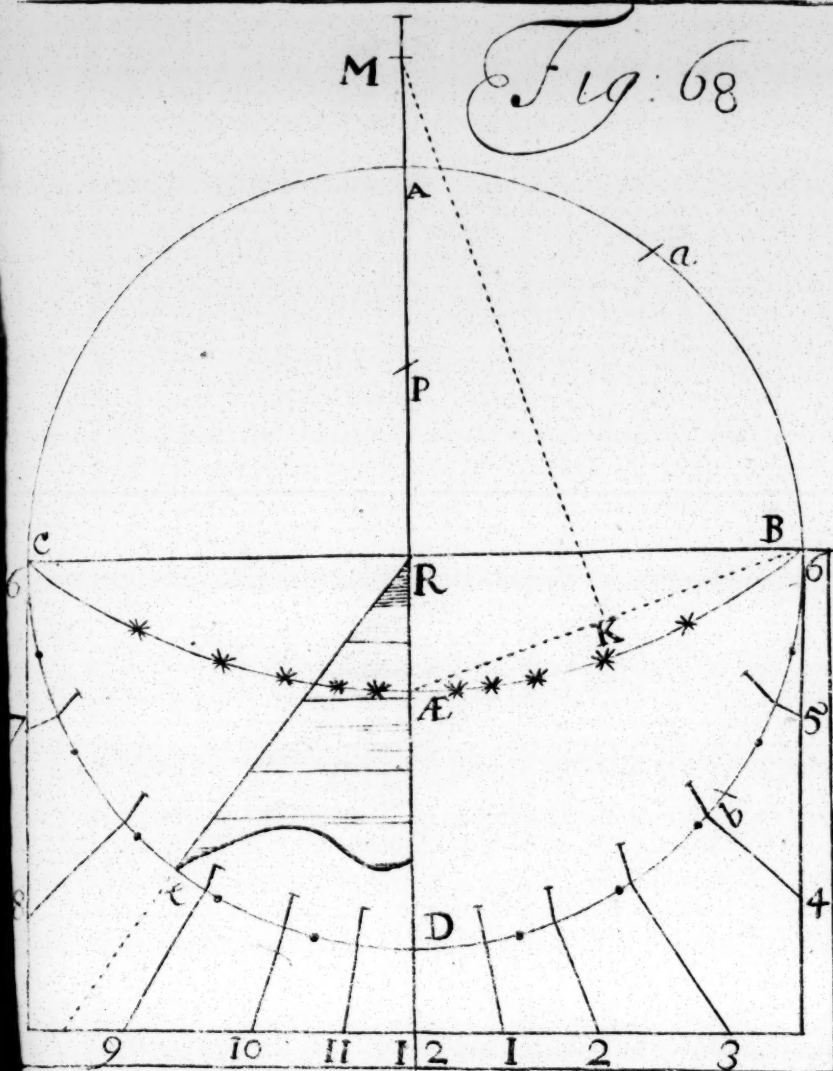
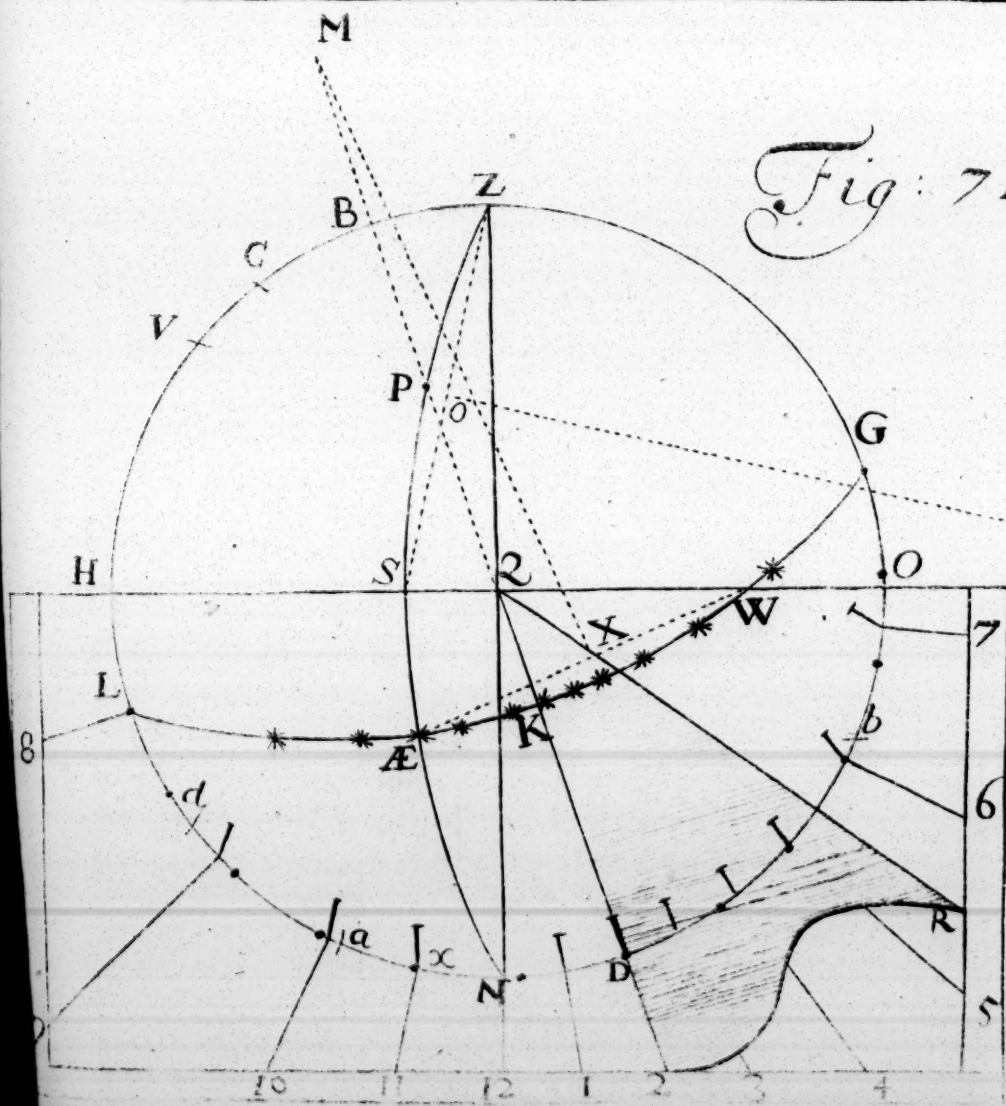


Fig: 71



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	D. M.
Truro	50 30
Winchester	51 03
Worcester	52 25
Warwick	52 30
Tork	54 00

	D. M.	§ l.
Skyrassin	58	38
Sterling	56	12

WALES.	D. M.
A nglesey	53 28
Barmouth	52 50
Brecknock	52 01
Cardigan	52 12
Caermarthen	51 56
Cornarvan	53 16
Denbigh	53 13
Flint	53 17
Landaffe	51 35
Monmouth	51 51
Montgomery	51 56
Pembroke	51 46
Radnor	52 19
St. David	52 00

SCOTLAND.	D M
A berdeen	57 30
Dunblain	56 21
Dunkel	56 48
Edinburgh	56 00
Glasgow	55 58
Kinfaile	57 44
Orkney	60 06
St. Andrews	56 40

IRELAND.	D. M.
A Ntrim	54 38
Arglas	54 10
Armagh	54 14
Carterlagh	52 41
Clare	52 34
Corke	51 55
Droghedagh	53 58
Dublin	53 55
Dundalke	53 52
Galloway	53 02
Kenney	52 30
Kildare	53 00
Kings Town	53 08
Knockfergus	54 40
Kynsale	51 41
Lymerick	52 30
Queens Town	52 52
Waterford	52 09
Wexford	52 18
Youghall	51 53

ISLANDS.	D. M.
W ight	50 48
Portland	50 30
Man	54 24
Lundey	51 22
Fersey	49 12
Guernsey	49 03

CHAP.

CHAP. XIII. Of FORTIFICATION.

THE Utility of this Mathematical Art called FORTIFICATION, or Military Architecture, is so well known, that it needs not my Commendation, and therefore to speak any thing thereto, were but to light a Candle before the Sun.

In the handling of this Part of the *Mathematics*, I shall be as brief as possible, yet as plain as can be desired: In the prosecution of which, I will use this Method. *First*, Give you the most principal *Definitions* or *Terms* belonging to this Art. *Secondly*, Prescribe the most conducting *Maxims* or *Rules* herein observed. *Thirdly*, Shew how to delineate the Ground-line of any *Fortification*, according to the several Proportions, used by the best and most experienc'd Engineers of *Italy*, *France*, *Holland* and *England*; *Fourthly*, Describe the Construction of the chief and principal Outworks now in use; and *Lastly*, lay down some general *Maxims* or *Rules*, by most *Modern Authors* observed in *Irregular Fortifications*.

S E C T. I.

Of the Definitions of the Lines and Angles, belonging to the Principal Ground-work of any Regular Fortification.

§ I. *Defin.* I. **T**HE Exterior or outward Line, which boundeth the Rampart, at the Foot next the

the *Ditch*, is the principal and only Line to be regarded in all *Regular*, or *Irregular Fortifications*, being the *Basis* on which all the other Lines and Parts of the *Fortification* do depend.

2. The *Exterior Polygon* is the outward side of any *Regular Figure*, as in the *Hexagon* (which Figure I shall make use of through this Tract) the side A A, is the *Exterior Polygon*.

3. The *Interior Polygon*, is the inward side of any *Regular Figure*, as in the *Hexagon* is noted by any of the sides between P and P. Fig. 73.

4. The *Bastion* or *Bulwark*, is that great Work of any *Fort*, that advanceth it self towards the *Campaigne*, and here are six all marked with B, the Lines which terminate them, are two *Gorges*, two *Flanks*, and two *Faces*.

5. The *Demi-Gorge* or *Gorge-line* P C, is half the Entrance into the *Bastion*, and terminates the Point C, whereby the *Flank* shall be raised. Fig. 73.

6. The *Flank* is another Out-line of the *Bastion* as C F, which terminateth the *Curtain* and *Face*.

7. The *Face* is the utmost Line of the *Bastion*, as F A, two Lines thereof do form the *Angle* of the *Bastion* A, or the *Flanked Angle*.

8. The Line forming the *Flank* F F, is a prick'd Line, made use of by the *Dutch Engineers*, and others.

9. The *Capital* is A P, part of the Line coming from the Center ©, terminated at the Point of the *Bastion* A. Fig. 73.

10. The *Curtain* is that Part of the *Interior Polygon* C C, which lieth betwixt the two *Bastions* B, and B.

11. The

§ 1.
Fig. 73.

* Because the length of a
Musket-shot doth not much
exceed that Distance.

11. The Line of Defence is AC, passing from A, the Point of the Bastion, to C the Angle of the Flank and Curtain, and ought never to exceed 800 English Feet *.

12. The Line Stringent, is the Line coming from the Point of the Bastion A, and prolonged on the Face AF, to the Curtain D, which sheweth that DC, the Part of the Curtain, (by some called the second Flank) will scour the Face.

13. The Diameter of the Interior Polygon, is the Line $\odot P$, coming from the Center thereof $\odot$.

14. The shortest Line from the Center unto the Curtain is $\odot m$. These are the Definitions of the principal Lines, appertaining to the Groundwork of any Regular Fortification, the Angles followeth.

Fig. 73.

15. The Angle of the Center of the Polygon is $P \odot P$.

16. The Angle of the Polygon PPP, is always the Complement of the Angle at the Center, or remainder unto 180 Degrees.

17. The Angle of the Triangle $PP \odot$ is always the one half of the Angle of the Polygon PPP.

Fig. 73.

18. The Angle of the Bastion, or the Flanked Angle FAF, is exposed unto the Batteries of the Besiegers, and formed by the two Faces, FA, and FA, which ought never to be less than 60, nor much above 100 Degrees.

19. The Angle of the Espaule, or Shoulder, is formed by the Face and Flank, as AFC.

20. The Angle of the Flank CCF, is formed by the Curtain and the Flank, and is most commonly a Right Angle, but by some later Ingenieurs,

neers, is made *Obtuse*, or more than a *Right* § 1.
Angle, or 90 Degree.

21. The Angle made by the two Lines *Fitchant* A t a, is called the Angle of the *Tenaile*.

22. The Angle forming the *Flank* is CPF, which Angle is made use of by most of the *Dutch Engineers*. Fig. 73.

S E C T. II.

Of General Maxims or Rules observed in Fortifications.

1. **T**HAT all the Parts of the Place be of § 2.
Cannon-Proof flanked, *i. e.* defended from another Place, which Place is no farther distant than the reach of a *Musket-shot* from the Place to be *Flanked* or defended *.

* Because the Defence ought to be easie, quick, certain, and of little charge, all which quali-

ties the *Musket* hath and the *Cannon* hath not, therefore the Defence of Fortification ought to be measured by the Port of a *Musket*, and not by that of a *Cannon*.

2. That in all the Place there may be no Part of the *Wall*, or Out-side of the *Rampire*, that is not seen from the Top to the Bottom of the *Mote* or *Ditch*.

3. That the *Bastions* are large, and full of Earth, and not empty; the bigger they are, they are the more to be esteemed, there being the more room to *Intrench*, in case of necessity: whose *Gorge* let be at least 35 *Fathoms*, and their *Flank* at least 18 *Fathoms*.

4. That

§ 2.

4. That the *Angle* of the *Bastion*, or *Flanked-Angle*, be not much above 90, nor much less than 60 *Degrees*, for in the former it would lie too very *Obtuse* and open at the *Point*; and in the latter it would be too slender, and so easily to be Battered down by the *Enemies Cannon*.

5. That the *Angle* of the *Flank* may be somewhat *Obtuse*; neither is there any more virtue in a *Right-Angle* than in any other for the Defence of the *Fort*.

6. That the Length of an extended *Curtain* be not above 135 *Fathoms*, nor the single above 80 *Fathoms*, nor be less than 40 *Fathoms*, to be well defended from two *Flanks*.

7. That the *Rampire* be so wide, that a *Parapet* of *Earth Cannon-proof* may be erected thereon, and a *Terraplana* left full wide for the *Ordnance* to be recoiled.

8. That the *Mote* or *Ditch* be at least 20 *Fathoms* broad, and as deep as possible. Now dry *Motes* in great *Cities* are to be preferred before others that are full of *Water*, to facilitate the *Sallies*, the *Relief* and *Retreat* of the *Besieged*; and in small *Fortifications* the *Motes* full of *Water* are the most Esteemable, because in such *Sallies* are not necessary, and *Surprises* are very much to be feared.

9. That the *Parts* that are most remote from the *Center*, be commanded by those which are nearest to it.

10. That the Defence of a *Face* is much stronger when the *Angle* made by the *Face* and *Exterior Polygon* is a *Great Angle*; this *Maxim* is so very essential, that it will try the Goodness

Goodness of any *Fortification* whatsoever: Thus I have described the 10 chiefest *Maxims* necessary for Good *Fertifications*.

S E C T. III.

Of the Construction and making of the principal Ground-Line of a Fort, according to the most Modern Ways, used by the Italian, Dutch, French, or English Ingineers.

I. Of the Italian Fortifications.

Gennaro Maria, Mathematician to the Catholick King, wrote at Florence, his *Elements of Military Architecture*, Entituled, *Breve Trattato delle Moderne Fortificazioni*. This Italian Author was a very Learned and Skilful Mathematician, and famous in his Nation. In his said Book, Printed 1665, he makes the Interior Polygon 800, and not less than 600 Feet, his Demi-Gorge he makes $\frac{1}{8}$ of it, and so for the Flank of the Quadrangle. But for the Pentagon, and all Figures above, he makes the Flanks $\frac{1}{5}$ Part of the Gorge more, and he placeth his Flank at Right Angles with the Curtain.

Supposing his Interior Polygon 1000 Parts, his Gorges will be 125, and in the Quadrangle the Flanks will be 125, but of the Pentagon, and all above 138 Parts. For the Faces, he makes them to fall on the third Part of the Curtain, unless in the Square, which he allows no second Flank.

S

P R O P.

§ 3.

§ 3.

P R O P. I.

To Fortifie a Hexagon according to this Author's Proportion.

Fig. 74. First describe the Hexagon PPP, &c. then divide the Interior Polygon PP, into 1000 equal Parts, take 125 for the Gorges, and set it from P to C. Then on C raise a Perpendicular, make it equal to 138 Parts, for your Flanks CF, then draw the Face AF, falling on the third Part of the Curtain CC, at D, and so do on every Bastion, until the Work is compleated.

II. Of the French Fortifications.

Monsieur De la Mont, in his Fortifications Offensive and Defensive, Printed 1671: And Monsieur Manesson Mallet in his late Work, Entituled, Travaux de Mars, Printed 1672, assign these Proportions for the laying down the Ground-Line of a Fort.

Both these Authors make the Interior Polygon 768 English Feet, which they divide into 5 Parts, and taking one for the Gorge $153\frac{1}{5}$ Feet. Both divide it into 3 Parts, and take one for the Capital, that is 256 Feet.

Fig. 74. Now our first Author De la Mont, makes the Flank to stand at Right-Angles and takes 115 Feet for it, which is $\frac{1}{4}$ of the Curtain, and so draws the Bastions, in all save the Quadrangle and Pentagon, which he makes to have no second Flank.

P R O P. II.

To Fortifie a Hexagon according to the Proportion of De la Mont.

First describe your Hexagon PPP, &c. Now supposing your Interior Polygon PP, 1000 Parts, the

the Capital 333, the Gorge 200, and the Flank § 3.
150 Parts, take out of your Triangular Scale, Fig. 74.
Fig. 75, (which is made for the more speedy
Delineation, according to this Proportion of
De la Mont) P A for the Capital, and prick it
off from P A on all the *Bastions*. Then take
P C, and prick off all the *Gorges* from P to C.
Then take F C and prick it off at *Right-Angles*,
from C to F. Lastly draw all the *Faces* A F,
A F, &c. so is your *Hexagon* compleat, as re-
quired.

P R O P. III.

To Fortifie a Hexagon according to Maneffon
Mallet's Proportion.

Now our Author *Monsieur Maneffon Mallet*,
in his Works Entituled, *Travaux de Mars*, de-
viates from our former Author, only in this :
that as *De la Mont* did place his *Flanks* at
Right-Angles, he places them at 98 Degrees
with the *Curtains*, and leaves no *second Flank*
in all his *Fortifications*.

Therefore having described the Polygon PP, Fig. 74.
&c. divide PP into 1000 Parts, prick off the
Capitals P A 333, and the Gorges P C 200, then
lay off the *Flanks* C F 150 Parts, at an Angle,
of 98 deg. with the *Curtain* C C (by Prop. 5.
S. 1. Chap. 4.) and draw all the *Faces* A F, A F,
&c. Falling on C the Point of the *Flank*
and *Curtain*, so shall your *Hexagon* be fortified,
as was required.

III. Of the Dutch Fortifications.

The Emperor *Ferdinand III.* hath learnedly Fig. 74.
altered the Method of *Fritach, Dogen, Goldman*
S 2 and

§ 3. and *Faulhaberus*, all which were Dutch *Engineers*, and wrote large Volumes on this Subject; In his Works Intituled, *Amuffis Ferdinanda*, Publish'd 1654; by turning their way of working by *Angles* into working by *Sides*.

Thus he setteth down a *Catholick* Way of delineating the *Sides* or *Lines* of any Fort by his 60 *Prop.* thus: the *Interior Polygon* to be 66, the *Capital* 24, the *Gorge* 15, and the *Flank* 12. Or in making the *Interior Polygon* 22, the *Capital* 8, the *Gorge* 5, and the *Flank* 4. Or yet making the *Interior Polygon* 1000, the *Capital* 363, the *Gorge* 227, and the *Flank* 181, this is an *Epitome* of all the Dutch Fortifications, and is general excepting for the *Square*, which

Fig. 74. must have no *second Flank*.

P R O P. IV.

To Fortifie a Hexagon according to the Emperor's Proportion.

Fig. 74. First describe the *Polygon* PPP, &c. divide PP, &c. into 22 Parts, take 8 for the *Capitals* PA, which prick off all round from P to A, take 5 for the *Gorges*, which prick off all round from C to P, then take 4 for the *Flank* CF, which prick off all round at *Right-Angles* from C to F, lastly draw the *Faces* AF, AF, AF, &c. So is the *Hexagon* compleated as was required.

IV. Of the English Fortifications.

His Late Majesty of Great Britain, Carolus II of Ever-Blessed Memory, hath much facilitated the Method of Count Pagan, who in his *Fortifications*, Printed at Paris, 1645, did place the
Flanks

Flanks at Right-Angles with the Line of Defence, and he works by the Exterior Polygon. Now His Majesty places the Flank at Right-Angles with the Line of Defence of the Interior Polygon, and works after another manner: Count Pagan makes the Proportion of the Grand Royal Fort. Supposing the Exterior Polygon to be 1000 Parts, will make the Perpendicular MT Fig. 74. to be 150, and the Complement of the Line of Defence TC to be 185, which may serve for a general Proportion be the Length what it will, only in a Square the Proportions must thus be altered in the Grand Royal Fort, the Perpendicular MT must be 162, in the Mean R 144, and in the Petty Royal 126, the Complement of the Line of Defence for the Grand Royal Fort is 228, and for the Mean Royal Fort 198, also for the Petty Royal Fort 198.

P R O P. V.

How to Fortifie a Hexagon according to Count Pagan's Proportion.

To delineate this Work draw a Line, about the Middle whereof, as at M, set off MA, the half of the Exterior Polygon 500 Parts, which makes the Exterior Polygon 1000, then on M by Prop. I. §. I. Chap. 4.) raise the Perpendicular Mm, which make Mt, MT Equal to 150, then draw ATC, and ATC, then take 185, and place it from T to C, and to C, and draw TC for the Curtain, then on the Points C raise Perpendiculars CF, to the Line of Defence CA, for the Flanks, so have you also the Faces FA. Then on the Points A set off half the Angle of the Figure, to wit 60° (as you see in the Table,

Of Fortification.

§ 3. in Page 38.) and draw the *Lines* OA and OA
 so shall O be the *Center* of the *Figure*, and PO
 the *Gorge*, and AP the *Capitals*: then finish
 each *Bastion* at your own *Discretion*, and the
 Work is finished as required.

P R O P. VI.

To Fortifie a Hexagon according to the way pre-
 scribed by His Majesty Carolus II.

His Late Majesty Charles II. hath much fa-
 cilitated this Work, as will appear in this fol-
 lowing *Example*, by making the *Line of Defence*
 stand at *Right-Angles* with the *Flank* of the In-
 terior Polygon, by this Table, which suppose

Fig. 74. the Interior Polygon to be 1000. Then

Polygons	4	5	6	7	8	9	10	Straight- Lines.
Capital	398	437	367	333	312	300	291	233
Gorge-Line	155	196	203	242	252	260	263	300

Now to describe the *Hexagon* PP, &c. The
 divide the *Interior Polygon* PP, into 1000 Parts
 take 367 and prick off all the *Capitals* PA
 Then take 203 and prick off all the *Gorge*
 from P to C. Now draw the *Lines of Defence*
 AC and AC, &c. Then at C set the *Flanks* at
Right-Angles with the *Line of Defence* AC,
 shall FC be the *Flank*, and FA the *Faces*, then
 finish every *Bastion*, and your *Hexagon* is For-
 tified as was required.

☞ That

Thus have I set down the several Ways and Rules for laying the Fundamental Ground Line, from the most considerable Engineers of this last Age, out of all which it's most agreeable to those Authors, and to Practice, to take $\frac{1}{3}$ of the Interior Polygon for the Capital, $\frac{1}{5}$ for the Gorge and Flank, which leaves $\frac{1}{5}$ for the Curtain, and let this be taken for a general Rule, where the Flank and Curtain stand at Right-Angles.

§ 3.
Observe
this for a
general
Rule in Re-
gular For-
tification.

P R O P. VII.

By the Semicircle to lay down on the Ground any of the former Fortifications.

Having drawn the Plot of your Fort on Imperial Paper, or Vellom, and if it be a Regular Fort you need not describe it but two Half Bastions from the Center, for that will be sufficient. Having such a Plate, whose Length is set down on each respective Line, and all proper Angles expressed, will not only be useful for laying down the Work, but for finding the Solidity of the Ramparts, Parapets, and the other Earth-Works. See Fig. 76.

If it be in such a Place, that from the Center of the Fort, all the Angles may be seen, place your Semicircle at Z, and lay off all the Angles of the Center, which here is 60° ; then mark out the Diametrical Lines, and making them their due Length, as by your Plate they appear to be, set Piquets on all the PP' upright with the Plane: Then take up your Instrument and place a Piquet at Z: Then lock-spit out all the Polygons PP: Then mark out the Gorges CP, then set out the Flanks CF, Fig. 76.

§ 3. either at *Right-Angles*, or as otherwise required: Then *lock-split* out the *Flanks* CF, and the *Face* AF, having first set off the *Capital* PA, so is the *Fort* lined out for the *Ground-Line*.

Case 2. But if there be *Houses* and *Obstacles* in the way, that from the *Center* all may not be seen, then must you mark out any one *Side* and measure it, and at each *End* set off the *Angles* of the *Polygon*, (which here is 120°) and draw *Side* after *Side* until all be finished: Then finish the *Bastions* as before, and here great *Care* must be had, or else you will run into infinite *Errors*.

 But you have *Liberty* Experimentally to alter any of the former *Proportions*, as you have occasion, and as will best serve the *Place*, as you see by the *Fortifying* a *straight-lined*

Fig. 77. Figure: Fig. 77. where in *Count Pagan's* or in *Manesson's* way it may not be allowed without some *Alterations*.

S E C T. IV.

The Dimensions and Measures of the Rampires, Parapets, Mote, Covert way, and its Glacis, with the Breast-work.

§ 4. THE *Rampire's* *Thickness* and *Height* must receive its *Determination* from the *Judgment* of the *Engineers* and *Purse* of the *Prince*.

Fig. 78. The *Height* TS, must not exceed 18 *Feet*, nor be less than 10; the *Thickness* may be from 50 to 80 in all *Royal Works*, and according as *Earth* is to be had; the *Slope* of the *Inward*

ward Side *Rampire* TR, is commonly a Foot for a Foot, therefore RS the *Talu*, will be equal to the *Height* TS; so if TS be 18, RS will be 18, if 15 then 15 Feet.

§ 4

The *Outward Slope* is generally proportioned a Foot for a Foot, so if the *Height* OQ be 8, the *Talu* OA will be 9 Feet, &c.

The *Height* of the *Parapet* ZD must always be 6 Feet, the *Exterior Height* PM must be 4 Feet, the *Thickness* of the *Parapet* DM in *light Earth* must be 20 Feet, in *stiff Earth* 16, and in *solid Rough Clay* 14 Feet. Suppose it be 18 Feet, PM will be 4 Feet, MQ 2 Feet, LD 1 Foot, so will the lower *Thickness* LQ be 21 Feet.

The *Height* of the *Foot-Bank* VX is 1 Foot, and *Thickness* VL 3 Feet.

The *Lizier* must be made so wide as to support the *Rampire* from slipping into the *Ditch*, and is taken from 3 to 10 Feet; the *Mote* or *Ditch* may be from 70 to 130 Feet, that is, from E to G, and the *Depth* IF may be 8, 10 or 12 Feet, the little *Ditch* at the Bottom of the *Mote* represented by c q g, must be as large and deep, as the Earth and Work will give leave.

The *Covert-way* is left about 18 Feet wide from G to C, on which is placed a *Parapet* and *Bank*, like that on the *Rampire*, which *Parapet* must slope so into the *Campaigne*, that a straight *Line* drawn from Z the Top of the *Rampire*, may terminate F d the Slope of the *Glacis*.

P R O P.

How to lay down the Profile of the Work, according to this Table.

	Feet
The Base of the Rampart R A	70
Height TS or OQ	16
Inward Talu RS	16
Outward Talu OA	8
Base of the Parapet LQ	21
Inward Height ZD	6
Outward Height MP	4
Outward Talu MQ	2
Inward Talu LD	1
Breadth of the Foot-bank V L	3
Height of it V X	1
The Terra Plana T V	22
The Berm or Lizier AE	3
The Mote's Breadth EG	112
The Depth of it IF	12
Breadth at Bottom FH	88
The Talu's EI or KG	12
Breadth of the little Ditch c g	18
Depth of it	5
The Covert-way GC	18
The Base of the Glacis or Counterscarp C d	60
The Height and Breadth of the Foot-bank, and Height of the Parapet, as before.	

Now to lay down this Profile do thus; draw a Base or Ground-Line of convenient Length, as RSOACG d, then by a Scale of 20 or 30 at most in an Inch, representing Feet, take out of it 70 for R A, 16 for R S, 8 for O A, 3 for A E, 112 for E G, 18 for G C, and 60 for

for CD, and mark them off on your Paper, as in Fig. 78. set also from E to I and K to G, 12 Feet for the *Talu's* of the *Mote*, at S, O and C raise, and from I and K let fall *Perpendiculars*. Then take 16 Feet for ST and OQ, 12 for IF and KH, and 6 for Cf, and draw RT, TQ, QA, EF, HG, Cf, fd. Then from Q set off QL 21, LV 3, QM 2, and LD 1, and raise the *Perpendiculars* MP 4, DZ 6, and VX $1\frac{1}{2}$. Then draw VX, XY, YZ, ZP and PQ, and make the little *Ditch* by its Measure, so is the *Profile* perfected.

§ 4.

The *Solid Content* of those *Earth-Works* may easily be attained by the former Rules, which *Content* being got in Feet, Divide the *Product* by 324, the *Quotient* shall be the *Solid Floors* contained therein, a *Floor* being 18 Feet Square and 1 Foot Deep.

S E C T. V.

Of the Dimensions and Construction of Platforms, Caveleers and Camezats in the Flanks,

1. **P**latforms are Plantations where Guns are to be placed, and are commonly made of Plank and Sleepers; there needeth for one Gun to be but one Platform, which must be 8 Feet broad next the Parapet, and 14 Feet wide at the other End, and their Length should be 18 Feet.

§ 5.

2. An *Embrasure* is the Port-Hole made in the Parapet, which towards the Gun must be 4 Feet wide, and towards the *Campaigne* 8 Feet wide, whose Height must be proportioned unto the Wheel of the Carriage, and are 16, 18 or 20 Feet asunder.

3. Ca-

§ 5.

Fig. 79.

3. *Cavaleers* or *Mounts*, are *Massy-pieces* of *Earth* raised on or near the *Rampart*, above the *Parapet*, on which *Ordnance* and *Small-shot* may be planted. As to their *Construction* I shall follow the *Method* of *Manesson*, who places them in the *Gorge* of the *Bastion*, and gives this Rule for it, [saith he] “Lengthen out the *Line* of *Defence* to E, until it cut the *Capital*, the *Center* of your *Cavaleer* shall be the middle Point betwixt P and E, to wit at F, then with the *Distance* of 84 Feet on the *Center* F strike a *Circle*, which shall be the *Base* of your *Cavaleer* : Now its *Height* ought to be at least 20 Feet ; and if the *Work* be to be faced with *Stone* or *Brick*, it needs not have a *Talu* above $3\frac{1}{2}$ Feet, so that the *Diameter* at the *Top* will be about 153 Feet, whereon set a *Parapet* of 20 Feet *Seat*, and 4 Feet *high*, and other *Dimensions* as aforesaid, in the *Rampire*, and there will be a *Terra-plana* at the *Top* of above 100 Feet, whereon six *Pieces* of *Ordnance* may be planted, making *Embrasures* and *Platforms* as was last directed.

4. *Cazemats* are made in the *Corners* of the *Flanks*, and are several *Platforms* for *Guns* to be planted on, thereby to be hid from the *Battery* of the *Enemy* : As to the *Construction* I shall follow *Manesson's* *Directions* ; First as to the *Form*, and also to the *Measure* : [saith he] “The *Cazemate* shall take up one half of the *Flank*, and no more : The *Grand Cazemate* D B is about 7, 8 or 9 Feet from the *Level* of the *Plane* of the *Fort*, and hath a *Passage* into it from within the *Fort*. A, C is its *Parapet* of 20 or 22 Feet *Seat*, and in it let there be 3 or 4

“ Em-

Embrasures; D is the Part thereof most hid from the Enemies Cannon; F is the Magazine for this Battery; H is the second *Cazemate*, G the Ladder, and L the Magazine, and M the Parapet; this is to hold but one Gun; M is the third *Cazemate* on the Level of the Bastion, which let be all firm, in which let there be no void place. The Dimensions and Construction, according to the Method of this our Author are thus. [Saith he] "Lengthen the Line of Defence from C to G some 40 Feet, then draw CD, parallel to C f, (by Prop. 4. § 1. Chap. 4.) let CF be half of c f, so that c F may be equal to F f, then from the middle of the Face opposed, draw K F, and let it cut GD, in I, then make I L, and FM equal to 6 Feet, then make MN 66 Feet, and draw NO parallel to the Flank, which let be 24 Feet: Lastly [saith he] for the Orillon or Blind, prolong the Face F T 36 Feet, and also F V 36 Feet, then join T V, and make that Part all solid: So is your *Cazemate* finished: Let the Height of the lower *Cazemate* be 6 Feet as before, and let all the rest be completed as you see in the Figure.

§ 5.

Fig. 79.

Fig. 79.

S E C T. VI.

Of the Dimensions and Constructions of those Out-Works called Ravelins, Horn-works, Crown Works, &c.

THE Ravelin is a certain Work lying beyond the Mote or Ditch, for the covering the Curtain, Bridge and Gate; the Angle of the Ravelin

§ 6.

§ 6. *Ravelin* must nor be less than 60, not much above 100 *deg*. The manner of delineating it is thus : Lengthen out the *Middle Line* of the

Fig. 80. *Curtain* OM unto a convenient Length, then take with your *Compasses* the Length of the *Curtain* CC, and setting one Foot in F, the Point of the *Face* and *Flank*, cross the *Middle Line* in q; then laying the *Ruler* at q, and to the Point F, draw the *Lines* of the *Ravelin* qR and qS, which shall be the *Ground-Lines* of the *Ravelin*. The *Mote* surrounding it must be half the Breadth of the *Great Mote*; the *Rampart* may be 30 Feet thick, and some 6, 7 or 8 Feet high, on which may stand a *Parapet* equal to that of the *Rampire*.

Fig. 81. Now if from the *Points* F you raise streight *Lines* into the *Campaign*, at *Right-Angles* to the *Curtain*, and from the *Points* F set off FE, and FE 720 or 750 Feet, then may you join EE either with, A;

Single Tenaile : which is done by joining EE, and dividing it into four equal Parts; take one and place from D to N, and so draw EN and EN, so have you a *Single-Tenaile* IENEL, which must have a *Mote Rampire*, and *Parapet* like the *Ravelin*. Secondly it may, if occasion require, be Fortified with, A;

Fig. 83. *Horn-Work* : which is done by joining the *Points* EE, and Fortifying the *Exterior Polygon* EE as is afore taught: Or divide EE into three Parts; make ME, and EN equal to MO; then draw NM, which divide so likewise at O and P; then draw EO and EP; then at P and O raise *Perpendiculars* OQ and PR, so shall M, E, Q, O, P, R, E, N, be the *Horn-Work* which

was

was desired : which must likewise have small § 6.
Rampires and *Parapets*, as afore.

For the *Crown-Work* : From the *Center* of the *Fort* O draw OMB of a convenient Length, then from the middle of the *Ravelin* set off 1000 or 800 Feet to B, then on q, strike the *Arch* DBE, set off the *Curtain* and *Demi-Gorge* PCC, from B to F, and G both ways, then draw CF and CE, to terminate the Points I and H on the *Counterscarp*; then take $\frac{1}{3}$ Part of BF or BE, and set it from B to M, and from F to L, and from E to M; then draw LM, and MN; then for your *Demi-Bastions* make NP and LO equal to NE, &c. Then for the *Demi-Gorges* of the whole *Bastion* in the Middle, let them be equal to $\frac{1}{3}$ of the *Interior Polygon* LM or MN, viz. MY or MX; then finish the *Bastions* by drawing the *Lines of Defence*, and raising *Perpendiculars*, or making *Angles* of 98° at O, X, Y, and P, then the *Crown-work* is finished as desired. You may make *Ravelins* and other Works (beforementioned) before these *Curtains*, if occasion require. There are some other Works which are used; as *Half-Moons*, *Bonnets*, *Double Tenails*, *Counter-guards*, *Horse-shoes*, *Priests-Caps*, &c. which would be superfluous to speak of in this Place.

5. *Cittadels* are *Castles* or *Forts* of the least sort, and are the *Out-works* lastly used, which are * commonly of 4 or 5 *Bastions*, and are placed in such Order, that there may be two *Faces* and a *Curtain* towards the *Town* : the *Construction* whereof is after this manner : Lengthen out the *Line* OM, and

Fig. 84.

* Built to bridle the Town or the Place, lest the Burgbers should be rebellious, and to be the last refuge or place of retreat.

therein

- § 6. therein find the *Center* of the *Cittadel*, the *Interior Polygon* of the *Pentagon* may be $\frac{3}{4}$ of the *Curtain* adjoining, or a little more; the *Center* of the *Square* may be on P the *Point* of the *Interior Polygon*, the *Center* of the *Hexagon* may be near the outward *Point* of the *Bastion* of the *Town*, taken away to make the *Cittadel* in, which may be delineated as afore: The *Motes* and other *Works* in proportion accordingly, and the *Rampires* as high as those of the *City* or *Town*.

S E C T. VII.

Of some Maxims or Rules necessary to be known in Irregular Fortification.

- § 7. **I**rregular Fortification is when any *Town* or *Place* is to be Fortified, which lieth in an *Irregular Form*; *i. e.* whose *Sides* and *Angles*

* The Engineer must first form a Map of the *Town* or *Place*, with all the *Ways*, *Passages*, *Old Walls*, *Rivers*, *Pools*, *Enclosures*, and all other matters fit to be known in the Draught, and then he is to design what *Works* he findeth most agreeing to the *Place* to be Fortified.

are unequal in the Fortifying of *Irregular Figures* *. I shall here say very little, only I shall lay down some Precepts that are of immediate concern in Fortifying of *Irregular Figures*, and shall refer you to peruse *Marlois*, *Dogen*, *Fritach*, *Tournier*, *Dilichius*, &c. which

will greatly satisfie and help you: To this end know,

- I. That the same *Laws* and *Maxims* for *Regular Fortifications* stand and be in Force for *Irregular*; *i. e.* that the *Line of Defence* must not

not exceed the *Port* of a *Musket*, nor the *Angles* of the *Bastion* be less than 60° , nor much above 90° , &c.

2. That no *Inward Angle* of the Place be less than 90° ; if it be so it must be altered, and that *Point* may be made the *Outward Point* of a *Bastion*.

3. That between *Regular* and *Irregular Fortifications*, there is no other Difference, but by rectifying the *Sides* that are too short, or too long, and altering the *Angles* that are too little; as for the *Sides*, if they be above 500, and under 1000 Feet, they may be Fortified by *Bastions* placed according to the usual manner, at the *Extreme Points* thereof; But if the *Sides* be between 1000 and 1700 Feet, then in the midst you may place a *Plat-Bastion*, and at the *Extreme Points* place two *Bastions*, as before: But if the *Line* be less than 500 Feet, you may lengthen it, by producing it into the *Plane*: As for the *Angles*, they are made greater or lesser, according as occasion requireth. For the Raising the *Rampires*, *Parapets*, and other *Out-works*, they are to be as in the *Regular*, and the *Out-work* may be placed before the *Curtains*, as was before mentioned.

4. That the *Capital*, in any *Regular* or *Irregular Bastion*, is found by dividing the *Angle* of the *Polygon* into two equal Parts (by *Prop. 7. Chap. 4.*) and by producing the *Line* of *Angular Division* or *Separation*, on which the due Length of the *Capital* must be placed, which observe for a general Rule.

T

S E C T.

S E C T. VIII.

Of the Dimensions and Construction of small Forts or Sconces, which are built for the Defence of some Pass, River, or other Place.

- § 8. **W**HEN they are made Regular, of 4, 5 or 6 Bastions, then they may be Fortified by the precedent Rules, but there are others of smaller Dimensions fit for the same Purpose: viz. Triangle with Demi-Bastions, Square with Demi-Bastions, Parallelograms with Demi-Bastions and Tong, Star-Redoubts of four, five or six Points, and Plain Redoubts.

P R O P. IX.

To Fortifie a Triangle with Demi-Bastions.

- Fig. 84.** This Triangle may consist and be comprehended of three equal or unequal Sides in this Example: let it be an Equilateral Triangle PPP. Now divide PP into three Parts, then take 1, and prick off the Capitals PA, &c. and the Gorges make equal thereunto, as PC, PC, &c. then make the Flanks FC to stand at Right Angles, and to be $\frac{1}{3}$ of PC or PA, then draw the Faces AF, AF, &c. and the Work is finished as required.

P R O P. X.

To Fortifie a Square with Demi-Bastions.

- Fig. 85.** The Sides of the Square may be from 100 to 200 Feet, let PP be 180 Feet, which divide into 3 Parts, take one for the Gorges PC, and for the Capitals PA, and prick them off all round as

as you see, then take $\frac{1}{2}$ of PP, and at Right-Angles prick off the Flanks CF, then draw the Faces AF, AF, &c. and the Figure is compleated.

§ 8.

P R O P. XI.

To Fortifie a Parallelogram with Demi-Bastions and Tong.

First describe the Parallelogram or Long-Square, PPPP, then divide PP into 6 Parts (the Side on which the Tong, or Tenaile, is placed) and make MC equal unto $\frac{1}{2}$ thereof, and also MG and MH. Draw CG, GC, and CH, HC, then finish the Demi-Bastions as before, so shall the Work be compleated as was required. A Long Square may also be Fortified, as Fig. 77. Fig. 86.

P R O P. XII.

To Fortifie a Star-Redoubt of 4, 5 or 6 Points.

1. A Star-Redoubt of Four Points may have his Side from 40 to 60 Feet: First describe the Square PPPP, then divide PP into two Parts at M, take $\frac{1}{4}$ of PM, (and by Prop. I. § I. Ch. 4.) raise Perpendiculars round at M, make MA equal to $\frac{1}{4}$ of PM, and draw all as in the Figure. Fig. 87.

2. A Star-Redoubt of Five Points is thus Fortified. Describe the Pentagon PP, &c. then divide PP into halves at M, raise the Perpendiculars MA, make MA equal to $\frac{1}{3}$ of PM, and draw the Fort in all respects as the Figure representeth. Fig. 88.

3. A Star-Redoubt of Six Points is thus Fortified. Describe the Hexagon PPP, &c. divide PP into two equal Parts at M, then raise Perpendiculars at the M's, then make MA equal to $\frac{1}{3}$ of PM, or $\frac{1}{4}$ of PP, and draw every respective Line, as you see in the Figure. Fig. 89.

T 2

P R O P.

Of Fortification.

P R O P. XIII.

To Delineate a Plain Redoubt.

Fig. 90. Plain Redoubts are called *Grand Redoubts*, which are used as *Batteries* in *Approaches*, whose Side may be from 60 to 80 Feet, or *Petit Redoubts*, which are used for a *Court of Guards* in the *Trenches*, and may be from 20 to 50 Feet, and are framed and delineated in all respects, as you see in Fig. 90.

The *Profiles* to be set on these several Works, and the *Motes*, are alterable and uncertain; for they being sometimes used in *Approaches*; then they do require the *Breast-work* at the Bottom to be 7 or 8 Feet wide, and the *Interior Height* 6, and the *Exterior* 5 Feet, and the *Mote* to be either 8, 10 or 12 Feet, sometimes 14 or 20 Feet wide at the Bottom, and the Height of 7, 8 or 9 Feet, to have two or three Ascents to rise to the *Parapet*. There are many other things belonging to this Art, which the Limitation I am bound to, will not permit here to be treated of.

C H A P. XIV.

Of Military Orders, or the *Embattelling* and *Encamping* of Soldiers.

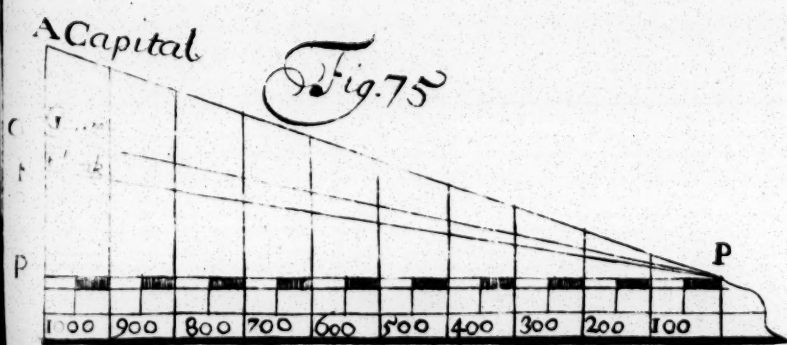
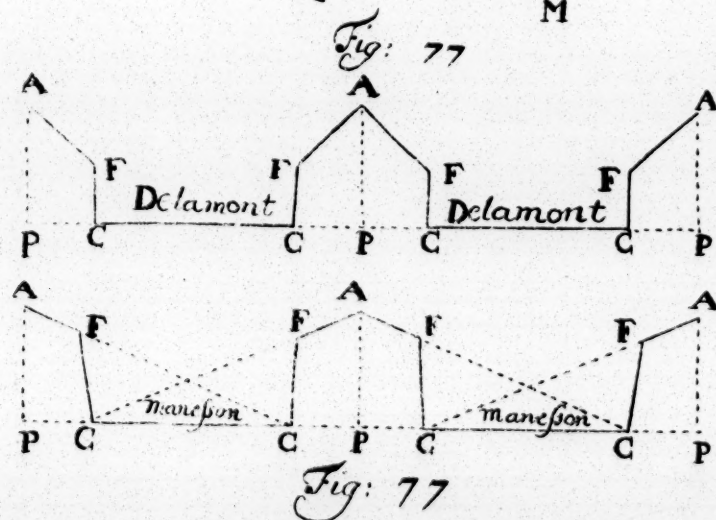
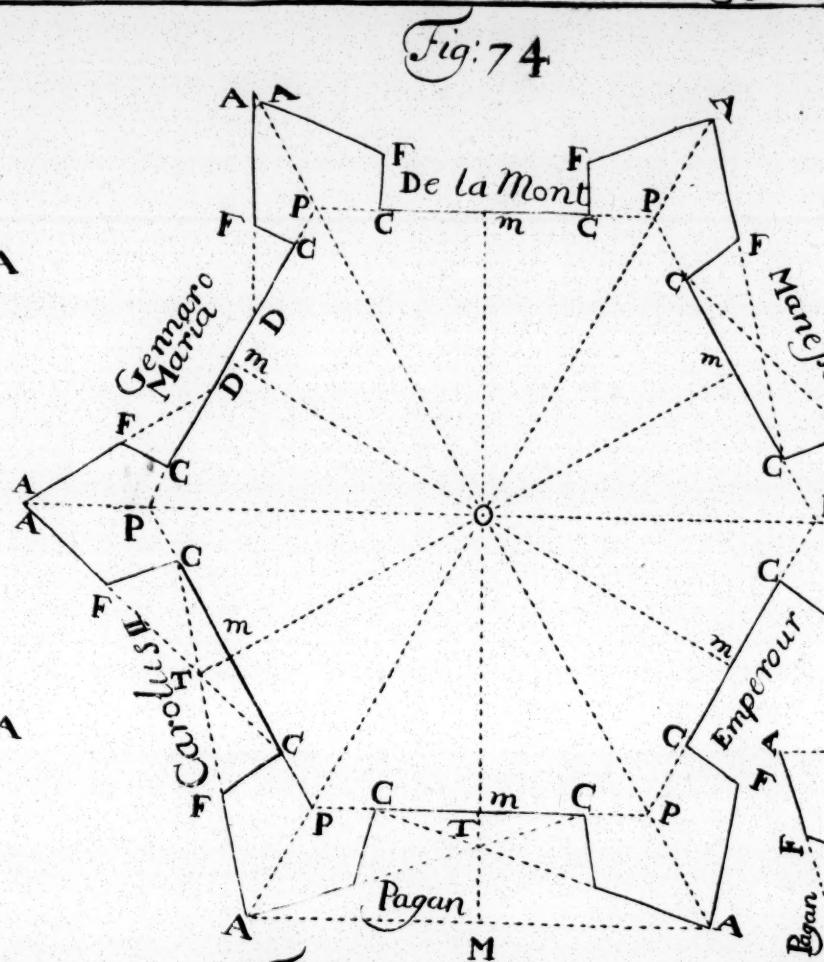
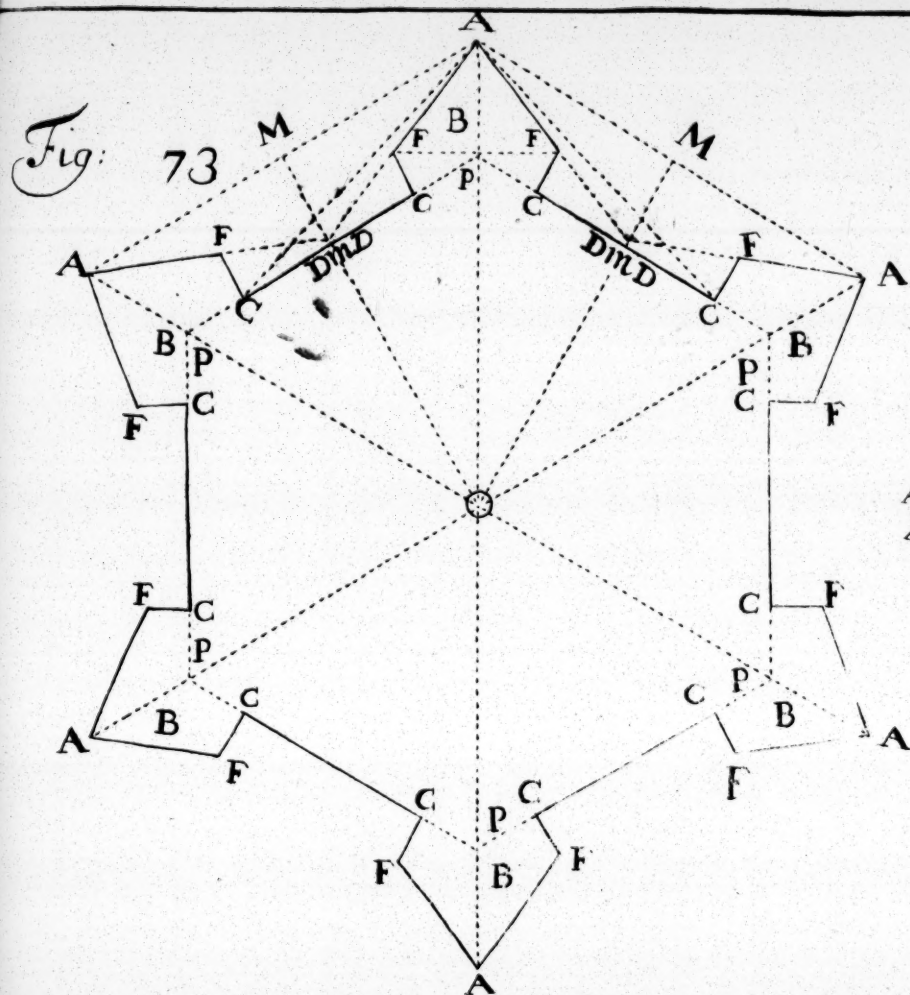
S E C T. I.

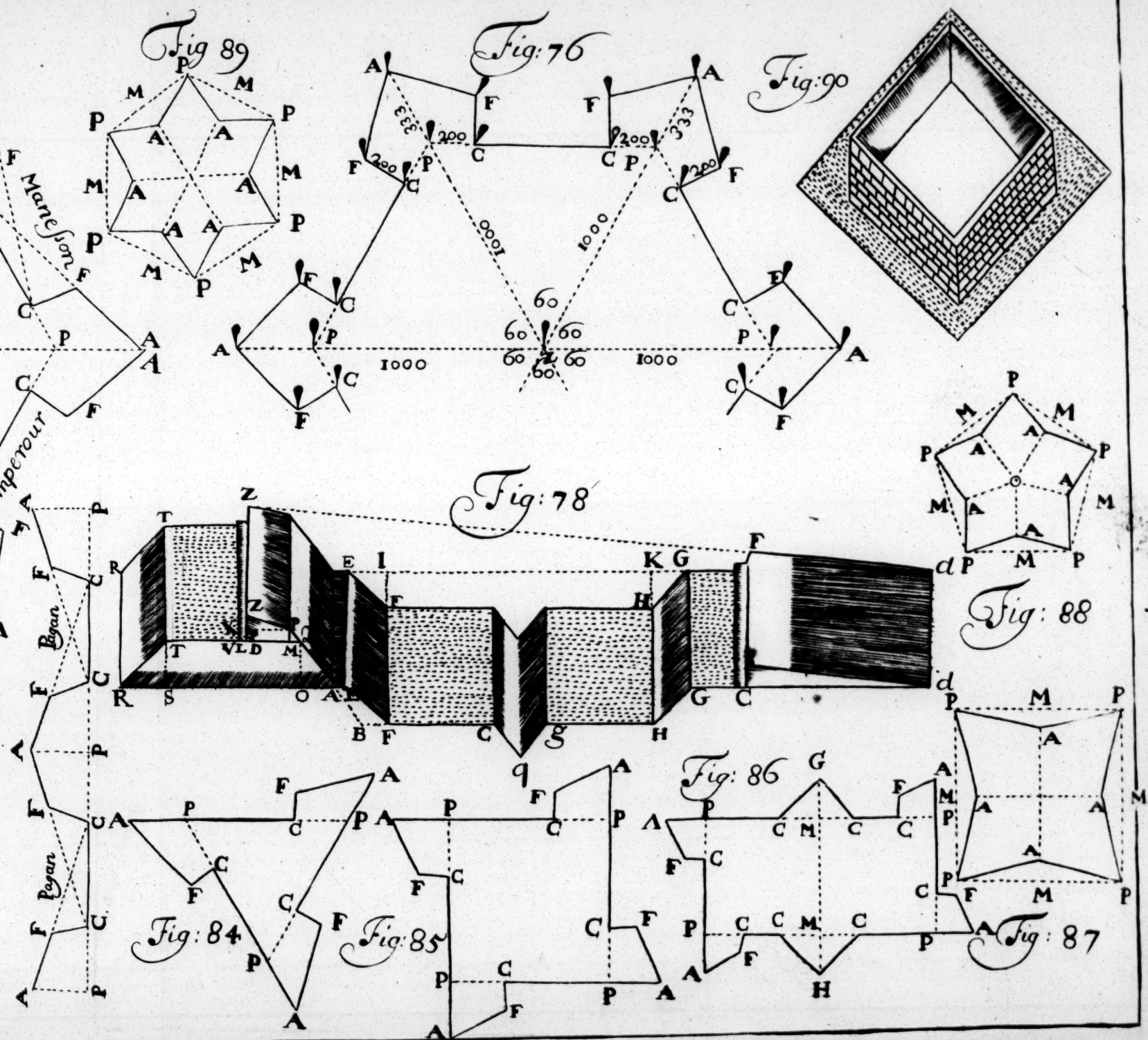
Of the *Embatteling* and Ordering of Soldiers.

§ 1.

BATTAILS are considered either in respect of the Number of Men, or in respect of the form of Ground. In the respect of the

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the Number of Men, it is either a Square Battail, § 1.
 a Double Battail, a Battail of the Grand Front,
 or a Battail of any Proportion, of the Number in
 Rank to the Number in File. In respect of the
 form of the Ground, the Battail is either a Geo-
 metrical Square of Ground, or a Long Square of
 Ground. For the Distance or Order of Soldiers,
 martialled in Array, is distinguished either into
 Open Order, which is when the Centers of their
 Places are 7 Feet distant asunder, both in Rank
 and File, or Order; which is when the Centers
 of the Places are $3\frac{1}{2}$ Feet distant both in Rank and
 File; or else $3\frac{1}{2}$ Feet in Rank, and 7 Feet in
 File.

PROP. I.

To place any Number of Soldiers (suppose 36000)
 in such Order of Battail, that there shall be
 any Number in Rank or File. Suppose 500
 in Rank, how many must there be in File?

Divide the Whole Number of the Army by
 the Assigned Men in Rank (or File) the Quote
 (72) is the Number that will be in File (or
 Rank.)

PROP. II.

To Order any Number of Soldiers into a Square
 Battail of Men.

Admit it were required to Martial into a
 Square Battail 16129 Men: To do which, ex-
 tract the Square Root of 16129 (by Prop. 8.
 §. 1. Chap. 1.) which is 127, therefore you are
 to place 127 Men in Rank, and also in File.

§ 1.

P R O P. III.

To Order any Number of Soldiers into a Double Battail.

Admit 16928 Men were to be Martialed into a *Double Battail*, extract the Root of half the Number of Men; *i.e.* of 8464, whose Root is 92, therefore I say that 92 Men must be placed in *File*, and 184 in *Rank*, to order that Number of Men propounded into a *Double Battail*.

P R O P. IV.

To Order any Number of Soldiers into a Battail of the Grand Front.

Admit 16900 *Soldiers* were to be Martialed into a *Battail* of the *Grand Front*, that is, *Quadruple*. Extract the *Square Root* of 4225 (that is $\frac{1}{4}$ of the Men) the Root is 65; therefore I say 65 must be placed in *File*, and 260 in *Rank*, to form a *Battail* of the *Grand Front*.

P R O P. V.

Any Number of Men, together with their Distance in Rank and File, being propounded, to Order them into a Square Battail of Ground.

Admit 2500 *Soldiers* were to be Martialed into a *Square Battail* of *Ground*, in such sort that their Distance in *File* should be 7 Feet, and in *Rank* 3 Feet, and 'tis required to know how many Men must be placed in *Rank* and in *File* to draw up 2500 Men into *Square Battail* of the *Ground*. According to *Prop. 1. §. 1. Ch. 1.* say, As — 7 to 3; So is 2500 to 1071, &c. whose *Square Root* is 32, &c. therefore I say 32 Men are to be placed in *File*. Now to find how

how many Men are to be placed in Rank, Divide 2500 by 32, the *Quotient* is 78, which are the Number of Men to be placed in Rank, and 4 Men to be disposed elsewhere. § 1.

P R O P. VI.

Any Number of Soldiers propounded, to Order them in Rank and File, according to the reason of any two Numbers given.

Admit 6400 Soldiers are to be Martialled into Array, in such Order that the Number of Men placed in *File*, shall bear such Proportion to the Number in Rank as 7 to 13; (according to Prop. I. §. I. Chap. I.) say as 7 to 13; so is 6400 to 11885, &c. whose Square Root is 109, &c. the Number of Men to be placed in Rank, by which divide 6400, it produces, 58, &c. the Number of Men to be placed in *File*, and 78 Men to be employed elsewhere.

S E C T. II.

Of Castrametation, or Quartering and Encamping of Soldiers.

IN Quartering and Encamping of Soldiers, it is requisite the Quarter-Master General, and all other under Quarter-Masters, be skilled at Foot-Measure, that so they may lay out their Quarters as directed. § 2.

The common Allowance for the Depth of Ground, that a Regiment of Horse or Foot will take up, the Wideness must be answerable to the Number of Men 200 Feet for the Huts in
T 4 Length,

§ 2. Length, and 100 for the *Commanders* and *Sutlers* before them; every two *Soldiers* to a *Hut*, 8 Feet broad, and 8 Feet deep, 2 Feet *Hut* from *Hut*, so that there may stand 20 *Huts* in the 200 Feet, the Alley betwixt *Hut* and *Hut* may be 8 Feet, that is 16 Feet in width, and 200 in Length for 40 Men, which is 3200 Feet, and for the 100 Feet more, 1600 Feet, in all 4800 Feet, and there must be 25 Rows of *Huts* for 1000 Men; so that for a *Regiment of Foot* containing 1000 Men, with *Officers* and *Sutlers*, will take up 120000 Feet, which is 2 *Acres* and 3 *Roods*, which because of Ways may be allowed 3 *Acres* of Ground for every *Regiment*, which may be 350 Feet deep and 370 Feet wide, or near 360 Feet Square: Now if 1000 Men, *Officers*, *Sutlers*, *High-ways*, and all take up a Square of 360 Feet, how many Feet shall the Side of a Square be wherein 10000 *Foot-men*, &c. may be encamped? say (by *Prop. 1. Chap. 1.*) as 1000 to 10000; so is the Square of 360, viz. 129600, to 1296000, the Square of 1138 Feet, which is very near 30 *Acres* of Ground.

For the *Quartering of Horse*, you must keep the same Depth of 300 Feet for all, and take 200 Feet for the *Huts*, the *Horse-Huts* must be 10 Feet deep and 4 wide; so that 12 *Horses* may stand in one *Hut* together, which is 48 Feet long and 10 wide, and 6 Feet a Street; The *Huts* for the *Troops*, will be 6 for 12 *Troops*; now conceive a *Regiment* to consist of 8 *Troops*, 50 to a *Troop*, it will take up, leaving 20 Feet Streets, and Cross-ways, very near as much Ground as a *Regiment of Foot*, Ways and all must be allow'd 3 *Acres*, near 360 Feet Square, so that 10 *Regiments*

ents of Horse will take up 30 Acres : Moreover, it will be needful and you may very well follow as much Ground as both Horse and Foot will take, for the Train of Artillery, Victuallers, Parade Places, &c. From these Considerations the young Beginner, may even the better practised Soldier may receive Help, and thereby be enabled to Encamp an Army if required.

§ 2.

CHAP. XV. Of GUNNERY.

SECT. I.

Of the Names of the Principal Members of a Piece of Ordnance.

i. Defin.

A CANNON is a long round Body, either of Brass or Iron, formed and made hollow by Art, and Proportion, to offend afar off, with a Ball of Iron, Stone, or any Artificial Substance, Charged with Gun-Powder, in it's charged Cylinder, which being fired, in an instant performs it's desired Effect. This Machine

was invented by an English-man, and first put in Practice by the Venetians against the Genoueses at Chiezza, Anno 1376.

This Military Engine Bombarda, Gun, Cannon, &c. so called from Bombo, a resounding Noise. Cannone, or Cannon, from the likeness it holds with

this Canna, Bore or Concavity; Artegleria, from Artiglio, the Talons or Claws of Ravenous Fowls, because its Shot flying afar off, tears and defaces all that it doth meet; from whence some Natures of this Machine are called Smeriglii, long-winged Hawks, Falconi, Falconets; Passa volanti, swift flying Arrows, &c.

2. The

§ 1.

§ 1. 2. The *Superficies* of the *Metal*, is the Outside round about the *Piece*.

3. The *Body* is the Substance of the whole Mass of *Metal*.

4. The *Chase* is the Concavity of the *Piece* in which they put the Charge.

5. The *Muzzel* is the Extremity of the *Chase* by which you load and unload the *Piece*.

Fig. 91. 6. The *Calibre* is A B the *Diameter* of the *Muzzel* or *Mouth*.

7. The *Touch-hole* is that little *Vent* which passeth from the *Convex Superficies* to the very *Chamber* of the *Piece*, made to give fire to the *Powder* within, as C; that which encloseth the Extremity of the *Chase* about the *Touch-hole* is called the *Breech* or *Coyl*.

8. The *Cascabel* is the *Pammel* at the *Breech* or *Coyl*, as D.

The *Trunnions* are pieces of *Metal* fixed unto the *Exterior Superficies* of the *Gun*, on which he moves in the *Carriage*, as E E.

Fig. 91. The *Body* of the *Piece* is that which is comprehended betwixt the *Center* of the *Trunnions* and the *Cascabel* E G.

The *Vacant Cylinder* is comprehended betwixt the *Center* of the *Trunnions* and the *Muzzel*, as E B.

The *Frees* or *Muzzel Ring*, is that thick *Cornish* which encompasseth the *Convex Superficies* of the *Piece* at I, the *Base Ring* is K L G, the *Reinforced Ring* is M, the *Trunnion Ring* is N, and the *Cornish Ring* is O.

The *Line of the Cylinder* is a direct *Line* imagined to be described along the *Chase Parallel* unto the *Middle* of the *Chase*, as X Z.

The

Of Gunnery.

283

The *Line of Metal* is a Line touching both Cornishes, as MNI. § 1.

Fig. 91.

The *Dispart Line* of the Piece, is the Difference betwixt the *Semidiameter* of the *Muzzel* and *Base Ring*, as the Line IH.

The *Vent* of the Piece is the Difference betwixt the *Diameter* of the *Shot* and the *Mouth* of the Piece, as e d.

The *Chamber* or *Charged Cylinder*, is that Part of the *Chase* towards the *Touch-hole* equally large, nor narrower in one Place than in another, and doth contain the *Powder* and *Ball*.

S E C T. II.

Of the *Dimension* of our *Usual English Cannon*, and other *Ordnance*, &c.

IN the following *Table* I have set down the Length and Weight of our most usual *English Ordnance*, the *Diameters* and Weight of their *Bullets*, the Length and Breadth of their *Ladles*, the Weight of *Powder* to Charge them, &c. § 2.

P R O P. I.

How to know the different *Fortification* of a Piece of *Ordnance*.

In *Fortifying* any Piece of *Ordnance* there are three Degrees observed, as first *Legitimate Pieces*, which are those that are ordinarily *Fortified*; secondly *Bastard Pieces*, which are such whose *Fortification* is lessened; thirdly *Double Fortified Pieces*, or extraordinary *Pieces*.

The

<i>The Names of the several Pie- ces of Ordnance.</i>	<i>Utmost Random</i>	Paces	<i>Shoots Level</i>	Paces	<i>Powder</i>	Ounces	weight	Pounds	<i>Ladles</i>	8 parts	breadth	Inches	<i>Ladles</i>	8 parts	length	Inches	<i>Bullets</i>	Ounces	weight	Pounds	<i>Bullets</i>	8 parts	diamet.	Inches	<i>Guns</i>	8 parts	bore	Inches	<i>Guns</i>	weight	Pounds	<i>Guns</i>	Inches	length	Feet	
<i>A Bafe.</i>		600	60	60	8	0	0	0	0	2	2	2	0	0	4	4	5	0	0	1	1	2	1	6	200	6	4									
<i>A Rabinet.</i>		700	70	70	2	0	0	0	4	2	2	2	1	0	5	4	8	0	0	3	1	4	1	300	6	5										
<i>A Falconet.</i>		500	50	50	4	1	1	1	0	4	4	4	4	1	7	7	5	5	1	1	2	2	2	400	0	6										
<i>A Falcon.</i>		1300	130	130	4	2	2	2	4	2	4	4	2	8	8	2	8	2	2	5	2	2	2	750	0	7										
<i>Minion ordinary.</i>		1200	120	120	8	2	2	2	0	2	2	5	4	4	7	7	4	3	3	7	7	2	3	800	0	7										
<i>Minion largest.</i>		1250	125	125	4	3	3	3	0	3	3	5	0	0	9	8	12	3	3	0	0	3	1000	0	8											
<i>Saker least.</i>		1500	150	150	6	3	3	4	4	4	6	6	6	4	9	9	12	4	4	2	2	4	3	1400	0	8										
<i>Saker ordinary.</i>		1600	160	160	0	4	4	6	6	6	6	6	4	10	10	0	6	3	4	4	4	6	3	1500	0	9										
<i>Saker old fort.</i>		1630	163	163	0	5	5	7	7	7	7	7	0	11	11	5	7	4	6	6	6	4	0	1800	0	10										
<i>Demiculver least.</i>		1740	174	174	4	6	6	6	0	8	8	8	0	12	12	9	9	0	9	0	0	2	4	2000	0	10										
<i>Demiculver ordinary.</i>		1750	175	175	4	7	7	8	0	8	8	8	6	12	12	11	11	2	10	4	4	4	4	2700	0	11										
<i>Demiculver old fort.</i>		1780	178	178	8	8	8	8	4	8	8	8	4	13	13	11	11	4	12	4	4	4	6	3000	0	11										
<i>Culverin least.</i>		1800	180	180	0	10	10	10	0	10	9	9	2	14	14	0	15	5	15	6	6	6	5	4000	5	11										
<i>Culverin ordinary.</i>		1810	181	181	6	11	11	11	4	11	9	9	0	16	16	5	16	0	17	5	5	2	5	4500	5	12										
<i>Culverin largest.</i>		1830	183	183	8	11	11	11	0	11	10	10	0	16	16	0	20	0	20	6	6	4	5	4800	5	12										
<i>Demicannon least.</i>		1560	156	156	0	14	14	14	4	14	11	11	0	20	20	0	30	0	30	6	6	2	6	5400	6	11										
<i>Demicannon ordinary.</i>		1620	162	162	8	17	17	17	0	17	12	12	0	22	22	0	32	0	32	6	6	4	6	5600	6	12										
<i>Demicannon large.</i>		1800	180	180	0	18	18	18	6	18	12	12	6	22	22	0	36	0	36	6	6	6	6	6000	6	12										
<i>Cannon Royal.</i>		1850	185	185	8	32	32	32	0	32	14	14	0	24	24	0	58	0	58	7	7	0	7	8000	8	12										

The Fortification of any Piece of Ordnance, is accounted, by the Thickness of the Metal, at the Touch-hole, Trunnions, and at the Muzzel, in Proportion to the Diameter of the Bore.

The Legitimate Pieces, or the ordinary Fortified Cannons, have $\frac{2}{3}$ at the Touch-hole, $\frac{1}{2}$ at the Trunnions, and $\frac{3}{4}$ at the Muzzel of the thickness of the Bore, in thickness of Metal. Bastard Cannons, or lesned Cannons, have $\frac{3}{4}$ at their Touch-hole, or $\frac{1}{2}$, and $\frac{2}{3}$ at their Trunnions, and $\frac{1}{2}$ at their Muzzel: the Double Fortified Cannons have full one Diameter of the Bore in thickness of Metal at the Touch-hole, and $\frac{1}{2}$ at the Trunnions, and $\frac{2}{3}$ at their Muzzel. Now all Double Fortified Culverins, &c. are $\frac{1}{3}$ at the Touch-hole, $\frac{1}{2}$ at the Trunnions, and $\frac{2}{3}$ at the Muzzel, and the Ordinary Fortified Culverins, are Fortified every way as Double Fortified Cannons, and lesned Culverins as Ordinary Cannons in all respects.

P R O P. II.

How to know how much Powder is fit for Proof, and what for Service for any Piece of Ordnance.

For Cannons take $\frac{4}{7}$ of the Weight of their Iron Bullet of good Corn Powder for Proof, and for Service $\frac{1}{2}$ the Weight of the Iron Bullet is sufficient, especially for Iron Ordnance, which will not endure so much Powder as Brasses ones will receive by $\frac{1}{4}$ in Weight, for Culverins allow the whole Weight of the Shoot for Proof, and $\frac{2}{3}$ for Service. For Sakers and Falcons, take $\frac{4}{7}$ of the Weight of the Shot, and for lesser Pieces the whole Weight may be used in Service, until they grow hot, but then there must be some Abatement

§ 2. ment made at Discretion, and take $1\frac{1}{2}$ of the Weight of their Iron Bullet for Proof.

P R O P. III.

To know what Bullet is fit to be used in a Piece of Ordnance.

* In his
Mathe-
matical
Manual.
Page 165.

The Bullet must be somewhat less than the Bore of the Gun, that so it may have Vent in the Discharge; some Authors affirm $\frac{1}{4}$ of an Inch less than the Bore will serve all Ordnance, but this Vent is too much for a Falcon, &c. and too little for a Cannon: therefore I approve thereof not, but commend Mr. Philip's Proportion to your Use, which is to divide the Bore of the Gun into 20 equal Parts; and let the Diameter of the Bullet be $\frac{19}{20}$ thereof, according to which Proportion the precedent Table is calculated.

P R O P. IV.

By knowing the Proportion of Metals one to another, and by knowing the Weight of one Ball to know what any other shall weigh.

The common received Proportions for Metals are these:

Lead is to Iron as 2 to 3.

Lead is to Brass as 24 to 19.

Lead is to Stone as 4 to 1.

Iron is to Lead as 3 to 2.

Iron is to Brass as 16 to 18.

Iron is to Stone as 3 to 8.

The more exact Proportion betwixt Metals are thus known: Admit a Cube, or Ball of Gold weigh 100 l. a Cube of any of those Metals ensuing of the same bigness, shall bear such Proportion as followeth, to the said Cube of Gold.

	li.	pts.		li.	pts.	§ 2.
Gold	100	00	Iron	42	10	
Quicksilver	71	43	Tin	38	95	
Lead	60	53	Stone	15	80	
Silver	54	39	Water	05	68	
Brass	47	37				

It is the Opinion of Dr. Wybard in his *Tactometria*, that a *Bullet* of Cast Iron, whose Diameter is 4 Inches doth weigh 9 l. *Averdupoize Weight*.

Now to find what any other *Bullet* or *Cube* shall weigh; say (as in *Prop. 4. Chap. 1.*) As the *Cube* of the *Bullet* propounded, is to his Weight, so is the *Cube* of another *Bullet* given, to his Weight, and so observe still this Proportion.

S E C T. III.

Of the Qualification of an able Gunner, and necessary Operations before Shooting, and in shooting.

A Gunner ought to be a Man of Courage, Experience, and Vigilant; he ought to have good Skill in *Arithmetick*, to know the Extraction of the *Roots*, &c. He ought to have Skill in *Geometry*, to take *Heights*, *Distances*, &c. to know the Divisions and Use of his *Circle*, *Quadrant* and *Quadret*; to know how to level and to lay *Platforms*, and to raise *Batteries*. He must know the Names of all sorts of *Ordnance*, their *Weight*, the *Height* of their *Bore*, the *Height* and *Weight* of their *Shot*, the *Length* and *Breadth* of their *Ladles*, how much *Powder* to use for
Proof

§ 3.

§ 3. *Proof and Action*; The *Shoots Level* and the *Shoots Random*; He must know the Names of all the Members of a *Piece of Ordnance*; he must also know the *Length, Thickness* and *Breadth* of all manner of *Carriages*, and must know all the Parts thereof: viz. the *Cheeks* or *Sides*, the *Ax-tree, Spokes, Nave, Hoops, Transomes, Bolts, Plates, Drawing-Hooks, the Clout, the Hole* for the *Linspin, the Shafts, the Thill* and *Thill-bolt*, the *Fore-lock* and *Fore-lock-keys, Cap-squares, the Fore-lock-pins* and *Chain, the Pintle* and *Bolt-hole, Fellows, Nails, Fellow-bars, Stirrops, the Ruts* of the *Wheel, Dowledges, Beds, Coins, Levers, Hand-screws, &c.* He must also know how to make his *Ladles, Sponges, Cartridges* whether of *Paper* or *Canvas*, and to have by him *Formeres* of all sorts, *Sheep-skins* undressed to make *Sponges, Powder, Shot, Needles, Thread, Paste* and *Starch, Marlin, Twine, Nails, Hand-spikes, Crows* of *Iron, Granado-shells, and Materials* for *Composition, Fasces, Budg-Barrels, Cannon-Baskets, &c.* These being general things he is to know, and at all times to have ready by him, and he is more particularly to know the following Parts of his Art: As,

P R O P. I.

How to Tertiate, Quadrate, and to Dispart a Piece of Ordnance.

1. *To Tertiate a Piece*, is to find whether it hath its due *Thickness* at the *Trunnions, Touch-hole* and *Neck*; if the *Trunnions* and the *Neck* are in its due *Order*, and the *Chase* straight.

2. *To Quadrate a Piece* mounted, is to see whether it be directly placed, and equally poized

the *Carriage*; which is known by finding in the *Convex Superficies* of the *Base* and *Muzzel* Ring; the Point which is *Perpendicular*, over the *Soul* of the *Piece*: which may be found by the *Gunner's Instrument*, called a *Level*; an Instrument whose Use is so vulgarly known, that it needeth not my Explanation.

3. To *Dispart a Piece*, is to fix or elevate on the *Convex Point* of the *Muzzel Ring*, a Mark far distant from the *Cylinder*, or *Soul* of the *Piece*, as is the Point of the *Base-Ring*; to the end that the *Visual-ray* which passeth by these Marks, may be *Parallel* to the *Chase*, *Soul*, or *Cylinder* of the *Piece*. Now the *Dispart*, i. e. the Difference of the *Semidiameters* of the *Cornishes*, may be by a pair of *Calliper Compasses* obtained: Which found, place on the Top of the *Cornish Ring* near the *Muzzel*, over the middle of the *Inferior Cylinder*.

P R O P. II.

To know how far any *Piece of Ordnance* will shoot, &c.

As to the several Shootings in *Artillery*, Authors differ much in their Judgments and Opinions, but they all unanimously agree that the Ball being shot forth flies through the Air, with *Violent*, *Mixt* and *Natural Motion*; describing a *Parabolical Line*, in whose Beginning and Ending are Lines sensibly streight, and in Middle Curved: In the Beginning the Imprest Force driving forward by the Fire, the *Natural Gravity* of the *Ball* doth describe a *Right-Line*, called the *Direct Line*, or *Ranges* of the *Ball's Circuit*.

Fig. 92.

U

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§ 3.

* See Mr. Diggs in his Pantometria, Page 179.

In the Middle that Force diminisheth, and the *Natural Gravity* prevaileth, so that it describeth a Curved Line, called the *Ball's Middle Horizontal* or *Conical Arch**; In the End the *Natural Gravity* overcoming the Imprest Violence, (which becomes altogether weak and faint) describes a new Right Line, called the *Ball's Declining Line* in which the *Ball* tends towards the Center of the Earth, as towards a Place natural unto all heavy Bodies: See Figure 92. These Motions are somewhat longer, according as the Piece is mounted from the *Level* unto the *Angle* of 45 deg. which is called the *Utmost Random*: The *Elevation* of which is regulated by the *Gunner's Quadrant*, the Use of which Instrument is generally known, and by so many Authorfully explained, that I here crave Leave to omit it: But take these for General Rules.

General Rules to be observed in the Batter-ing down of a Place, or making of Breaches.

1. That a Shot at Right-Angles strikes more violently and furiously than at Oblique Angles therefore Gunners use, when they are to Batter down a Tower, Wall or Earth-work, to shoot Point Blank at the Object, Tire by Tire; by discharging all the Pieces in the Battery against the self same Object in the same Instant, holding it for a Maxim, that Ten Cannons discharged together do far more Execution than discharged one after another. Now at Oblique Angles they shoot either Cross-ways or by rebounding.

2. That the speediest way to make a Breach in a Wall, &c. is by shooting at the Object from Two Batteries, which ruins far more speedily than by striking the Object with one Battery at Right Angles, although that one Battery hath as many Cannon as the other Two hath.

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3. That if you were to batter a Flank covered with an Orillion, (which because you cannot possibly Batter it right forward) you must therefore of necessity Batter it Obliquely, by way of Rebound; thus : Chuse a fit Place in the Curtain to be your Object, on which you may play with your Battery Obliquely, so that by a Rebound the Shot may leap into the Flank, holding for a Maxim, in this Operation, * That the Angles of Incidence and Reflexion are Equal.

§ 3.
* Accord-
ing to
Learned
D' Chales,
on the 4th
Prop. of
the first
Book of
Euclid.

Now we come to shew the Length of the Right Range, of all our Common English Ordnance, which is set down in the precedent Table, in which the Cannon exceed not 185 Paces, &c. Esteeming the Pace 5 English Feet, nor his utmost Random above 1850 Paces, which Table sheweth for all other Natures.

As for the Ranges and Randoms, to the several Degrees and Points of Mounture of the Quadrant, I have hereunto annexed some Tables calculated by the Experiments of sundry most Eminent Artists, whose Works will perpetuate their Worth and Name to succeeding Generations.

The Use of the following Table of Randoms.

This Table is most agreeing to Cannons and Culverins, and the greatest sort of Ordnance, the Use whereof is thus :

Admit a Saker to be mounted to 3 deg. shoots the Bullet 323 Paces, how far will it shoot being mounted unto 7 deg? Say (by Prop. 1. Chap. 1.)

As 510 the Tabular Distance for 3 deg. of Mounture, to 323, the Distance found ;

So is 934 the Tabular Distance for 7 deg. of Mounture, to 591 $\frac{1}{10}$, the Distance required,

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which

§ 3. which the *Saker*, according to this Experiment shall shoot at 7 deg. of *Mounture*.

A Table of Ranges and Randoms, to the several Degrees of *Mounture* of the *Quadrant*.

A TABLE OF				
Right Ranges, or Point Blanks.		Randoms, or the First Graze.		
<i>The Degrees of the Pieces Mounture.</i>	0	192	0	192
	1	209	1	298
	2	227	2	404
	3	249	3	510
	4	261	4	610
	5	278	5	722
	6	285	6	828
	7	302	7	934
	8	320	8	1044
	9	337	9	1129
	10	354	10	1214
<i>The Right Range in Paces of 5 Feet.</i>	20	454	20	1917
	30	693	30	2185
	40	855	40	2289
	50	1000	50	2283
	60	1140	60	1792
	70	1220	70	1214
	80	1300	80	1000
	90	1350	90	

Mr. NYE in his Book of *Gunnery*, Printed Anno 1647. faith he made an Experiment by *Saker* of 8 Feet long, which he loaded with Three Pounds of *Powder*, of an exact Weight both *Powder* and *Wad* at every Charge, every time ramming it down with three equal strokes as near as possible; but on the *Bullet* he put no *Wad*, because the *Saker* was mounted; And thus

thus he made four Shoots, each of them half an Hour after the other, that so the Piece might be of equal temper, and mounted his Piece to these four Degrees of Mounture, viz. 1 deg. 5 deg. 7 deg. 10 deg. and found these Randoms.

At 1 Deg. the Random was 225
 At 5 Deg. the Random was 416
 At 7 Deg. the Random was 505
 At 10 Deg. the Random was 630

} Paces.

According to which Experiment, he framed this Table of Randoms.

Deg.	Paces	Deg.	Paces
0	206	6	461
1	225	7	505
2	274	8	548
3	323	9	589
4	370	10	630
5	416		

Captain HEXAM in his Book of Gunnery, shews how by finding out the Random of a Cannon, for the first Degree of Mounture, thereby to find the Random for every Degree to 45 deg. or utmost Random, and this is his Rule to perform it. First find how many Paces the Cannon will shoot being laid level by the Metal, (which by him is accounted 1 deg.) Then divide the Distance found by 50, then multiply the Quotient by 11, so shall the Product be the greatest Digression, or Difference betwixt Ranges and Ranges; which being divided by 44, the Quotient giveth the Number of Paces, which the Bullet will lose in the other Ranges, from

U 3 Degree

§ 3. Degree unto Degree; according to this Rule this Table is calculated.

A Table of Randoms to 45 Degrees, accounting a Pace $2\frac{1}{2}$ Foot.

<i>D. Mount.</i>	<i>Paces.</i>	<i>Diff.</i>	<i>D. Moun.</i>	<i>Paces.</i>	<i>Diff.</i>
0	0775	225	23	4685	110
1	1000	220	24	4795	105
2	1220	215	25	4900	100
3	1435	210	26	5000	95
4	1645	205	27	5095	90
5	1850	200	28	5185	85
6	2050	195	29	5270	80
7	2245	190	30	5350	75
8	2435	185	31	5425	70
9	2620	180	32	5595	65
10	2800	175	33	5560	60
11	2975	170	34	5620	55
12	3145	165	35	5675	50
13	3310	160	36	5725	45
14	3470	155	37	5770	40
15	3625	150	38	5810	35
16	3775	145	39	5845	30
17	3920	140	40	5875	25
18	4060	135	41	5900	20
19	4595	130	42	5920	15
20	4325	125	43	5935	10
21	4450	120	44	5945	5
22	4570	115	45	5950	

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§ 3.

I have hereunto also annexed the *Table* calculated by *Alexander Bianco*, for all sorts of *Ordnance*, (which *Table* I account one of the best that was ever yet found Extant) In his Work Printed 1648.

A Table of Randoms for the first Six Points of the Gunner's Quadrant.

Points.	1	2	3	4	5	6
<i>Falconet.</i>	375	637	795	885	892	900
<i>Falcon.</i>	550	935	1166	1254	1309	1320
<i>Minion.</i>	450	765	954	1026	1071	1080
<i>Saker.</i>	625	1062	1325	1125	1487	1500
<i>Demi-culver.</i>	725	1232	1537	1653	1725	1740
<i>Culverin.</i>	750	1275	1590	1710	1785	1800
<i>Demi-cannon.</i>	625	1062	1325	1425	1487	1500
<i>Cannon of 7.</i>	675	1147	1431	1489	1606	1620
<i>Double Cannon.</i>	750	1275	1590	1710	1785	1800

S E C T. IV.

Of Shooting in Mortars.

A *Mortar-Piece* is a short *Piece* with which they shoot *Bombs*, *Grânado-Shells*, *Stone-Balls*, &c. and their *Flight* is not in a *Streight* *Line*, but in a *Curve* *Line* from on high, so that it may fall where it should be desired; This *Mortar* is placed in the *Carriage* in all respects, as you see in *Fig. 93.* in which *A* signifies the *Carriage*, *B* the *Mortar*, *C* the *Course*

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§ 3. Degree unto Degree; according to this Rule this Table is calculated.

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5	1850	200	28	5185	85
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7	2245	190	30	5350	75
8	2435	185	31	5425	70
9	2620	180	32	5595	65
10	2800	175	33	5560	60
11	2975	170	34	5620	55
12	3145	165	35	5675	50
13	3310	160	36	5725	45
14	3470	155	37	5770	40
15	3625	150	38	5810	35
16	3775	145	39	5845	30
17	3920	140	40	5875	25
18	4060	135	41	5900	20
19	4595	130	42	5920	15
20	4325	125	43	5935	10
21	4450	120	44	5945	5
22	4570	115	45	5950	

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S E C T. IV.

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the *Shot* flies, and *D* the *Place* on which it falls.

Bombs are great *Hollow Balls* or *Carcasses* of *Iron* in which are put fine Sifted *Gun-Powder*, which by a *Fuse* they proportion to them a due *Fire* that so they may break in such a determinate *Time*, or at such a determinate *Distance*; these *Fuses* are small *Trunks* of *Wood*, *Tin* or *Iron* filled with a prepared *Composition* for that *Purpose*. *Granadoes* are of the same *Form*, and for the like *Use* with *Bombs*, only smaller, and many times are thrown by the *Hand*, and are made not only of *Iron*, but of *Brass*, *Glass* or *Earth*.

Now in *Order* to the right and true *Shooting* in those kind of *Machines*, 'tis requisite to let you know that *Authors* have differed very much in their *Opinions*, but now they all unanimously agree that all *Projectiles* are in the *Curve* of a *Parabola*, as has been prov'd by *Galileus*, *Torricellius*, and of late by the Ingenious *Captain Halley*, in a *Discourse of Gravity* Published in *Philos. Transact.* Numb. 179. In which *Discourse* he *Mathematically* Demonstrates that the *Motion* of all *Projects* is in the *Curve* of a *Parabola*; from the *Properties* of which *Curve* these useful *Propositions* have been deduced.

First, That the *Distance* of *Horizontal Ranges* are one to another, as the *Sines* of the *Double Angles* of *Elevation*.

Secondly, That the *Heights* of *Shots* in the *Air* are one to another, as the *Versed-Sines* of the *Double Angles* of *Elevation*.

Thirdly,

Thirdly, That the Times of Continuance of a Shot in the Air at different Elevations, are one to another as the Sines of the Angles of Elevation.

From these Three Propositions, all that is necessary in the Practice of Gunnery may be known.

As First: Suppose I make a Shot upon some given Elevation, and then measure the Length of the Horizontal Range of that Shot (which is only the Distance between the Mouth of the Piece and where the Shot falls) you may from thence, by one of the preceding Propositions tell the Length of the Ranges of all Shots upon any other Elevation, using the same Force or Charge of Powder.

But before we proceed to the Propositions for Shooting, it will be necessary to give the following Rules.

1. *That before you make a Shoot at any Place, you find the Distances thereof from your Mortar, which may be obtain'd by Prop. 3. §. 4. Chap. 9.*
2. *That the Bombs, or other Bodies that are to be shot be of equal Weight, otherwise the Shots will vary.*
3. *That the Carriage in Breadth be always on a Level, and without any Descent, that so it may not leap in Discharging.*
4. *That the Powder with which the Mortar is Loaded, be always of the same Force and Weight.*
5. *That the Charge of the Mortar, as well in Powder as in Wadding, be always Rammed in with Blows equally Heavy, and of equal Number.*
6. *That the Wads be always either of Wood or Tampeons, or else of Oakum, for the strongest drives it farthest.*
7. *That*

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7. That the Fuses be newly made, that is, in those Days they are to be used, and that they be made of a Composition proportionable to the Range that the Shot will make in the Air, so that the Bomb may break in the very Moment of its Fall; which Composition must be such, that though it fall in the Water yet not to Extinguish, but the Bomb there to break. And therefore, before we proceed any further, 'twill be necessary to shew how to Compose the Ingredients for your Fuse.

P R O P. I.

To make Fuses for Bombs, &c.

The Composition for Bombs must be of a slow Motion, that so time enough may be given to throw either Bombs, Granadoes, Fire-balls, Thundring-Barrels, &c. They are Compounded of these Ingredients, thus: Take a Pound of Gun-Powder, $\frac{4}{15}$ of Sulphur, $\frac{4}{15}$ of Salt-Peter, well beaten, dry, and sifted separately, then mix it and make up your Fuse thereof: Or take Powder of Benjamin and Small-Coal, all well beaten and mixed together with some Oil of Peter, and so fill your Fuse therewith.

P R O P. II.

Suppose a Shot with a Mortar upon 15 Degrees of Elevation throws a Bomb 30 Yards, How far will it throw it upon 35 Deg. of Elevation?

Say, As Sine 30 deg. Double Angle Elev. 9.698970

To Sine of 70 deg. Double Angle Elev. 9.973986

So is 30 Yards the first Range 1.477121

11.451107

To 57 Yards the second Range 1.752137

So

So that upon 35 Deg. of Elevation the same Mortar with the same Charge will throw a Bomb 57 Yards. And after this way are the Distances or Ranges upon all Elevations found.

§ 4.

Again: Admit a Shot with a Mortar upon 20 deg. Elevation sends a Bomb 420 Yards, What is the greatest Distance such a Mortar will throw a Bomb.

 Note, The greatest Distance a Mortar can throw a Bomb, is, when the Mortar is Elevated to 45 deg. for the Sine of double 45 deg. which is 90, is the greatest of all Sines.

Say, As Sine of 40 deg.

9.808067

To Sine of 90 deg.

10.000000

So is 420 Yards the first Range

2.623249

To 652 Yards the second Range

2.814182

So the greatest Space such a Mortar can throw a Ball, is 652 Yards.

P R O P. III.

Suppose I had a mind to let fall a Bomb 400 Yards distant from me, what Elevation is requisite thereto, from a Mortar whose greatest Range is 700 Yards.

This Proposition is the Converse of the former, and therefore you must say,

As 700 Yards

2.845098

To 400 Yards

2.602060

So is Sine of 90 deg.

10.000000

To the Sine of 34 deg. 51 min.

9.756962

The Half of this 34° 51' is 17° 25' $\frac{1}{2}$ the true Angle of Elevation.

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The same may be found from any other Range as well as the greatest, if the *Angle of Elevation* were given.

From these *Proportions* is the *Table of Ranges* made.

P R O P. IV.

Suppose the greatest Range of a Shot be 600 Yards, 'tis requir'd to find how high such a Piece will throw a Ball upon 40 deg. of Elevation.

 *Note: The greatest Height that a Ball will ascend to when shot Perpendicular upright, is $\frac{1}{2}$ the greatest Range.*

Say therefore: *As the Versed Sine of 180 deg. (viz. twice the Rad.) is to the Versed Sine of 80 deg. (viz. twice 40 deg.) So is $\frac{1}{2}$ the greatest Range, to the Altitude required.*

Multiply therefore the *Versed Sine* of 80 deg. which is 8263 by 300 Yards, which is half the greatest Range, and dividing the *Product* by 20000 the *Versed Sine* of 180, the *Quote* is 124 Yards, the Perpendicular Height. See the following Operation.

$$\begin{array}{r}
 8263 \\
 300 \\
 \hline
 2 \mid 0000 \quad) \quad 247 \mid 8900 \quad (\quad 124 \text{ Prope.}
 \end{array}$$

And so on the Contrary: *If the greatest Horizontal Range was given, and the Perpendicular Height of any Shot known, to find the Angle of Elevation*

Say: As 300 Yards half the greatest Range, to the *Versed Sine* of 180 deg. viz. 20000; so is the given Perpendicular Height of a Shot; Suppose

Of Gunnery.

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pose 124 Yards to 8263 the *Versed Sine* of 80 deg. half of which is 40 deg. the *Angle of Elevation* required. § 4.

P R O P. V.

Suppose a Shot made upon 45 deg. of Elevation, and by a Watch with a Hand to show Seconds, I find it continues 12 Seconds in the Air, 'tis requir'd to know how long it will continue in the Air when shot at 75 deg. of Elevation.

Say, As Sine of 45 deg.	9.849485
Is to 12 Seconds	1.079181
So is Sine of 75 deg.	9.984944
	11.054125
To 16 Seconds	1.214640

By this Rule *Fusees* are adjusted as to their time of Burning.

The Contrary of this Proposition is also manifest, viz. That if upon 54 deg. a Shot continues in the Air 14 Seconds, how many Degrees was the Piece Elevated to when the Shot continued 16 Seconds in the Air?

Say, As 14 Seconds	1.146943
To 16 Seconds	1.204120
So is Sine 54 deg.	9.907958
	11.112073
To the Sine of 67° 35'	9.965130

which 67° 35' is the *Angle* the Piece must be set to for the *Ball* to continue 16 Seconds in the Air.

Further I might have gone in shewing from some *Data's* to find the *Powder requisite*, *Length* of the *Chase*, *Diameter* of the *Ball*, &c. But these being not so absolutely necessary to be acquired

§ 4.

acquired in this Case I omit them, and will conclude with a useful Table by which may be found the *Horizontal Random* of any Shot from some given *Elevation*; as also from a given *Elevation* to cast a Shot to any Place at Demand, provided it be within the reach of the Piece, and that without the Help of *Logarithms*.

The Use of the following Table is thus: The left Column is the *Degrees of Elevation*, and the Top are the *Minutes*, being to every 6 *Minutes*, beginning with 6, 12, 18, &c.

Suppose a Piece Elevated $13^{\circ} 18'$, lays a Ball 1837 Paces, what is his greatest Random?

In the Table you will find against 13 deg. and under 18 min. 896. Say therefore, As 896 to 2000; So is 1837 Paces to 4100, the greatest Random of such Piece.

If the *Elevation* had been $76^{\circ} 42'$, which is as much above 45 deg. as $13^{\circ} 18'$ is under, the same would have come out.

The contrary of this is also evident; for, If the greatest *Horizontal Range* be 4100 Paces, what must be the *Elevation* of the Piece to throw a Ball 1837 Paces?

Say, As 4100 to 2000; So is 1837 to 896, against which in the following Table you have in the left hand Column 13 deg. and in the Head of the Table over the Number you have 18 Minutes, which is the *Elevation* of the Piece to lay a Ball at 1837 Paces distance. Or if you look in the Right-Hand Column, at the Bottom of the Table you will find $76^{\circ} 42'$ the *Elevation* over 45 deg. to throw a Ball the aforesaid Distance.

A TA-

A TABLE of the Horizontal Distances of Projections.

Deg. Elev.	0	6	12	18	24	30	Min.
0	0	7	14	21	28	35	89
1	70	77	84	91	98	105	88
2	140	146	153	160	167	174	87
3	209	216	223	230	237	244	86
4	278	285	292	299	306	313	85
5	347	354	361	368	375	382	84
6	416	423	429	436	443	450	83
7	484	491	497	504	511	518	82
8	551	558	565	571	578	585	81
9	618	625	631	638	645	651	80
10	684	691	697	704	710	717	79
11	749	756	762	769	775	781	78
12	813	820	826	833	839	845	77
13	877	883	889	896	902	908	76
14	939	945	951	957	964	970	75
15	1000	1006	1012	1018	1024	1030	74
16	1060	1066	1072	1078	1083	1089	73
17	1118	1124	1130	1136	1141	1147	72
18	1176	1181	1187	1192	1198	1204	71
19	1231	1237	1242	1248	1253	1259	70
20	1286	1291	1296	1302	1307	1312	69
21	1338	1343	1349	1354	1359	1364	68
22	1389	1394	1399	1404	1409	1414	67
23	1439	1444	1448	1453	1458	1463	66
24	1486	1491	1496	1500	1505	1509	65
25	1532	1537	1541	1545	1550	1554	64
26	1576	1580	1585	1589	1593	1597	63
27	1631	1622	1626	1630	1634	1638	62
28	1658	1662	1666	1670	1674	1677	61
29	1696	1700	1703	1707	1711	1714	60
30	1732	1736	1739	1742	1746	1749	59
31	1766	1769	1772	1776	1779	1782	58
32	1798	1801	1804	1807	1810	1813	57
33	1827	1830	1833	1836	1838	1841	56
34	1854	1857	1860	1862	1865	1867	55
35	1879	1882	1884	1886	1889	1891	54
36	1902	1904	1906	1908	1911	1913	53
37	1923	1924	1926	1928	1930	1932	52
38	1941	1942	1944	1946	1947	1949	51
39	1956	1958	1959	1961	1962	1963	50
40	1970	1971	1972	1973	1974	1975	49
41	1981	1981	1982	1983	1984	1985	48
42	1989	1990	1990	1991	1992	1992	47
43	1995	1996	1996	1996	1997	1997	46
44	1999	1999	1999	1999	2000	2000	45
Min.	60	54	48	42	36	30	Deg. Elev.

The TABLE continued.

Deg. Elev.	36	42	48	54	60	Min.
0	42	49	56	63	70	89
1	112	119	126	133	140	88
2	181	188	195	202	209	87
3	251	258	265	271	278	86
4	320	327	334	340	347	85
5	388	395	402	409	416	84
6	457	463	470	477	484	83
7	524	531	538	545	551	82
8	591	598	605	611	618	81
9	658	664	671	677	684	80
10	723	730	736	743	749	79
11	788	794	801	807	813	78
12	852	858	864	870	877	77
13	914	920	927	933	939	76
14	976	982	988	994	1000	75
15	1036	1042	1048	1054	1060	74
16	1095	1101	1107	1113	1118	73
17	1153	1159	1164	1170	1176	72
18	1209	1215	1220	1226	1231	71
19	1264	1269	1275	1280	1286	70
20	1317	1323	1328	1333	1338	69
21	1369	1374	1379	1384	1389	68
22	1419	1424	1429	1434	1439	67
23	1467	1472	1477	1482	1486	66
24	1514	1519	1523	1528	1532	65
25	1559	1563	1567	1572	1576	64
26	1601	1606	1610	1614	1618	63
27	1642	1646	1650	1654	1658	62
28	1681	1685	1689	1692	1696	61
29	1718	1721	1725	1729	1732	60
30	1753	1756	1759	1763	1766	59
31	1785	1788	1791	1795	1798	58
32	1816	1818	1821	1824	1827	57
33	1844	1846	1849	1852	1854	56
34	1870	1872	1875	1877	1879	55
35	1893	1896	1898	1900	1902	54
36	1915	1917	1919	1921	1923	53
37	1934	1935	1937	1939	1941	52
38	1950	1952	1953	1955	1956	51
39	1965	1966	1967	1968	1970	50
40	1976	1978	1979	1980	1981	49
41	1986	1987	1988	1988	1989	48
42	1993	1994	1994	1995	1995	47
43	1998	1998	1998	1999	1999	46
44	2000	2000	2000	2000	2000	45
Min.	24	18	12	6	0	Deg. Elev.

Of Gunnery.

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§ 4.

More *Examples* might be given, but these are sufficient for *Practice*: what I have else to say, is, That the Resistance of *Air* is some Impediment in *Nice-shooting* (for by the Resistance of that *Medium* all Projects are diverted from flying in a true *Parabola*) and consequently that the greatest *Random* is not made upon 45 *Degrees of Elevation*, but at some few *Minutes* under, and so will cause some small Difference in the *Distances*.

But considering that in ordinary and common Practice (especially in Service) that with the common Instruments we can scarce come to the Certainty of a *Degree* or two, the foregoing *Table* will be near enough, and serve very well for ordinary Practice, although the Allowance for the *Air's* Resistance be not therein Considered.

Vive, vale : si quid novisti rectius istis,

Candidus imperti : si non his utere mecum.

Horat. lib. I Epist.

A SHORT
TREATISE
 OF THE
 DESCRIPTION and USE

Of the following

Mathematical Instruments;

V I Z.

Globes,
 Sector,
 Quadrant,
 Forestaff,

} Nocturnal,
 } Universal Dial,
 } Gauging Rod,
 } Coggeshal's Rule.

By W. ALINGHAM.

The Description and Use of the GLOBES.

Notwithstanding *Astronomical* Definitions have already been given before, I must upon the *Description and Use* of the GLOBES, re-iterate some of them here again as followeth.

A *Sphere* or *Globe*, is a perfect round *Solid* contained under one *Surface*, in the midst of which is a Point called the *Center*, from whence all *Lines* drawn to the outside are equal.

The Description and Use, &c. 307

In this Form the *Heavens* and *Earth* are prov'd nearly to be, and therefore *Astronomers* have contriv'd Two *Artificial Globes*, one to represent the *Heavens*, on which is depicted the Principal fixt *Stars* in such Proportion and Distance as they are found to have in the *Heavens*; on the other is described the *Earth* and *Water* in all their Parts, and according to their true Situation, Form and Distance, as found by the late Discoveries of our Modern *Navigators*, so that in reference to our present Business a *Sphere* or *Globe* is only an Artificial Resemblance of *Heaven* and *Earth*.

The Line the *Globe* turns about on is termed the *Axis* of the *World*, the Extremities of it are the *Poles* of the *World*, one of which is the *North Pole*, the other the *South Pole*. It is represented in the *Artificial Globe* by the two *Wires* the *Globe* turns upon in the *Meridian*.

Of the Circles of the Sphere.

Circles of the *Sphere* are Lines imagined to be drawn in the *Cavity* of the *Heavens*: Their Use is to determine several Propositions in *Astronomy*, *Geography* and *Navigation*.

These *Circles* are *Greater* or *Lesser*; *Greater* are such as cut the *Globe* into two equal Parts, *Lesser* cut the *Globe* unequally.

Every *Circle*, whether Great or Small, is conceived to be divided into 360 equal Parts called *Degrees*, and every *Degree* they again divide into 60 *Minutes*, and every *Minute* into 60 *Seconds*, &c. so that one *Degree* is the $\frac{1}{360}$ of a *Circle*, and one *Minute* is the $\frac{1}{2160}$ of a *Degree*.

The Principal *Great Circles* are the *Horizon*, the *Meridian*, the *Equinoctial* or *Equator*, the *Zodiack* or *Ecliptick*, and the two *Colures*.

The *Horizon* divides the Visible Part of *Heavens* from the Invisible, and when at *Sea*, or on Plain Ground, is the *Circle* that seemingly parts *Heaven* and *Earth*; this *Circle* limits the Length of the Artificial Day, for when the *Sun* is above this *Circle* 'tis then Day, when under 'tis Night. It bounds the Rising and Setting of the *Sun*, *Moon* and *Stars*, and shews the Point of the *Compass* they Rise or Set at. 'Tis artificially represented by the broad Wooden *Circle* in which the Brass *Meridian* and the *Ball* turns; Upon this *Circle* is a *Calendar*, shewing the Place of the *Sun* by having the Day of the Month, according to either Account; as also next the outer Edge is set the *Winds* or *Points* of the *Compass*.

The *Meridian* passeth through the *Zenith* and *Nadir*, and both *Poles* of the *World*, and cuts the *Horizon* at *Right Angles*, and shews the North and South Points thereof. It divides the Visible Part of the *Heavens* into two equal Parts, one of which is the Eastern, the other the Western *Hemisphere*; it also determines the *Noon* and *Midnight* of all Places, for when the *Sun* is upon this *Circle* in the Day-time it is *Noon*, when upon it in the Night, 'tis *Midnight*.

It is represented on the *Artificial Globe* by the Great Brass Hoop the *Ball* hangs in, and is divided into 360 *Degrees*.

The *Equinoctial* or *Equator*, is equally distant from both *Poles* of the *World*; It divides the *Globe* into two *Hemispheres*, that next the *North Pole* is the *Northern Hemisphere*: the other next the *South* is the *Southern Hemisphere*, All Places lying under this *Circle* have their *Day* and *Night* always equal; for in all Positions of the *Globe*, this *Circle* is half above and half under the *Horizon*. When the *Sun* comes to this *Circle*, which is twice a Year, he makes the *Day* and *Night* equal to all the Inhabitants of the *Earth*; 'tis upon this *Circle* we number the *Right* and *Oblique Ascension* of the *Sun* and *Stars*, this *Circle* is delineated or depicted on the *Ball* of an *Artificial Globe*, and is actually divided into 360 equal Parts.

The *Zodiack* is one of the greatest *Circles* of the *Sphere*, because by *Astronomers* it has allowed 18 *Degrees* of Breadth, which Breadth is limited by the wandring of the *Planets*; upon it is placed the 12 *Signs*; In the middle of the *Zodiack* runs the *Ecliptick Line*, so named from the *Greek Word Ecliepo*, which signifies to fail or want; 'tis under this *Line* the *Sun* and *Moon* suffer their *Eclipses*, this *Circle* cuts the *Equinoctial* obliquely, and is called *Via Solis*, because the *Sun* describes it by his Yearly Motion.

This *Circle* is described on the *Artificial Globe* with the *Pourtraiture* of the 12 *Signs*, and is divided into 360 equal Parts, as was the *Equator*.

The *Colures* are Two great *Circles* of the *Sphere*, being drawn through both *Poles* of the *World*, in which Points they cut one another, and also the *Equator*, at *Right Angles*. One of

these is named the *Solstitial Colure* from its passing through the first Points of *Cancer* and *Capricorn*, in which Points the *Sun* seems to stand still, the other is called the *Vernal Colure*, from its passing through the first Points of *Aries* and *Libra*. They determine the Four Seasons of the Year, for when the *Sun* passes over the *Solstitial Colure* he shews the Beginning of *Summer* and *Winter* Quarters, the first when he is in his greatest *North Declination*, the latter when he is in his greatest *South Declination*. When he passes the *Vernal Colure* he shews the Beginning of *Spring* and *Autumn* Quarters, the former beginning when he passes over it in his *Ascension*, the latter when he passes over it in his *Descension*.

These Two *Circles* are described on an *Artificial Globe*, and actually divided into 360 equal Parts or Degrees.

Of the Lesser Circles.

The *Lesser Circles* are the Two *Tropicks* and Two *Polars*.

The *Tropicks* are distant on each Side the *Equinoctial* or *Equator* $23^{\circ} \frac{1}{2}$; That on the North Side is called the *Tropick of Cancer*, the other on the South Side is named the *Tropick of Capricorn*; these *Circles* limit the *Torrid Zone*: they also bound the *Sun's* Course, he never swerving so far North or South, as to go beyond either of them*.

* These are delineated on an Artificial Globe by a red or gilded Line.

The *Polars* are as far distant* from the *Poles* of the *World* as the *Tropicks* are from the *Equator*, that at the North is termed the *Arctic Circle*, that next the South the *Antarctic*. They

They bound the *Frigid Zones*, and shew us what Places have their *Artificial Day* longer than 24 Hours. These *Circles* are also actually described on an *Artificial Globe* by such Lines as were the *Tropicks*.

Besides the *Circles* before mentioned, there are some other both of the greater and lesser sort, which are very necessary for Solving *Astronomical* and *Geographical Problems*, and are as followeth.

Hour Circles or *Meridians*, are great *Circles* meeting in the *Poles* of the *World*. They are on the *Artificial Terrestrial Globe* drawn thro' every 15 deg. of the *Equator*, and at *Right Angles* thereto.

Azimuths or *Vertical Circles*, are great *Circles* all meeting in the *Zenith* and *Nadir*, and cut the *Horizon* at *Right Angles*. These are never described on an *Artificial Globe*, because the *Quadrant of Altitude* supplies their Place.

Circles of Longitude, or *Position*, are great *Circles* meeting in the *Poles* of the *Ecliptick*, and therefore cut it at *Right Angles*. They are drawn on the *Artificial Cælestial Globe* through every 30 Degrees of the *Ecliptick*.

Circles (or as commonly called *Parallels*) of *Declination*, are small *Circles* parallel to the *Equator* on both Sides thereof, and on the *Terrestrial Globe* are drawn through every 10° of the *Meridian*, gradually diminishing till they end in the *Pole*.

Almicantbars, or *Circles of Altitude*, are small *Imaginary Circles* parallel to the *Horizon*, but are never described on an *Artificial Globe*, the *Quadrant of Altitude* shewing the Points thro' which they must pass.

Parallels of Latitude are small *Circles* imagined to be drawn parallel to the *Ecliptick*, and are sometimes delineated on an *Artificial Cælestial Globe*.

Rhomb Lines are neither *Circles* nor *Straight Lines*, but *Spiral*, imagin'd to proceed from the Place where we stand, and wind about the *Globe* till they lose themselves in the *Pole*. They are drawn on an *Artificial Terrestrial Globe*, from the Four Points where the *Colures* cut the *Equator*, and their Use is to shew the bearing of Places one from another.

Of the Appendants to an Artificial Globe.

To a *Globe*, (besides the forementioned *Circles*) belongs a small *Brass Circle* called the *Hour Circle*, whose *Center* is the *Pole* of the *Globe*; 'tis divided into 24 equal Parts, representing the *Hours* in a *Natural Day*. This *Circle* is fixed in a *Grove* on the *Brass Meridian*; on the *Pole* is put a small *Bit* called the *Index*, which, as the *Globe* turns about, points to the Divisions on the *Circle*: Their Use is to shew the Time of the *Sun*, *Moon*, or a *Star's* Rising and Setting, and what a *Clock* 'tis in any Part of the *World*.

There is also another *Appendant* called the *Quadrant of Altitude*; 'tis a thin *Brass Plate* divided into 90 equal Parts or *Degrees*, to one End is fitted a *Nut* and a *Skrew* for it to move, and then to be fixt to any *Degree* upon the *Brass Meridian*. Its Use is to limit *Azimuths* and *Almicantbars*, and to measure *Distances* and *Altitudes*.

Moreover to some *Globes* belong a small *Compass Box* and *Needle* for the setting the
Globe

Globe due North or South, also half a *Brass* Circle called the *Semicircle of Position*; but in regard none of the following *Problems* require them, they being but for some particular *Uses*, we shall omit a further Description of them.

Astronomical Terms Explained.

Declination is the *Distance* of the *Sun*, *Moon* or *Stars* from the *Equinoctial*, if they be on the North Side they have *North Declination*, if on the South Side, *South Declination*.

Right Ascension is an *Arch* of the *Equinoctial* contained betwixt the Beginning of *Aries*, and that Point of the *Equinoctial* which comes to the *Meridian* with the *Sun*, *Moon* or *Star*.

Oblique Ascension is an *Arch* of the *Equinoctial* comprehended betwixt the Beginning of *Aries* and that Point of the *Equinoctial* which arises with the *Sun*, *Moon* or *Star*.

Oblique Descension is an *Arch* of the *Equinoctial* lying betwixt the Beginning of *Aries* and that Point of the *Equinoctial* which sets with the *Sun*, *Moon* or *Star*.

Ascensional Difference is the *Difference* betwixt the *Right* and *Oblique Ascension* which, if converted into *Time*, gives the *Hour* and *Minutes* of the *Sun's* Rising and Setting before or after Six.

Amplitude is an *Arch* of the *Horizon* betwixt the true East or West Point, and that Point against which the *Sun*, *Moon* or *Star* rises, and is named according as it is, viz. either *Amplitude* of Rising or Setting.

Azimuth is the same with *Amplitude*, but only 'tis reckon'd when the *Planets* or *Stars* are above the *Horizon*. Alti-

Altitude is the *Height* of the *Sun*, *Moon* or *Stars* above the *Horizon*.

Longitude of a *Star* or *Planet*, is an *Arch* of the *Ecliptick* contain'd between the Beginning of *Aries* and the Point where any *Star* or *Planet* is; or else 'tis the Place where a Circle (that passeth thro' the *Star* and *Poles* of the *Ecliptick*) cuts it.

Latitude of a *Star* or *Planet*, is the Distance of it from the *Ecliptick*.

Longitude of a Place, is the Distance in *Degrees* and *Minutes* of the *Equator* from the *Assign'd* or *Fixt Meridian*.

Latitude of a Place is its Distance from the *Equator*, or 'tis an *Arch* of the *Meridian* contain'd betwixt the *Elevated Pole* and *Horizon* of the Place.

The *Zenith* is the Point directly over our Heads where-ever we be.

The *Nadir* is opposite to the *Zenith*, and therefore the Point directly under our Feet.

Of the Superficies of the Cœlestial Globe.

On the Surface of this *Globe* is depicted in Proportion and Distance most of the Principal *Fixt Stars*, in that Order and Position as they appear in the *Heavens*; and tho' the Number of them seems Infinite, yet the greater are not only numbred but have Names to distinguish them. The Number at present taken notice of are about 1400, the Biggest and Brightest of them are called *Stars* of the *First Magnitude*, the next to them *Stars* of the *Second Magnitude*, and so gradually decreasing to the *Sixth Magnitude*, which are the least that are pictur'd on the *Globe*.

Now

Now *Astronomers* are desirous to bring these Bodies into such Order whereby they might be more readier be known and found, having divided them into certain Forms and Shapes of Men, Beasts, Birds, and other Creatures called *Constellations*, by which Method of Division they know where to seek them in the *Natural Sphere*, and being found, how to express them on the *Globe* or *Hemisphere*. The Number of *Constellations* depicted on the *Globe* are 62, of which 23 are on the North Side of the *Ecliptic*, 12 upon it, and 27 on the South Side; in all which *Constellations* the Stars are express'd according to their several *Magnitudes*.

Astronomical Problems solv'd by the Glöbe.

PROB. 1. *Day of the Month (suppose April 1.) given, to find the Place of the Sun.*

Find the Day of the Month on the *Horizon*, according to the *Old* or *New* Account, then right against the same you have (*Aries* 22°) the Sine and Degree the *Sun* is in.

PROB. 2. *The Sun's Place (suppose 15° ♈) given, to find the Day of the Month.*

Seek for his Place on the *Horizon*, against which you have (*Jan.* 24.) the Day of the Month, according to the *Old* or *New* Account.

PROB. 3. *To rectifie the Globe fit for Use in any particular Latitude; (suppose London, which is $51^{\circ} 30'$).*

Count from the *North Pole* downwards on the *Brass Meridian* $51^{\circ} \frac{1}{2}$, and bring that Point to the *Horizon*, the *Quadrant of Altitude* must be screw'd to the *Zenith*, which is found by telling

telling the *Complement*, which is $38^{\circ} \frac{1}{2}$, from the *Elevated Pole* to the *Top* of the *Globe*, the *Place* of the *Sun* must be brought under the graduated *Edge* of the *Brass Meridian*, and then the *Index* must be set to point at 12 on the *Hour Wheel*.

PROB. 4. *Latitude of the Place* ($51^{\circ} \frac{1}{2}$) and *Place of the Sun* ($\cong 23^{\circ}$) given;

To find his $\left\{ \begin{array}{l} \text{Declination,} \\ \text{Right Ascension,} \\ \text{Height at Noon.} \end{array} \right.$

First, Rectifie the *Globe* by the foregoing *Problem*, then find out the *Place* of the *Sun* on the *Ball*, which *Point* bring under the graduated *Edge* of the *Brass Meridian* and there fix it. This done count on the *Brass Meridian* the *Degrees* betwixt his *Place* and the *Equinoctial*, it gives $8 \frac{1}{2}$ *Deg.* his *South Declination*.

Secondly, The *Degrees* on the *Equinoctial* betwixt the first *Point* of *Aries*, and the *Point* where the *Brass Meridian* cuts the *Equinoctial* is ($201 \frac{1}{2}$) his *Right Ascension*.

Thirdly, The *Degrees* on the *Brass Meridian* betwixt the *Sun's Place* and the *Horizon* is ($30^{\circ} 00'$) his *Height at Noon*.

PROB. 5. *Elevation of the Pole* ($51^{\circ} \frac{1}{2}$) and *Sun's Place* (27° *Cancer*) given,

To find $\left\{ \begin{array}{l} \text{His Amplitude of Rising,} \\ \text{Time of Rising,} \\ \text{Length of the Night,} \\ \text{Oblique Ascension,} \\ \text{Point of the Compass he riseth on,} \\ \text{Difference of Ascension.} \end{array} \right.$

Having

Having rectified the *Globe* turn the *Ball* Eastward till the Point of the *Sun's* Place comes right against the *Horizon*, then count the *Degrees* on the *Horizon* from the East Point thereof to the Point right against the *Sun's* Place, it gives ($35^{\circ} 00'$) his *Amplitude* of *Rising*. Secondly, The *Hour* the *Index* points to is (4 a Clock) the Time of *Rising*. Thirdly, Double the *Hour* and *Minute* of his *Rising*, it gives (8 *Hours*) the Length of the *Night*. Fourthly, The *Degrees* on the *Equinoctial* betwixt the first Point of *Aries*, and Point where the *Horizon* cuts it is (90°) his *Oblique Ascension*. Fifthly, The Point of the *Compass* he rises on is (N. E. by E.) found upon the *Horizon* right against his Place on the *Ball*. Sixthly, Subtract the *Right Ascension* from the *Oblique Ascension* or *Contra*, the Remainder (29°) gives the Difference of *Ascension*.

PROB. 6. Elevation of the Pole ($51^{\circ} \frac{1}{2}$) and Place of the *Sun* ($21^{\circ} 4'$) given,

To find { *His Amplitude* of *Setting*,
Time of *Setting*,
Length of the *Day*,
Oblique Descension,
Point of the *Compass* he *Sets* on.

The *Globe* rectify'd as before, turn the *Ball* Westward till the *Sun's* Place be just against the *Horizon*, then tell the *Degrees* on the *Horizon* betwixt the West Point and the *Sun's* Place, for that is ($38^{\circ} 00'$) his *Amplitude* of *Setting*. Secondly, See what time the *Index* points to, for that is (4 a Clock) the *Hour* of his *Setting*. Thirdly, Double the Time of his *Setting*,

Setting, it gives (8 Hours) the Length of the Day. *Fourthly*, The Degrees on the Equinoctial betwixt the Point where the Horizon cuts and the Beginning of Aries, is (318°) his Oblique Descension. *Lastly*, The Point of the Compass he Sets on is (S. W. by W.) found on the Horizon right against his Place on the Ball.

PROB. 7. Sun's Place ($25^{\circ} 00' \text{ N}$) Latitude of the Place ($51^{\circ} \frac{1}{2}$) given, and, any Morning Hour (suppose 9 a Clock) proposed;

To find $\left\{ \begin{array}{l} \text{His Azimuth from the East,} \\ \text{His Height,} \\ \text{Point of the Compass he is upon.} \end{array} \right.$

The Globe and all things rectified, move round the Ball Eastward till the Index points to the Hour propos'd, and there fix it; Then if you lay the Quadrant of Altitude's graduated Edge over the Sun's Place, and tell the Degrees from the East Point of the Horizon to the Point where the Quadrant cuts the Horizon, it will give you ($32^{\circ} 00'$) his Azimuth from the East. *Secondly*, You have his Altitude ($38^{\circ} 00'$) by counting the Degrees on the Quadrant betwixt the Horizon and his Place. *Lastly*, Just against the Quadrant's Edge on the Outer Circle of the Horizon you have (S.E. by E.) the Point of the Compass he is then upon.

If an Afternoon Hour had been propos'd, the Globe must have been turn'd Westward, and the Azimuth found from the West.

Note, 'Tis by this Problem we find the Sun's Azimuth at 6 in the Morning or Afternoon, which he has only in the Summer Half Year to all Northern Inhabitants.

PROB.

PROB. 8. Again, Place of the Sun ($17^{\circ} 00' 00''$) Latitude of the Place ($51^{\circ} \frac{1}{2}$) and any Azimuth (suppose 28 Degrees) from the East Southward given ;

To find { His Altitude,
Hour of the Day,
Point of the Compass he is upon.

All things rectified, bring the Quadrant of Altitude about till it's graduated Edge stands just against the given Azimuth, then turn about the Ball till the Sun's Place comes under the Quadrant's Edge and there fix it ; then the Degrees on the Quadrant betwixt the Sun's Place and Horizon is ($39^{\circ} 00'$) his Height. Secondly, The Hour the Index points to is the Time of the Day. Thirdly, The Point of the Compass is (S. E. by E.) had on the Outer Circle of the Horizon right against the Sun's Place on the Ball.

Note: If the Azimuth given be Westward, the Hour found will be Afternoon, otherwise 'tis Morning.

By this Prob. the Sun's Height or Hour of the Day is found when he is due East or West, which can only be in the Summer Half Year, in Places that have North Latitude.

PROB. 9. Sun's Place ($6^{\circ} 00' 00''$) and Elevation of the Pole ($51^{\circ} \frac{1}{2}$) with any Afternoon Altitude (suppose $32^{\circ} 00'$) given ;

To find { The Hour of the Day,
Azimuth from the West,
Point of the Compass he is upon.

ALL

All things rectified, bring the *Quadrant* of *Altitude* on the West Side of the *Horizon*, then turn the *Ball* and it together till the *Altitude* counted on the *Quadrant* from the *Horizon* lies exactly over the *Sun's Place*, where fix it, then the *Hour* the *Index* points to is ($\frac{3}{4}$ after 3) the *Time* of the *Day*; also the *Degrees* on the *Horizon* from the West to the Divided Edge of the *Quadrant* is ($20^{\circ} 00'$) his *Azimuth* from the West. Thirdly, Just against the *Quadrant's Edge* on the *Outer Circle* of the *Horizon* is (W.S.W.) the *Point* of the *Compass* he is then upon.

If it had been *Morning Altitude* the *Globe* must have been turn'd Eastward, and then the *Index* would have pointed to a *Morning Hour* and the *Azimuth* will be *Easterly*.

PROB. 10. *Latitude of the Place* ($51^{\circ} \frac{1}{2}$) and *Place of the Sun* ($20^{\circ} 00'$) given, to find the *Length of the longest present* (which suppose Jan. 29.) and *shortest Day*.

From several of the preceding *Problems* and *Nature of the Sun's Motion*, 'tis manifest the *Sun's Declination* is the Cause of the Increase and Decrease of the *Artificial Day*, for his Increasing in *North Declination* lengthens the *Day* to all *Northern Inhabitants*, but the contrary happens to all *Southern*; and consequently, when he is in his greatest *North Declination*, the *Longest Day* is made to all that have *North Latitude*, but when in his greatest *South Declination*, then the *Longest Day* is made to all *Southern Inhabitants*.

Rectify therefore the *Globe* to the *Latitude* of the *Place*, and find the *Place of the Sun* for the

the present Day (which is $20^{\circ} 00' \approx$) on the Ball, this done, bring his said Place to the East Side of the *Horizon*, and set the *Index* to the lower 12 on the *Hour Wheel*. Then (having made two Marks on the *Tropicks* where the East Part of the *Horizon* cuts them,) turn about the Ball Westward, till the Mark on the *Tropick of Capricorn* comes right against the West Side of the *Horizon*, so the *Hours* the *Index* has past over from the Point 'twas set at is (7 *Hours* $\frac{1}{2}$) the Length of the *Shortest Day*. This done, turn it more Westward till ($20^{\circ} 00' \approx$) the *Sun's* Place for the *present Day* comes against the *Horizon*, then, as before, the *Hours* the *Index* has past over from the Point 'twas first set at, which (is 9 *Hours* $\frac{1}{2}$) will be the Length of that Day in our *Latitude*. *Thirdly*, Turn the *Globe* more Westward till the Mark on the *Tropick of Cancer* come just against the West Point of the *Horizon*, so will the *Index* point out (16 *Hours* $\frac{1}{2}$) the Length of the *Longest Day* in the *Latitude* of $51^{\circ} 00'$ North.

The Length of the *Night* is found by taking the Length of the Day from 24 *Hours*.

PROB. II. *Latitude of the Place* ($51^{\circ} 30'$) and *Place of the Sun* ($20^{\circ} 00' 8$) given;

To find { *Sun's Depression at Midnight,*
Time of Twilight's Beginning,
Time of Twilight's Ending,
Time of its Continuance, and
Length of the quite dark Night.

All things rectified, bring the opposite Point to the *Sun's* Place under the *Brass Meridian*,
Y (which

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(which Point is had in the *Ecliptick* under the *Brass Meridian* below the *Horizon*.) Then the *Degrees* on the *Meridian* betwixt that Point and the *Horizon* is ($21^{\circ} 00'$) his Depression at Midnight. Secondly, Move the *Ball* and *Quadrant* together Westward, till the aforefaid opposite Point of the *Sun's* Place lies just against 18 *Degrees* of the *Quadrant*, then the *Hours* the *Index* has pass'd is ($1 \text{ Hour } \frac{3}{4}$) the Time after *Midnight*, *Twilight* begins at; Or, if you move the *Ball* and *Quadrant* Eastward till the said Point be under 18 *Degrees* of the *Quadrant*, 'twill give you ($10 \text{ Hours } \frac{1}{4}$) the Time *Afternoon Twilight* ends at. Fourthly, If the *Hour Twilight* begins at be taken from the *Hour* of *Sun-rising*, 'twill leave ($5 \text{ Hours } \frac{3}{4}$) the Time of *Twilight's* Continuance. Fifthly, The Time that *Twilight* begins at, doubled, gives ($3 \text{ Hours } \frac{1}{2}$) the Length of the quite dark Night.

PROB. 12. The Day of the Month, (suppose Aug, 17.) according to either Account, given, to find what other Day in the Year is of the same Length.

From Prob. 10. 'tis evident that in the *Sun's* decreasing in *Declination*, there is one Day in which he had the same *North Declination* as he had in his Encreasing, which two Days are of equal Length. To find which, look the Place of the *Sun* for the given Day on the *Horizon*, which got, find it on the *Ball*, and note the *Declination*; then turn about the *Ball* till some other Point of the *Ecliptick* comes under the afore-found *Declination*, which Point note, and find out on the *Horizon*, and right against it

is

is (Apr. 4.) the Day of the Month that is of the same Length with the given Day.

PROB. 13. To find in what Latitude the Longest Day is an Hour longer than with us at London.

Set the *Globe* to London's Latitude, and bring the *Solstitial Colure* under the *Meridian*, and set the *Index* to 12, then at the Intersection of the *Horizon* with the *Tropick of Cancer* make a *Prick* on the said *Tropick*. After this move the *Ball* Westward till $7^{\circ} 30'$ of the *Equinoctial* have pass'd the *Meridian*, and against the *Horizon* on the *Tropick* make a second *Mark*, then bring back the *Colure* to the *Meridian* and there fix it; after which Elevate the *Globe* 'till the second *Prick* appears, so will the *Globe* rest at (56°) the Latitude of the Place Inquir'd.

PROB. 14. To find how much the Sun's Declination must increase or decrease, to make the Artificial Day an Hour longer (or shorter) than it is at (March 8.) the Time propos'd.

Having Elevated the *Globe* to the Latitude of the Place, find the Sun's Declination (for March 8.) this Declination mark on the *Vernal Colure*, which *Mark* bring to the East Part of the *Horizon*, then if the Days lengthen, turn the *Globe* Eastward, but if they shorten, turn it Westward, till $7^{\circ} 30'$ of the *Equator* has pass'd the *Meridian*, and where the *Horizon* now cuts it makes another *Mark* upon it, so the Distance on the *Colure* betwixt these two Marks is (6) the Degrees that the Declination must

Y 2

increase

increase or decrease to make the Day an Hour longer or shorter than it was at the time propos'd. *If the Days have not an Hour to lengthen or shorten, then this Prob. is improper.*

PROB. 15. *Having the Latitude of the Place, to find the Number of Days contain'd betwixt the Days of lengthening or shortening one Hour at any time propos'd. Suppose from March 8.*

Make two Pricks on the Colure, as in the last Example, then find the Days on which the Sun's Declination answers to those Pricks, so the Time included betwixt those two Days are the Days that he is lengthening or shortening one Hour at that time of the Year. In our Example 'twill be about 31 Days.

PROB. 16. *To find the Height of the Sun by the Globe, when the Sun shines.*

Set the Globe on some Plain (where the Sun shines) as level as you can, then turn the North Pole of the Globe to the Sun, moving it up and down till the Pole has no Shade, then will the Globe rest at the Altitude counted on the Brass Meridian betwixt the Pole and Horizon.

PROB. 17. *To know whether 'tis before or after Noon.*

Having made an Observation of the Sun's Height, make a second a little while after, then if the Altitude increase 'tis before Noon, if it decrease 'tis after Noon.

PROB

PROP. 18. To find the Hour of the Day by the Globe when the Sun shines.

Rectify the *Globe* and set it upon some plain truly Level, with the *North Pole* towards the North Part of the *Heaven*, then when the *Sun* shines, observe the Shade of the *Axis* upon the *Hour Circle*, 'twill give you the Hour of the Day.

Note: This is only for the *Summer Half-Year*, and therefore for to find the Hour for the other Half-Year, depress the said *Pole* as much below the South Part of the *Horizon*, by which means 'tis directly opposite to its first Position, so will the Shade of the *Axis* give the time of the Day in the *Hour Circle* as before.

Of the STARS.

PROB. 19. To find the Longitude and Latitude of any Star.

Lay the *Quadrant of Altitude* over that *Pole* of the *Ecliptick* nearest the *Star* whose *Longitude* you seek. Then place the *Quadrant's* Edge over the *Center* of the *Star*, and look what *Degrees* of the *Ecliptick* (reckoned from the Beginning of *Aries*) it cuts; for that is the *Star's Longitude*: Also the *Degrees* on the *Quadrant* betwixt it and the *Star* is its *Latitude*. Proceed thus and you'll find the *Longitude* of *Acturus* in *Bootes* to be about 20 *Degrees*, and his *Latitude* near 34 *Degrees*.

PROB. 20. To find the Distance of 2 Stars
wherever the Stars be.

Lay the *Quadrant of Altitude* over them, and if the *Quadrant* be too short, elevate or depress the *Globe* till they both come against the *Horizon*, so the *Degrees* betwixt the Stars, either on the *Quadrant* or *Horizon*, is their Distance. Proceed thus and you will find the Distance betwixt *Aldebaran* and *Cor Scorpio* 171 *Degrees*.

PROB. 21. To know what Stars will be upon the *Meridian* at any Hour of the Night and any time of the Year, the Latitude being given.

The *Globe* and all things rectified, bring the *Sun's* Place for the given Day under the *Brass Meridian*, and set the *Index* to 12, then turn the *Ball* Westward till the *Index* points to the Hour propos'd, where fix it; then all the Stars that are under the *Brass Meridian*, are then upon the *Meridian*. Proceed thus and you shall find that *Scheat* and *Marshal* both in *Pegasus* with some other small Stars, will be under the *Meridian* on Oct. 8. at $\frac{1}{4}$ past 9 at Night.

PROB. 22. To know what Day in the Year and Star will be upon the *Meridian* at 9 at Night.

Bring the *Star* to the *Meridian*, then turn the *Ball* Eastward till the *Index* points at as many Hours before Noon as the *Star* is to be on the *Meridian* after Noon, that is, in our Example, till the *Index* points at 3; the *Globe* resting in this Position, see what Point of the *Ecliptick* is under the *Brass Meridian*, which Point is the Place of the *Sun*, and if sought out

on the *Horizon*, right against it is the Day of the Month the *Star* propos'd will be on the *Meridian*. Proceed thus and you shall find that *Aldebaran* will be on the *Meridian*, December 30. near 9 at Night.

PROB. 23. *Latitude, Day of the Month, and Altitude of a Star given, to find the Hour of the Night.*

All things rectified, and opposite Point of the *Sun's* Place brought under the *Brass Meridian*, move the *Ball* till the *Altitude* propos'd lies just under the *Quadrant's* Edge, then will the *Index* point at the Hour of the Night.

Note, That if the *Sun's* Place and *Quadrant* be brought on the East Side, the Hour found is after *Midnight*, but if on the West Side, 'tis before *Midnight*. Proceed thus, and you'll find that *Cor Leonis* on November 1. is 25 deg. high at $\frac{1}{2}$ an Hour past 2 in the Morning.

PROB. 24. *To find how many Hours any Star continues above or under the Horizon in any Latitude.*

The *Globe* set to the *Latitude*, bring the *Star* to the *Meridian* and set the *Index* to 12, then turn the *Globe* Westward till the *Star* propos'd comes just against the *Horizon*, then shall the Hour the *Index* points to be half the Time, which doubled gives the whole Time; as suppose it points at 7 Hours $\frac{3}{4}$, this doubled gives 15 Hours $\frac{1}{2}$, the time it continues above the *Horizon*. Proceed thus and you shall find that *Fomahaut* in *Aquarius* will continue 5 Hours above the *Horizon* in our *Latitude*.

The Description and Use

PROB. 25. *At what Hour any time of the Year, any fixt Star rises or sets in any Latitude.*

Let it be propos'd to find what Hour any *Star* rises or sets *Sept. 14.* The *Globe* being rectified to the *Latitude* and Day propos'd, that is, the *Sun's* Place brought under the *Brass Meridian*, turn about the *Globe* till the *Star* enquir'd after comes just against the East Part of the *Horizon*, then will the *Index* point at the Hour of its rising. If you turn it Westward till it comes against the *Horizon*, the *Index* will shew the time of its setting. Proceed thus, and you shall find that in our *Latitude*, *Syrius* the *Dog-Star*, on the 14. of *September*, will rise at 7 a Clock.

PROB. 26. *Latitude of the Place given, to find what Stars never rise or set.*

Observe what *Stars* have $38^{\circ} \frac{1}{2}$ North Declination, for all such *Stars* never set in our *Latitude*, neither doth any *Star* rise that hath above $38^{\circ} \frac{1}{2}$ of South Declination.

PROB. 27. *To find the Cosmical and Acronical Rising and Setting of the Stars.*

The *Cosmical* Rising of a *Star* is when it riseth with the *Sun*, and the *Cosmical* Setting when it sets in the Morning or goes down when the *Sun* is rising. The *Acronical* Rising is when a *Star* riseth when the *Sun* sets, and the *Acronical* Setting is when a *Star* sets with the *Sun*.

The *Globe* rectified, bring the *Sun's* Place for the Day propos'd to the East Part of the *Horizon*,

Horizon, then all the *Stars* that appear on the *Ball* against the Eastern Edge of the *Horizon*, do that *Day* rise *Cosmically*; and those *Stars* which are against the Western Rim of the *Horizon*, do that *Day* set *Cosmically*. But if you bring the *Sun's* Place to the Western Side of the *Horizon*, then all the *Stars* against the East Part of the *Horizon* be rising *Acronically*, and all the *Stars* against the West Part of the *Horizon* be setting *Acronically*.

Thus proceeding, you shall find that on *May* the 27th, *Aldebaran* or the *Bull's Eye*, with other small *Stars* rise *Cosmically*, and the *Right Leg* of *Serpentarius*, with several smaller *Stars* setting *Cosmically*.

Also upon the 18th of *October* you shall find a *Star* in the *Whale's Tail*, and other smaller *Stars* *Acronically* rising, and the *Tail* of the *Lion*, *South Ballance*, and other little *Stars* setting *Acronically*.

The Use of the Globes in Geography and Navigation.

Before we enter on the *Geographical* and *Nautical Problems*, it will be necessary to inform the *Reader*, that among the *Meridians* before described to be on the Surface of the *Globe*, one is more generally noted than the rest, which *Meridian* is (as it were) the *Landmark* of the whole *Sphere*, and is the Bounds from whence the *Longitude* of all Places is accounted quite round the *Globe*. In our common *English Globes* it is made to pass thro' *St Michael's*, one of the *Azores Islands*.

In

In Navigation we count that the *Grand Meridian* from whence we begin our Reckoning.

PROB. I. *Any Place being found on the Globe to find its Longitude and Latitude.*

Bring the Place under the *Brass Meridian* then the *Degrees* the *Meridian* cuts on the *Equator* is its *Longitude*; also the *Distance* on the *Brass Meridian* betwixt the Place and *Equator* is the *Latitude*.

Thus proceeding, you will find

Jerusalem $\left\{ \begin{array}{l} \text{Longitude of } 66^{\circ} 40'. \\ \text{Latitude of } 32^{\circ} 10'. \text{ N.} \end{array} \right.$

The *Converse* of this *Problem* is easily perform'd, that is, by having the *Longitude* and *Latitude*, to find out the Place on the *Globe*.

PROB. 2. *To find the Bearing and Distance of one Place from another; suppose London.*

Elevate the *Globe* to the *Latitude* of *London* and bring *London* to the *Meridian*, so will it fall just in the *Zenith*, where fix the *Quadrant* of *Altitude*; then lay its graduated *Edge* over any other Place, whose *Bearing* and *Distance* you require from *London*. Then right against the *Quadrant* on the *Horizon*, is the Point of the *Compass* the said Place bears from *London*; also the *Degrees* on the *Quadrant*, betwixt the *Zenith* and Place, is their *Distance* in *Degrees*, which Multiplied by 70, gives their *Distance* in *English Miles*.

Proceed thus, and you shall find the *Bearing* and *Distance*

From

From London of	{	Tangier	{	S.S.W.	{	1225
		Aleppo in Syria		E. by S.		2170
		Babylon in Caldea		E. by S.		2660
		Surrat		East		4620

If the *Quadrant of Altitude* will not reach both Places, take their Distance with a Pair of *Compasses*, and apply that Extent to the *Equator* or *Ecliptick*, and you'll have their Distance in *Degrees*, as before.

PROB. 3. To find what Hour it is in any Part of the Earth, when 'tis Noon with us at London.

By reason of the *Diurnal Motion* of the Sun (or Earth) once in 24 Hours (the Sun never Enlightening but half of the Earth at one time) it comes to pass, that when 'tis Morning in one Place, 'tis Noon in a second, Night in a third, and Midnight in a fourth, according to their several Situations, in respect of East and West one from another, and this *Temporary Difference* is known by the *Degrees* in the *Equator* between the *Meridian* of any two Places converted into Time, allowing 15 *Degrees* to an Hour. And further note, that if the Place lies East from the Place whose *Noontide* was given, then 'twill be Noon at that, before 'tis at the other Place; if it lies West, then their Noon will be after the Noon of the Place whose *Noontide* is given. As if London be brought under the *Brass Meridian*, then all Places in the *Eastern Hemisphere* are said to lie East of London, and those in the *Western Hemisphere* West of London. This being premis'd, suppose at 12 a Clock at London I would know what 'tis at Port-Royal in Jamaica. Bring

Bring *London* to the *Meridian* and set the *Index* to 12, then turn the *Globe* Eastward, because *London* is East of *Jamaica*, till *Port-Royal* comes under the *Brass Meridian*, then the Time the *Index* points to (which is 6 Hours 37 Min.) so that 'tis 37 Minutes past 6 in the *Afternoon* at *Port-Royal* when 'tis 12 a Clock at *London*.

If you would know what Hour 'tis at any Place when 'tis any other Hour at *London*; take the Difference of the *Noontides* of the two Places from the Hour proposed, if the Place lies Westward of that you enquire; but if it lies Eastward, add the Difference to the Hour proposed, the Remainder or Sum is the Hour of the Day in that other Place.

EXAMP. When 'tis 1 a Clock at *London*, what is it at *Jerusalem*, their Difference of *Noon-tides* being $2\frac{1}{2}$ Hours?

So that 'tis 30 Min. past 1 at *London*,
 3 in the *Afternoon* at *Ferusa-* } $2\frac{1}{2}$ add
lem when 'tis 1 at *London*. } $3\frac{1}{2}$ at *Ferusalem*.

Again, when 'tis 2 at *London*, what is it at *Bermudas*, their Difference of *Noon-tides* being $4\frac{1}{4}$ Hours?
 2 at *London*,
 $4\frac{1}{4}$ Subtract
 $9\frac{3}{4}$ a Clock at *Bermudas*. } So that 'tis $9\frac{3}{4}$ a
 Clock at *Bermudas*
 when 'tis 2 at *London*.

Note: When you cannot take the Difference from the Hour proposed, add 12 to it.

PROB. 4. To know what Parts of the Earth have Noon when 'tis any Hour with us.

Note: If the Hour given at *London* be before Noon, the Places that have then Noon lie East

East of London, if the given Hour be *Afternoon* they lie West of London.

The *Globe* rectified, and the *Index* set at the Given Hour, turn the *Globe* about till the *Index* points to 12, then all Places under the South Part of the *Brass Meridian* have Noon at the Hour proposed, thus proceed, and you will find that when 'tis 9 a Clock in the *Morning* at London, 'tis Noon at *Babylon* in *Caldea*; and when 'tis $\frac{1}{4}$ past 4 in the *Afternoon* at London, 'tis Noon at *Bermudas*.

But note, when the *Sun* is in one Degree of *Cancer*, he Enlightens $23^{\circ} 30'$ of the *Meridian* on the other Side of the *North Pole*, and consequently $23^{\circ} 30'$ Dark on this Side the *South Pole*, understand the contrary when he is in the first Point of *Capricorn*, from which Consideration may easily be seen the Limitation in the preceding Case.

PROB. 5. To find, at any time of the Year and Hour of the Day, what Part of the Earth has the *Sun* in their Zenith.

No Part of the *Earth* has this Property but such as are in the *Torrid Zone*, Find therefore the *Sun's Declination* for the Day proposed, then rectify the *Globe* to the *Latitude* of the Place in which you propose the Question, and bring the said Place to the *Brass Meridian*, setting the *Index* to point at the Hour proposed, this done, turn the *Globe* Westward, if before Noon, else Eastward, till the *Index* points at 12: then if you look for the Degree of *Declination* on the *Brass Meridian*, you will have just under it the Place which on that Day and

and at that Hour has the *Sun* in their *Zenith*.

Thus proceed, and you shall find that on *Apr.* 10th and *Aug.* 17th, both which Days the *Sun* has $10^{\circ} 00'$ of *North Declination* at 30 Min.

past { 5 Afternoon } *Nombre di dias* on the 15th-
 { 6 Morning } *Sumus* of *Panama*; also some small Islands on the West of *Molucca* in *East India* to lie under the *Brass Meridian*, so that both those Places at the time propos'd have the *Sun* in their *Zenith*.

PROB. 6. To find what Parts of the Earth have the *Sun* Rising, what Setting, what have Noon, what have Midnight, also where *Twilight* is beginning, where ending, any time of the Year and Hour of the Day.

Find by the last Problem what Part of the Earth has the *Sun* in their *Zenith* at the time propos'd. Then Elevate the *Globe* to that Place's *Latitude*, after which bring it under the *Brass Meridian*, so shall all Places cut by the Eastern Part of the *Horizon* have the *Sun-rising*, those cut by the Western Part of the *Horizon* will have him setting, those under the Elevated Half of the *Meridian* will have *Noon*, those under the deprest Half will have *Midnight*, those 18° below the East Part of the *Horizon* will have *Twilight* beginning, and those Places 18° below the West Side of the *Horizon* will have *Twilight* ending.

PROB. 7. To find the *Antæci*, *Periæci*, and *Antipodes* of any Place.

Antæcians are those that lie in the same *Longitude* but have *Different Latitudes*, that is, they

They have as much *South Latitude* as we have *North*, hence they have the Hours with us, but contrary Seasons, their *Summer* being our *Winter*.

Periæciæ are those that lie in the same Degrees of *Latitude* but 180° Difference of *Longitude*, they have the same Seasons with us but contrary Hours, viz. *Noon* for *Midnight*.

Antipodes are those that are wholly opposite, going with their Feet to ours, having 180° Difference of *Longitude*, and $51^{\circ} \frac{1}{2}$ of contrary *Latitude*, and so have both Seasons and Hours contrary.

To find them by the *Globe*, bring *London* to the *Meridian*, then count from the *Equator* Southward on the *Brass Meridian* $51^{\circ} 30'$, and you have *London's Antæci*. Secondly, Count from the *North Pole* the Complement of the *Latitude* $38^{\circ} 30'$ downward and you have *London's Periæci*. Thirdly, reckon $38^{\circ} 30'$ from the *South Pole* downward on the *Meridian* and it gives the Point of *London's Antipodes*.

PROB. 8. Having the Sun's Declination and Meridian Altitude, to find the Latitude of the Place.

Count the Declination from the *Equator* on the *Brass Meridian* either toward the *North* or *South Pole*, as it is, noting the Point; then turn the *Brass Meridian* up and down in the Notches of the *Horizon*, till the Point on the *Meridian* be so far distant from the *Horizon*, as is the given *Altitude*; so will the *Globe* rest at the *Latitude*. Thus, if the Declination be $11^{\circ} 30'$ North, and the *Meridian Altitude* 50° Degrees, the *Latitude* will be $51^{\circ} 30'$.

Note,

Note, If the Observ'd Altitude be on the South Side of the Meridian, the Latitude is generally North; but if on the North Side, the Latitude is generally South.

PROB. 9. *By the Meridian Altitude of any fix'd Star, to find the Latitude.*

Bring the *Star* to the *Meridian*, and from its *Center* count downwards the given *Altitude*, and note the Point on the *Brass Meridian*; then bring the said Point to the South Part of the *Horizon*, if it was observ'd on the South Part of the *Meridian*, (but to the contrary Side of it, if it be observ'd on the North Part) and the *Globe* will rest at the *Latitude* of the Place.

Thus if the *Meridian* $\left\{ \begin{array}{l} \text{Algol} \\ \text{Spica Virginis} \end{array} \right\}$ be $\left\{ \begin{array}{l} 62^\circ \\ 32^\circ \end{array} \right\}$
Height of

the *Latitude* will be $\left\{ \begin{array}{l} 67^\circ 30' \\ 47^\circ 30' \end{array} \right\}$ North.

PROB. 10. *To find the Latitude by two Stars, one being on the Meridian, the other rising or setting.*

Bring the *Star* observ'd on the *Meridian* to the *Meridian* and there fix the Body of the *Globe*, then turn the *Meridian* up and down in the *Horizon* till the other *Star* be just against the East or West Side of the *Horizon*, then will the *Globe* stand at the *Latitude*.

Thus if *Aldebaran* be on the *Meridian*, and *Danebin* the *Lion's Tail* rising, the *Latitude* will be about $50^\circ 10'$ North.

PROB.

PROB. 11. To find the Latitude by the Height of 2 Stars being both on the same Azimuth or Point of the Compass.

Lay the Quadrant over both Stars at the Degree of Altitude you observ'd them, then turn the Brass Meridian up and down in the Horizon till the Quadrant's Edge rests at the Given Azimuth, so shall the Globe stand at the Latitude.

Thus if Capella and Scheder be both on the North East Azimuth, the first 20° high and the last 66° high, you will find the Globe to rest at 40° deg. which is the Latitude desired.

PROB. 12. Having the Sun's or a Star's Declination, to find the Amplitude.

Elevate the Globe to the Given Amplitude, and count from the Pole on the Meridian the Complement of the Amplitude to get the Zenith, and thereto screw the Quadrant of Altitude, and bring the End thereof to the East or West Side of the Horizon, -then count the Co-Declination of the Sun or Star from the Pole on the Equinoctial Colure, and bring the Degrees to the Quadrant of Altitude, then the Distance on the Quadrant betwixt that Point and the Zenith is the Latitude you are in.

Proceed thus and you shall find that if the Sun or a Star have 12 Degrees of Declination and 20 Degrees of Amplitude from the East Northward, that the Latitude will be about 51° Degrees North.

The following Table will serve as a Proof for most of the Astronomical Operations by the Globe.

Z

A T A.

A TABLE shewing from the first Day of *January*, to every 10th Day afterwards, the Sun's place also his Declination, Right Ascension, Oblique Ascension, Amplitude of Rising and Setting, Time of Rising and Setting, and Time of Twilight's Beginning and Ending.

Month	Days.	Sun's place.	De- clin.	R. A. cent.	Obli. Ascen.	Amp. of ris. & set. from S.	Time of ris. & set.	Begin of twi- light.	End
		° ' S. ° ' N.				° ' h. m. h.			
Jan.	1	21.25	21.51	292.42	323.03	050.48	8.01.4	5.54	7
	11	01.35	19.45	304.16	331.27	056.15	7.48.5	5.43	7
	21	11.44	17.14	314.28	337.27	062.20	7.31.5	5.35	7
	31	21.51	14.12	324.22	342.58	065.43	7.14.5	5.17	7
Feb.	10	01.55	10.47	334.00	347.53	072.21	6.55.6	5.00	7
	20	11.57	07.04	343.24	352.25	078.50	6.35.6	4.45	8
Mar.	2	22.55	02.47	353.35	357.06	085.14	6.16.6	4.20	8
			N.						
	12	02.50	01.11	002.45	001.14	091.35	5.56.7	3.59	9
	22	12.41	05.08	011.57	005.26	097.58	5.36.7	3.34	9
Apr.	1	22.29	08.35	020.19	009.46	104.24	5.17.7	3.05	9
	11	02.14	12.11	029.48	014.10	109.17	4.48.8	2.38	10
	21	11.56	15.28	039.32	019.10	114.17	4.40.8	2.02	10
May	1	21.35	18.18	049.34	024.58	119.24	4.24.8	1.30	11
	11	01.11	20.24	058.50	030.54	123.45	4.08.8	0.30	12
	21	10.46	22.08	069.25	038.35	128.23	3.57.9	Twilight	
	31	20.20	23.07	079.06	046.26	129.12	3.50.9	all	
June	10	29.52	23.30	090.00	056.48	130.26	3.47.9	Night	
	20	09.24	23.11	099.47	067.10	129.12	3.49.9		
	30	18.27	22.08	110.34	079.46	128.23	3.56.9		
July	10	28.30	20.24	121.09	093.13	123.45	4.07.8	0.42	12
	20	08.04	18.18	130.25	105.48	119.24	4.21.8	1.22	11
	30	17.35	15.28	140.27	120.03	114.17	4.36.8	2.00	10
Aug.	9	27.17	12.32	149.13	132.58	107.37	4.53.8	2.21	10
	19	06.58	08.57	158.43	147.17	104.24	5.12.7	3.00	9
	29	16.41	05.08	168.02	161.32	097.53	5.31.7	3.30	9
Sept.	8	26.27	01.35	176.19	174.20	093.11	5.51.7	3.59	9
			S.						
	18	06.17	02.22	185.30	188.30	089.49	6.11.6	4.19	8
	28	16.10	06.18	194.43	202.44	080.27	6.30.6	4.48	8
Octo.	8	26.06	10.04	204.05	217.00	073.49	6.50.6	5.00	7
	18	06.06	13.33	213.40	231.20	067.24	7.09.5	5.18	7
	28	16.08	16.40	223.31	245.39	072.20	7.27.5	5.37	7
Nov.	7	26.14	19.18	233.35	259.59	058.51	7.43.5	5.45	7
	17	06.22	21.21	244.06	273.37	055.18	7.57.5	5.54	7
	27	16.31	22.51	255.52	287.57	050.48	8.07.4	5.57	7
Dec.	7	26.42	23.27	266.43	299.52	050.48	8.12.4	6.00	8
	17	06.54	23.18	277.37	310.30	050.48	8.12.4	5.58	7
	27	17.05	22.24	288.36	319.44	051.37	8.06.4		

The Description and Use of the SECTOR.

THE SECTOR is a *Rule* that opens and shuts like a common two Foot *Joint-Rule*, they are from 4 Inches and $\frac{1}{2}$ long when shut, to 18 Inches. Upon this *Rule* are put several *Lines*, the principal of which are put *Sector-wise*, that is, they are drawn from the Center of the *Sector* upon both Legs to near the Ends of the Legs, the most useful of which is the Line of *Lines*, the Line of *Chords*, the Line of *Sines*, the Line of *Tangents*, and the Line of *Secants*.

The Line of *Lines* is no more than a Line of *Equal Parts* divided into 100 or 1000 Equal Parts, according as the *Sector* will bear it.

These *Lines* are drawn from the Center on each Leg, and are numbred with the Figures 1, 2, 3, &c. on each Leg to 10, which is near the End of the Leg. These *Lines* are noted by the Letter L.

And here note, that this 1 may be taken for 10, or for 100, 1000, &c. as Occasion requires, and then 2 will signifie 20, 200, 2000, &c. and consequently that 10 at the End of each Leg will be 100, 1000, 10000, &c.

The Line of *Chords* is a Line divided after the usual manner of the Line of *Chords*, from a Circle whose *Radius* is nearly the Length of one of the Legs; this Line is plac'd upon each Leg of the *Sector*, beginning at the Center and running toward the End thereof. It is numbred with 10, 20, &c. to 60, and to this Line on each Leg is set the Letter C.

The Description and Use

Note : This Line on some *Sectors* runs to 90 Degrees.

The Line of *Sines* is a Line of common natural *Sines*, divided from a *Circle* of the same *Radius* that the Line of *Chords* was ; these are also plac'd on each Leg of the *Sector*, and numberd with the Figures 10, 20, 30, &c. to 90, at the End of which on each Leg is set the Letter S.

The Line of *Tangents* is a Line of common *Tangents*, divided from a *Circle* of the forementioned *Radius* and is placed upon each Leg and runs to 45 ; it has the Numbers 10, 20 &c. to 45, placed upon it with the Letter T for *Tangents*.

Besides this is another small Line of *Tangents*, divided from a *Radius* of about 2 Inches and is plac'd upon each Leg of the *Sector* ; it begins at 45, which stands at the Length of the *Radius* from the Center and runs to about 75 Degrees or farther, having the Numbers 45, 50, &c. to 75, with the Letter T set there to. The Use of this Line (as hereafter shall be shown) is to supply the Defect in the Great Line.

The Line of *Secants* is only a Line of common *Secants*, divided from a *Circle* of about 2 Inches *Radius* ; these are placed upon each Leg of the *Sector*, beginning not at the Center but at 2 Inches distance therefrom, and runs to 75 Degrees. To these are set the Numbers 10, 20, 30, &c. to 75, with the Letter S at the End thereof for *Secants*.

These are the principal Lines now put on *Sectors*, and what will supply the Places of most

most of the other Lines, the Use of which is as follows.

The Use of the SECTOR.

The Excellency of this Instrument above the Common *Scales*, that it may be made to answer all Scales within the greatest opening and Closest shutting the *Sector*, for by it you may divide a Line, not exceeding the Distance between 10 and 10 when open, nor less than the Distance between 10 and 10 when shut into any Number of Equal Parts.

Also from the Lines of *Chords*, *Sines*, *Tangents*, &c. placed on the *Sector* as afore directed, you have a Line of *Chords*, *Sines*, *Tangents*, &c. to any *Radius* betwixt the Breadth of the *Sector* when shut, and Length when open'd, by which Contrivance a *Sector* is made one of the Usefulest *Mathematical Instruments* in being; As to the Invention and Contrivance of this Instrument, it, no doubt, arose from a thorow Consideration of the 4. Prop. of *Euclid's* 6th, which proves, *That Similar Triangles have their corresponding Sides proportional*; from which Consideration the Reason of all the Operations on this Instrument will be Evident.

But before we begin with its *Use*, 'tis necessary to acquaint the *Artist*, that the Lines on the *Sector* are used two sorts of ways, (*viz.*) *Laterally* and *Parallelly*. *Laterally* is when any Length is taken from the Center upon one Leg of the *Sector*; *Parallelly*, when you Extend from one Leg to the other, that is, quite cross the *Sector*.

Note ; The *Inmost Parallel* is the true Divided Line, and therefore in using the *Compasses* you must always set them upon the *Inside Line* both in *Lateral* and *Parallel Entrance*.

Secondly ; The Lines plac'd upon the *Sector* now made, are after a different way to what they formerly were ; for instead of putting the same Lines at equal Distances from the Inward Edge, they now put them at unequal Distances so that upon one Side of the *Sector* one Leg has the Line of *Chords* Innermost, the other Leg the Line of *Tangents* Innermost ; that is the Innermost Line of *Chords* and *Tangents* are equally distant from the Inward Edge, and so are the Outermost Line of *Chords* and *Tangents*. The Advantage of this *Contrivance* is this ; When you have set the *Sector* for any one Line, as the Line of *Lines*, or the Line of *Chords*, it will serve also for the Line of *Sines* and the Line of *Tangents*, for the *Parallel* betwixt 10 and 10 upon the Line of *Lines*, 60 and 60 upon the Line of *Chords*, 90 and 90 upon the Line of *Sines*, and 45 and 45 upon the Line of *Tangents* are all Equal. Hence the Reason is evident why they now make *Chords* run but to 60 upon the Leg of the *Sector*, whereas they used to run to 90, but by Practice the Inconveniency is found out.

The Use of the Line of Lines.

PROB. I. To Divide a Line into any Number of Equal Parts, suppose 9.

Take the Given Line betwixt the *Compasses*, and make it a *Parallel* betwixt 9 and 9 (which you may call 90 and 90) then shall the *Parallel*

parallel Distance of 8 and 8 cut off 8 of those Equal Parts, also the Parallel Distance of 7 and 7; mark out 7 of those Equal Parts, and so by taking the Parallel Distance of 5 and 5, 4 and 4, 3 and 3, 2 and 2, 1 and 1, you may divide the Line into 9 Equal Parts. If you would divide a Line into 37 Equal Parts; take the Line betwixt your *Compasses* and make it a Parallel betwixt 37 and 37 on the Legs of the *Sector*, then the Parallel Distance betwixt 36 and 36 will cut from the said Line 36 of those Parts, and leaves a $\frac{1}{37}$ Part remaining, and betwixt 35 and 35; also 34 and 34, &c. cuts from the said Line 35 and 34 of such equal Parts as the whole Line contains. After the like manner may any other Line, not exceeding the *Sector's* Length when open'd, be divided into any Number of Equal Parts.

PROB. 2. *To Increase or Diminish a Line in a given Proportion; that is, to find two Lines in any Proportion, as 5 to 7.*

Take the Given Line betwixt your *Compasses* and open the *Sector* till that Extent lies betwixt 5 and 5 on the Line of *Lines*, then if you take the Parallel Distance betwixt 7 and 7, 'tis the Line requir'd.

If it were to be *Diminished*, suppose as 7 to 5, then make the given Line a Parallel betwixt 7 and 7, and the Parallel Extent betwixt 5 and 5 is the Line requir'd.

PROB. 3. *Two Lines given, to find a Third in continual Proportion, Increasing or Diminishing.*

Take the Two Lines given, and set upon each Leg on the Line of *Lines* from the Center,

Z 4

marking

marking the Points where they fall, then if the Proportion be Increasing, take the Longest Line betwixt your *Compasses*, and apply that Extent Parallel-wise in the terms of the shortest. The *Sector* remaining at this *Angle*, the Parallel Distance betwixt the terms of the longest, taken and apply'd to the Leg of the *Sector* from the Center, will give the Length of the third Proportional; If it be *Diminishing*, then make the shortest of the two given Lines a Parallel in the terms of the longest, then the Parallel Distance betwixt the terms of the shortest apply'd as before, will give the third Proportional; if the Lines be 18 and 24, then the fourth Proportional *Increasing* is 32, and the fourth *Decreasing* is $13\frac{1}{2}$.

PROB. 4. *Three Lines given, to find a fourth in Proportion, either Increasing or Diminishing.*

If it be Increasing, set the two shortest Lines upon each Leg of the *Sector*, then open it till the longest Line comes betwixt the terms of the shortest. The *Sector* thus remaining, take the Parallel Distance betwixt the terms of the longest, and apply that Extent to the Leg of the *Sector*, it will give you the fourth Proportional *Increasing*.

If it be Diminishing, lay the two longest Lines on the Legs of the *Sector* from the Center, and make the shortest of the three a Parallel in the terms of the longest, then the Parallel betwixt the terms of the shortest, is the fourth Proportion *Diminishing*.

PROB.

PROB. 5. To find the Proportion betwixt two given Lines.

Make the greater a Parallel betwixt 10 and 100 on the Line of Lines, then take the lesser and carry it Parallel to the greater Line till it rests in like Points on each Leg, the Number where it rests is the Proportion to 10, 100, 1000, &c. requir'd.

So that from this Prob. if the Length of one Line be given, the Length of any other Line may be found in such Parts.

PROB. 6. A Right Line, as AB, any how divided, being given; to find how to divide any other Line, as BD, into the like Parts.

Set the Line AB on both Legs of the Sector, and in the Points where this Line reaches, apply the Line BD to be divided Parallel-wise; this done, lay the Divisions, Ae, Af and Ag on both Legs of the Sector from the Center, then the Parallel Extent of these Divisions, (Viz.) ee, ff, gg, being apply'd to the Line BB, will divide it into the like Parts, as was the Line AB.

The Use of the Line of Chords, Sines, Tangents and Secants.

The Convenience of putting these sort of Lines on the Sector, Sector-wise, is of vast Advantage, for by disposing them after this manner, we can have Lines of Chords, Sines, Tangents and Secants, to all Radius's not exceeding the Sector's Length.

For

For Example ; If you would have the *Chord* *Sine* or *Tangent* of 12 Degrees to a *Radius* of 4 Inches : Take the 4 Inches betwixt your *Compasses*, and make it a Parallel betwixt 60 and 60 on the *Chords*, then, as I told you before the *Sines*, *Tangents* and *Lines* are set to the same *Radius* ; the *Sector* remaining in this Position, take the Parallel Distance betwixt 20 and 20 on the *Chords*, is the *Chord* of 20 Degrees to that *Radius*, 30 and 30 is the *Chord* of 30 Degrees to the same *Radius*, &c. Understand the same by the *Sines* and *Tangents*, that is, Parallel betwixt 20 and 20 on the *Line* of *Lines*, is the *Sine* of 20 Degrees to that *Radius*, and 30 and 30 the *Sine* of 30 Degrees, &c.

If you want the *Chord* of 70 Degrees, Take the Parallel Distance of $\frac{1}{2}$ the *Arch* proposed, that is, the *Chord* of 35 Degrees, and repeat that Distance twice on the *Arch* you lay it down on, and you have the *Chord* of 70 Degrees.

If you want the *Tangent* of 70 Degrees to this *Radius*, you must use of the small *Line* of *Tangents* (the great one running to 45 Degrees) thus ; Take the *Radius* of 4 Inches, and make it a Parallel betwixt 45 and 45 on this small *Line* of *Tangents*, then the Parallel Extent of 70 Degrees on the said *Line* is the *Tangent* of 70 Degrees to the Three-Inch *Radius*.

If the *Secant* of any *Arch* be requir'd ; Take the given *Radius* and make it a Parallel betwixt the Beginning of the *Line* of *Secants*, that is 0 and 0, to the Parallel Distance be-
twixt

twixt 40 and 40, 70 and 70 on the said *Secant Line* will give you the *Secant* of 40 or 70 *Degrees* to the *Radius*.

Note : If the *Radius* be so large that it cannot be made a *Parallel* betwixt 45 and 45 on the small *Tangents*, then no *Tangent* of an *Arch* above 45 *Degrees* can be found, nor the *Secant* of any *Arch* to such a *Radius*, because all *Secants* exceed the *Semi-Diameters* of the *Circle*.

If the *Reverse* of any of these Things be requir'd ; as if I require the *Radius* to which a given *Line* is the *Chord*, *Sine*, *Tangent* or *Secant* of any *Arch*, suppose of 30 *Degrees*, then 'tis but making that *Line* (if it be a *Chord*) a *Parallel* on the *Line* of *Chords* betwixt 30 and 30, then the *Parallel Distance* betwixt 60 and 60 is the *Radius*. Understand the like if it be *Sine*, *Tangent* or *Secant*.

Thus you see 'tis very easie to find the *Chord*, *Sine* or *Tangent* to any *Radius*, which is the great *Advantage* of this *Instrument*, and is of great *Use* not only in *Delineating* and *Projecting* on *Paper*, but also on the *Ground* in laying out any *Parcel* in any *Form* or *Shape*.

To open the *Sector* to any *Angle*, or the *Sector* being open'd, to find the *Quantity* of the *Angle* it is open'd at.

Suppose I would open the *Lines* of *Chords* on the *Sector* to any *Angle* of 40 *Degrees*. From the *Center* of the *Sector* take the *Chord* of 40 *Degrees*, and make it a *Parallel* betwixt 60 and 60 on the *Line* of *Chords* set to an *Angle* of 40 *Degrees*, but also the *Line* of *Lines*,
Line

Line of *Sines* and Line of *Tangents*, they all be Equi-distant from the Inward Edges of the *Sector*.

If you would open the two Inward Lines suppose *Chords* and *Tangents*, or *Sines*, and *Lines*; 'tis done by making the *Lateral Chord* of 40 *Degrees* (or any other) a *Parallel* betwixt 60 and 45 on one Side of the *Sector* or else betwixt 90 and 10 on the other Side and it sets those Inner Lines to the *Angle* of 40 *Degrees*.

If you would open the very Edges to a given *Angle*, shut the *Sector* close, take the *Parallel Extent* betwixt the Innermost *Chords* and *Tangents*, and apply that *Extent Laterally* on the *Chords* from the Center, it shews you the *Angle* those Lines make when the *Sector* is shut, which suppose 6, then 'tis *Evident* if I would open the *Sector's* Edges to an *Angle* of 40 *Degrees*, I must open the Inward Lines to an *Angle* of 46 *Degrees*, for then will the *Sector's* Edges stand at an *Angle* of 40 *Degrees*. To open the Line of *Lines* to a *Right Angle*, take the Line of 10 Parts and make it a *Parallel* betwixt 8 and 6 of the same, on the Line of *Sines* is open'd to a *Right Angle*, if the *Sine* of 45 *Degrees* be apply'd over in the *Sines* of 30 *Degrees*.

To find the Angle the Sector is open'd to.

Take the *Parallel Distance* between the *Chords* of 60, and apply that *Extent Laterally* from the Center on the *Chords*, the *Degrees* it reaches to is the *Angle* those Lines are open'd to; for with a *Sector* of 2 or 3 Feet when open, we can fur-

with our selves with Scales of all sorts, and fitting for any Purpose or Design whatsoever.

The Description and Use of the Common
SEA-QUADRANT.

THIS Instrument was first Invented by Capt. Davis, an English-man, and is the most Convenient and Commodious Instrument used at Sea, for observing the Sun's Meridian Altitude.

The Form is sooner seen in an Instrument-maker's Shop than here describ'd; however, the Component Parts are Two Arches, Three straight Pieces and Three Vanes, they are now commonly made about 26 Inches long.

One Arch is called the 60th Arch, because formerly it contained but 60 Degrees, tho' now it is made to contain 65 Degrees. This Arch is of small Radius, being so design'd, that the Shade coming from the Vane used on this Arch might appear very plain on the Horizon Vane to the Eye of the Observer; this Arch is divided into no less than Whole Degrees, and is numbred with the Figures 5, 10, 15, 20, 25, &c. Increasing by 5 to 65, which 65 stands just upon the long straight Piece.

The Great Arch is called the 30th Arch, tho' now it contains but 35 Degrees, viz. the Complement of 65; It's of a large Radius that it might contain the Sub-divisions of a Degree. This Arch is so broad, that it hath 10 Concentrick Circles described upon it, which Circles are

are Intersected by Three *Diagonal Lines*, by means of which each *Degree* is actually divided into 2 *Minutes*, the outer Limb of the *Arch* is actually divided into every 10 *Minute* by long *Strokes*, and are numbred with the *Figures* 5, 10, 15, 20, 25, &c. which stand on the long straight *Piece*.

One of the Three *Vanes* is called the *Horizon Vane*, because it respects the *Horizon* in time of observing; this is fixt upon the End of the long straight *Piece*. The *Vane* which gives the Shadow is called the *Shade Vane*, and is placed on the 60th *Arch*, and the *Angle* thro' which you look is called the *Sight Vane*, and its Place is upon the 30th *Arch*, sometimes the *Shade Vane* has a *Glass* in it, that Observations might be taken in *Hazy Weather*.

The Use of this Instrument is to take the *Sun's Meridian Altitude*, which is done thus The *Vanes* being put on in their proper Places viz. the *Sight Vane* on the 30th *Arch*, the *Shade Vane* on the 60th *Arch*, and the *Horizon Vane* on the End of the long straight *Piece*; set the upper Edge of the *Shade Vane* to a Number of *Degrees* less than 15 or 20 than you judge the *Complement* of the *Latitude* to be: This done turn your Back to the *Sun*, and taking the *Quadrant* in your Hand, look thro' the *Sight Vane*, and cause the upper Edge of the *Shade Vane* to fall on the upper Edge of the *Slit* in the *Horizon Vane*; if then at the same time the *Horizon* appears thro' the *Slit* in the *Horizon Vane*, the *Vanes* on the *Quadrant* stand at the *Complement* of the present *Altitude* of the *Sun*, but if the *Sky* appear instead of the

Horizon

Horizon, slide the *Sight Vane* a little higher; but if the *Sea* appears, then bring the *Vane* lower, thus must you do till thro' the *Horizon Vane* the *Horizon* appears. This done, see what the *Vanes* cut upon their *Arches*; suppose the inside of the *Sight Vane* cuts 10 *Degrees*, and the upper Edge of the *Shade Vane* cuts the Number 42 *Degrees*, these added together give 52 *Degrees*, the *Complement* of the *Sun's Altitude*, for this Instrument always takes the *Complement*, and not the *Altitude*, that is, his *Zenith Distance*.

Note First: When you would obtain the *Meridian* (which is always the greatest) *Altitude* the *Sun* will have that Day, begin to observe a little before Noon, and continue observing, as the *Sun* rises the *Sea* will appear thro' the *Horizon Vane*, so the *Sight Vane* must be slid lower.

Secondly: When the *Sun* begins to fall the *Sky* will appear thro' the *Horizon*, instead of the *Horizon*, upon which leave observing that Day.

Having found the *Co-Altitude*, or *Zenith Distance* as before directed, the *Altitude* is got by taking it from 90, from which *Altitude* you must take 16 *Minutes*, (because the upper Edge of the *Sun* shows the *Shade*, and that is distant from the *Sun's Center* 16 *Minutes*) then there will remain the *Sun's Meridian Altitude* near enough for finding the *Latitude at Sea*.

The

*The Description and Use of the FORESTAFF
by some called the Cross-Staff.*

THIS Instrument is so called from the Position of the Observer in using it, whose Face is generally towards the thing observed but for the Sun 'tis contriv'd to be used backward, for the preserving the Eyes of the Observer. It's called a *Cross-Staff* from its Form being a *Square Staff* with 3 or 4 Pieces a-cross it; this *Staff* is commonly 2 Feet $\frac{1}{2}$ or 3 Feet long; add $\frac{3}{4}$ of an Inch *Square*, having 4 Sides and each of them a *Distinct Cross*, but generally they make the shortest *Cross* serve two Sides of the *Staff*, that is, the Breadth is the 10 *Cross* and the Length the 30 *Cross*.

The *Staff* being Square there are 4 Sides; One is called the 10 *Side*, because the Gradations begin at 3 *Degrees* of *Altitude* and end at 10 *Degrees*, increasing by 10 *Minutes*; to this *Side* belongs the least *Cross*. The next is the *Second Side*, where the Divisions begin at 10 *Degrees* and end at 30 *Degrees*; to this *Side* belongs the *Second Cross*; the *Sixty Side* where the Divisions begin at 20 *Degrees* and end at 60, to which *Side* belongs the next biggest or *Sixty Cross*. Lastly the *Ninety Side*, where the *Degrees* of *Altitude* begin at 30 *Degrees* and end at 90; to this *Side* belongs the biggest or *Ninety Cross*. At the End of these Divisions is writ the Word *Alt.* for *Altitude*, and generally under the *Altitudes* are set the *Complements*, and the Word *Com.* writ thereto. If the Divisions of the *Staff* be true and the Length of the *Crosses* right, there will be this Agree-

ment betwixt them; Half the 10 *Cross*s being laid on the *Ten Side*, will reach from 10 *Degrees*, to 9 *Degrees* 12 *Minutes*, and the whole from 10 *Degrees* to 8 *Degrees* 31 *Minutes*. Secondly; Half the *Thirty Cross*s laid on the *Thirty Side*, will reach from 30 *Degrees* to 23 *Degrees* 2 *Minutes*, and the whole from 30 *Degrees* to 19 *Degrees* 47 *Minutes*. Thirdly; Half the 60 *Cross*s laid on the *Side* proper, will reach from 60 *Degrees* to 40 *Degrees* 13 *Minutes*, and the whole from 60 *Degrees* to 30 *Degrees*. Lastly; Half the *Ninety Cross*s measur'd on the *Ninety Side* will reach from 90 *Degrees* to 53 *Degrees* 7 *Minutes*, and the whole from 90 *Degrees* to 36 *Degrees* 52 *Minutes*.

The *Use* of this Instrument is to find the *Altitude* of the *Sun* or *Stars*, which may be done both by a fore and back Observation; it also finds the *Distance* of two *Stars*.

The *Crosses* are used but One at a time, according as the *Altitude* is. If the *Altitude* be less than 10 *Degrees*, use the least *Cross* and its proper *Side*; if above 10 *Degrees*, and under 30 *Degrees*, use the *Thirty Cross* and *Side* proper to it: So if it be above 30 and under 60 *Degrees*, use the 60 *Cross*s and *Side* belonging; if above 60 *Degrees*, use the biggest *Cross*s and *Side*, but the last is not so exact as other Instruments.

To use this Instrument you must first Consider, that is, guess at the *Sun's Altitude*, and then use the *Cross* and *Side* proper thereto; thus, put the flat *Side* of the *Cross*s toward you, then taking the *Staff* in your Hand, place the flat End, that is, the End of the *Staff* where the Divisions end, to the Corner of your Eye
[A a] and,

and there hold it steady, sliding the *Cross* backward and forward till you see the Center of the *Sun* or *Star* by the upper End of the *Cross* and the *Horizon* by the lower End, then the *Degree* and *Minute* cut by the Inner Edge of the *Cross*, upon the Side peculiar to the *Cross* you use, is the *Altitude*.

Note: 'Twill be difficult to cut exactly the Center of the *Sun*, unless in *Hazy Weather*; that you break his Rays by a *Colour'd Glass* fix'd on the Top of the *Cross* to defend the Eye from the *Sun's Force*: But you may observe the upper Limb of the *Sun*, and then must take *Minutes* from the *Altitude* so found: Or you may take the *Sun's* lower Limb, and add *Minutes* to the *Altitude* found.

To observe the *Distance* of two *Stars*, Place the *Staff* to your Eye, as before directed, look to both Ends of your *Cross*, moving it farther or nearer till you see one *Star* at one End of the *Cross* and the other at the other, the *Degrees* and *Minutes* cut by the *Cross* (as aforesaid) is the true *Distance* sought.

To use this Instrument for a Back Observation which is chiefly for the *Sun*. 'Tis made fit for this Purpose thus; There is a Piece of *Wood* or *Ivory* put a-cross the *Ten Cross*, so that the lower End of this Piece of *Wood* or *Ivory* lies even with the middle of the Square Hole in the *Ten Cross* which also answers to the middle of the *Staff's* Thickness. There is also a Piece of *Brass* with a Slit in it, so fitted that it will slide off and on the Ends of the *Crosses*. This done, consider the *Altitude* you are going to take, and accordingly use the *Cross* proper for it, putting it on the *Staff*, the flat Side of it even with the

at End of the Staff, where skrew it fast, and at one End of the Cross put on the forementioned Piece of Brass, so as to leave a Slit at the lower End of the Cross, then put the Ten Cross (with the Piece a-cross it) on the Staff, the flat Side of it towards the other Cross; This done, turn your Back to the Sun, and look thro' the Slit at the lower End of the Cross, till the Horizon lies even with the Shade on the Cross-Piece in the Ten Cross, at which time the Shade of the upper End of the Cross must fall on the Middle Line in the Ten Cross. If instead of the Horizon you see the Water, draw the Ten Cross nearer; but if you see the Sky, you must put it further from you, till at last you make the Horizon and Shadow meet, so the Place where the Ten Cross now rests, on that Side of the Staff belonging to the Cross fixt at the End, in which is the Brass Sight, is the Altitude or Complement as before, in a Forward Observation.

The Description and Use of the NOCTURNAL.

ON the Fore-side of the great Piece of this Instrument are Two Circles, the innermost is divided into twice 12, to represent the Hours in a Natural Day; the outermost is a Circle of Time, and contains the Days in each Month and the Months in the Year, each Division being 2 Days. The next Piece contains Two Circles; the innermost is divided into 29 equal Parts to shew the Age of the Moon, the outermost contains twice 12, for the Hours in a Natural Day.

The Nature of this *Instrument* is only to find the *Right Ascension* of a *Star*, which it evidently doth, if you consider the *Pole* is the Center of the *Diurnal Motion*, and the *Index* of the *Instrument* a *Meridian* passing thro' the *Pole* and *Two Pointers*, and consequently cuts the *Equator* at *Right Angles*.

Upon the Middle Piece is a *Tooth* that comes out, and has the Letters *GB* for the *Great Bear*, against which is the Number 12, this *Tooth* set to the *Day* of the *Month*, and the *Natural* held up *Perpendicular* before you, til thro' the *Hole* you see the *Pole-Star*, and by the *Index* the *Pointers*, then will the *Index* stand at the *Hour* of the *Night*, which is only the *Difference* betwixt the *Right Ascension* of the *Sun*, and the *Right Ascension* of the *Pointers*.

The *Right Ascension* of the *Sun* changes every *Day* about a *Degree*, but the *Right Ascension* of any fixt *Star* is the same *insensibly* for some *Years*.

The Description and Use of the Common Foot GAUGING-ROD, in Gauging Casks and other Vessels.

FROM the End not *Bevell'd*, on one Side a Line of *Inches*, running from 1 to 48 each *Inch* being divided into *Tenths*; adjoining to this is another Line which gives the *Area* of any *Circle* in *Ale-Gallons*, and *Hundredth Part* of a *Gallon* upon one *Inch* deep, having the *Diameter* in *Inches* and *Tenths* of an *Inch*; on this Line is writ *Area in Ale-Gallons*, and is called *Oughtred's Line*.

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Upon another Side of the *Rule* are Two *Diagonals*, one for *Ale* and another for *Wine Gallons*, by which is shewn the Number of *Ale* or *Wine Gallons* any Cask contains. The Method is thus; Put the Bevell'd End of the *Rule* in at the *Bung-hole*, and run it down to the Head *Diagonal-wise*, then the Number on either of these Lines against the Middle of the *Bung*, is the Content of the Cask either in *Ale* or *Wine Gallons*.

Note : You must take care that the *Bung-hole* be in the Middle of the Cask, and that the *Rule* be put to the Middle of the *Bung-hole*.

This Line will serve very well for a Cask of 30 or 40 Gallons, and indifferent well for a Cask of 150 Gallons.

On a Third Side are Three Lines, beginning at the Bevel End with 1, 2, 3, 4, &c. and running half way the *Rule*; the first of them running to 9, and has *Firkins* writ upon it; the second has the Word *Kilderkin* writ upon it, and runs to 18; the third runs to 36, and has the Word *Barrel* upon it.

These Lines are term'd *Ullage Lines*, and their Use is to find what a Cask has in it when it is not full; thus, Put the Bevel End down into the *Bung-hole* to the opposite Stave of the Cask, and so far as the *Rule* is wet, on the proper Line for the Vessel, so many Gallons is in the Vessel.

Or the Number from the Inside of the *Bung-hole* to the Surface of the Liquor, shews how many Gallons will fill the Vessel.

Upon the same Flat on the other Half of the *Rule* are Two Lines, the one is *Numbers*, the other is *Segments*; they both begin together

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with

with the Number 100 just at the middle Joint and run to the blunt End of the Rule where the Words *Numbers* and *Segments* are writ to them.

Their Use is to find the remaining Quantity in any Cask whose *Axis* lies parallel to the *Horizon*, and this is done, not only more exactly than by the *Ullage Lines*, which serve only for a *Firkin*, *Kilderkin* or *Barrel*, but also for all sorts of Casks; *the Method is thus.*

First find the *Content* of the whole Cask, which may be done near enough by the *Diagonal Line* then find how many Inches are wet of the *Bung Diameter*, as also the Length of the *Bung Diameter* it self, by putting the Rule Perpendicular in at the *Bunghole*.

Then the Extent on the *Line of Numbers* from the *Bung Diameter* to the Inches wet, will reach, being laid the same way on the *Line of Segments*, from 100 to a fourth Number, which remember; Next the Extent on the *Line of Numbers* from 100 to the Vessel's whole *Content* in *Wine* or *Ale Gallons* will reach, laid the same way on the *Line of Numbers*, from the Number remember'd to the Gallons in it, or else wanting to fill the Cask. ▀

EXAMPLE. *The Content of a Cask is 37 Gallons, the Inches wet are 17, the whole Diameter 25, how many Gallons will fill the Cask?*

Ans. 9 Gallons 7 Tenths.

Upon the Fourth Side of the Rule are Three Lines, beginning at the *Bevel End* with 5, and runs towards the other End; the first goes better than half way, and has the Word *Hogshead* writ upon it; the second runs almost $\frac{3}{4}$ its Length, and has the Word *Butt* lying writ upon

upon it, and the third runs near to the other End and has the Words *Butts Standing* writ upon it; these also are *Ullage Lines* for these Casks so posited, and their Use is to find what a Cask has in it when 'tis but partly full. The manner is thus; When the *Hoghead* or *Butt* is lying, put the Bevell'd End of the *Rule* downright into the *Bung-hole*, to the opposite Stave of the Cask, and so far as the *Rule* is wet on the proper Line for the Vessel, it shows how many Gallons are in the Vessel.

Or the Number from the Inside of the *Bung-hole* to the Surface of the Liquor, shews how many Gallons will fill the Cask.

In Case the *Butt* be *Standing* the *Rule* must be put in at the *Head*, and so far of it as is wet upon the *Butt Standing Line*, so much is the Number of Gallons in the Cask.

So also the Dry Inches betwixt the *Butt's* Head and Surface of the Liquor, shews the Gallons wanting to fill the Cask.

These and the former *Ullage Lines* are thus put upon the *Rule*; Cause any of those Casks to be filled, and then draw out the Liquor Gallon by Gallon, and so mark the *Rule* as the Liquor sinks in the Vessel.

*The Description and Use of Coggershal's-Rule,
in the Measurement of Board and Timber.*

THE Principal Contrivance of this *Rule* is for the *Practical Measuring* of *Timber*, which it doth the best of all others. It consists of Two Pieces of a Foot long, sliding one by another like a *Glazier's Rule*. On each Flat of the *Rule* are Four Lines, Two next

the Outer Edges, and Two next the Inner; those on one Flat next the Outer Edges are *Lines of Inches* divided into Halves and Quarters; they are Figured from 1 to 12 on one Piece, and from 12 to 24 on the other.

Those next the Inward Edges of the same Side are *Lines of Numbers* in two Lengths both alike: They are Figured from 1 at the Beginning to 1 in the Middle, and then from 1 in the Middle to 10 at the End.

On t'other flat Side, the Lines next the Outer Edges are *Lines of Measure*, one being a *Line of Inches*, divided into 10 equal Parts for *Gauging*; the other a *Foot* divided into 100 equal Parts, called the *Line of Foot-Measure*.

On one of the Inward Edges of this Flat is a *Line of Numbers*, Number'd and Figur'd as before described.

On the other Inward Edge is a Line called the *Square* or *Girt-Line*, Figured from the Beginning with 4, 5, 6, 7, 8, 9, 10, 20, 30, and 40 which stands at the End; this is a *Line of Numbers* at a double *Radius*.

First; To Measure Board, Glass, &c. A Piece of Board is $1\frac{1}{4}$ Foot broad, $24\frac{1}{2}$ long, What's the Content in Square Feet?

Set 1 to $1\frac{1}{4}$, the Breadth on the *Line of Numbers*, then against $24\frac{1}{2}$ Feet is $30\frac{1}{2}$ Feet, the Content near.

More Examples.

		Feet Inch. Parts	
Length	$37\frac{1}{2}$ Feet, Breadth $\frac{1}{4}$ Foot	Ans ^w .	9 4 6
—	$16\frac{1}{4}$ Feet, — $3\frac{1}{2}$ Feet	56	10 6
—	$60\frac{3}{4}$ Feet, — $2\frac{1}{4}$ Feet	136	8 3
—	$15\frac{1}{2}$ Feet, — $8\frac{1}{2}$ Feet	131	9 0
—	29 Feet, — $7\frac{1}{4}$ Feet	210	3 0
—	$35\frac{1}{4}$ Feet, — $9\frac{3}{4}$ Feet	343	8 3
		Next	

of Coggerhal's-Rule.

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Next for *Painting, Plaistering, Paving and Wain-*
scotting, which is Measured by the *Square Yard*.

A Piece of Paving is $22\frac{1}{2}$ Feet long, and 14 Feet
broad, how many Square Yards is the Content?

Set 9 to 14 on the Line of Numbers, then
against $22\frac{1}{2}$ is 35 Square Yards, the Content.

More Examples.

		Yards.
Length $34\frac{3}{4}$ Feet, Breadth $13\frac{1}{2}$ Feet	Ans.	$52\frac{1}{4}$
— $15\frac{1}{4}$ Feet, — $11\frac{1}{2}$ Feet		$19\frac{1}{2}$
— $19\frac{1}{2}$ Feet, — $9\frac{1}{4}$ Feet		20
— $52\frac{1}{4}$ Feet, — $20\frac{1}{2}$ Feet		119
— $36\frac{1}{2}$ Feet, — 30 Feet		122
— $27\frac{3}{4}$ Feet, — $24\frac{1}{2}$ Feet		76

To Measure Square Timber. A Piece of Timber
is 1 Foot 2 Inches Square, and 20 Feet long,
what's the Content?

Note: Instead of 1 Foot 2 Inches, take 14
Inches.

Set 12 on the Girt-Line, to 20 the Length of
the Piece on the Line of Numbers, then right
against the Side of the Square, or $\frac{1}{4}$ of the Girt
being 14 reckoned in Inches (which must al-
ways be done) on the Girt-Line, is 27 Feet,
the Content required.

Note: If the Piece be not Square, but broader
than 'tis thick, you may reduce it near to a
Square, by adding the Breadth and Thickness
together, and taking half the Sum.

To Measure round Timber. Girt the Tree in the
Middle, and take $\frac{1}{4}$ of that Girt for the Side of
the Square.

Then suppose the Girt of a Tree be 64 Inches,
and the Length 30 Feet, what's the Content?

Then

The Description and Use

Then as before : Set 12 on the Girt-Line to 30 Feet the Length, then against 16, the $\frac{1}{4}$ of the Girt, is 53 $\frac{1}{2}$ Feet the Content.

If the Tree be much *tapering*, take the Side of the Square, or $\frac{1}{4}$ of the Girt, about $\frac{1}{3}$ from the greater End.

If you have *Wainy Timber*, that is, such as is neither round nor square, take one *Wain* in and leave the other out.

More Examples.

		Feet	Parts
$\frac{1}{4}$ of the Girt, or the Side of the Square.	10 In. Length 40 $\frac{1}{2}$ Feet	Ans ^r . 28	$\frac{1}{4}$
	15 In. — 39 $\frac{1}{2}$ Feet	61	$\frac{3}{4}$
	13 $\frac{1}{2}$ In. — 50 Feet	63	$\frac{1}{2}$
	20 In. — 48 Feet	130	$\frac{1}{2}$
	7 $\frac{3}{4}$ In. — 36 $\frac{1}{4}$ Feet	15	$\frac{1}{4}$
	30 In. — 24	150	

These Dimensions are actually cast up by a Rule, and therefore a small Errour must not be heeded.

The Description and Use of the EQUINOCTIAL, commonly called the Ring-Dial.

Def. **T**HIS Dial consists of Five Pieces, viz. Two Rings or Circles, One Bridge or Piece going cross the Outermost Ring, and Two Sliding Bits of Brass that move in the Grooves, one on the Outermost Ring, the other on the Bridge or Cross Piece.

The Outermost Ring represents the Meridian of the Place where-ever you are, having on one Side grav'd twice 90 Degrees, with N and S for North and South Latitude; on the other Side is a Line of Altitudes, beginning at 1 and ending at 90 Degrees. Upon this Ring, in a Groove on the outer Edge, moves one of the Sliding Bits

its, upon one Side of which is a *Line* or *Stroke* drawn to shew the Center or Middle of those moving *Bits*. In this *Bit* is a little *Ring* to hold it up by in time of *Observation*.

There is also in this *Outer Ring* a *Pin-hole* to put a *Pin* in when the *Altitude* is observ'd.

The *Innermost Ring* denotes the *Equinoctial*, and is to turn out upon a *Joint* at *Right Angles* to the *Meridian*, on one *Face* is nothing, on the other the 24 Hours of the *Day* and *Night*, number'd by twice 12, which is continu'd to the Edge and mark'd also upon the Inner Rim of the *Circle*, each *Hour* is divided into *Quarters* on some of these *Dials*, upon others into every 5 *Minutes*, according as the Largeness is.

The *Bridge*, or *Cross Piece*, denotes the *Axis* of the *World*, in the middle of which is a *Slit* cut quite through for a *Sliding Piece* to move in, which *Sliding Piece* has a small *Hole* in it, and has a *Line* or *Stroke* on both *Faces* crossing the middle of the said *Hole*.

On one *Face* of the *Bridge* is the *Months* and *Days* of the *Month*, on the other *Face* opposite against each particular *Day*, is the *Sun's Declination* proper for the *Day*.

PROB. 1. To find the *Hour* of the *Day*.

Turn out the *Equinoctial* as far as it will go, or then it stands at *Right Angles* to the *Meridian*, and set the *Streak* in the *Sliding Piece* on the *Outer Ring* to the *Latitude* of the *Place*; also the *Stroke* crossing the little *Hole* in the other *Sliding Bit* to the *Day* of the *Month* in the *Bridge*; then hold up the *Dial* by the little *Ring* in the *Outer Sliding Bit* and turn the *Bridge* toward the *Sun*, then move the whole *Dial* this and that way, till the *Sun* shining through the little

364 *The Description and Use, &c.*

little Hole in the *Bridge* you see the Spot of Light to fall on the middle of the Inside of the *Equinoctial* among the *Hours*, so the *Hour-Line* it falleth upon is the *Hour* of the *Day*. Now for 2 or 3 *Days* when the *Sun* is upon the *Equinoctial*, that is, about the 10th of *March* and 12th of *September*, this *Dial* is useless.

PROB. 2. *To find the Sun's Height.*

Set the Stroke in the *Sliding Bit* with the Ring to the Beginning of the *Degrees*, that is to 00 *Degrees*, betwixt N and S on the *Latitude Side*, and put a Pin stiff in the Hole on the Back-side of the Ring, and the Shade of the Pin will shew the *Altitude* in the large *Quadrant* on the Back-side of the *Meridian*.

PROB. 3. *To find the Sun's Declination.*

Set the Stroke crossing the Hole of the *Sliding Bit* in the *Bridge* to the *Day* of the *Month* as before, and against it on the Back-side is the *Sun's Declination*.

PROB. 4. *To find the Latitude of the Place.*

First find the *Sun's Meridian*, or greatest *Altitude* for the *Day* as before, then find the *Sun's Declination* for the same *Day*; this done, take Notice if you make your *Observation* betwixt the 10th of *March* and 12th of *September*, you must Subtract the *Declination* from the *Altitude*; but if you observe betwixt the 12th of *September* and 10th of *March*, then you must add the *Declination* to the *Altitude*, and either of those *Numbers* is the *Height* of the *Equator*, which Subtracted from 90 *Degrees*, leaves the *Latitude* of the *Place*.

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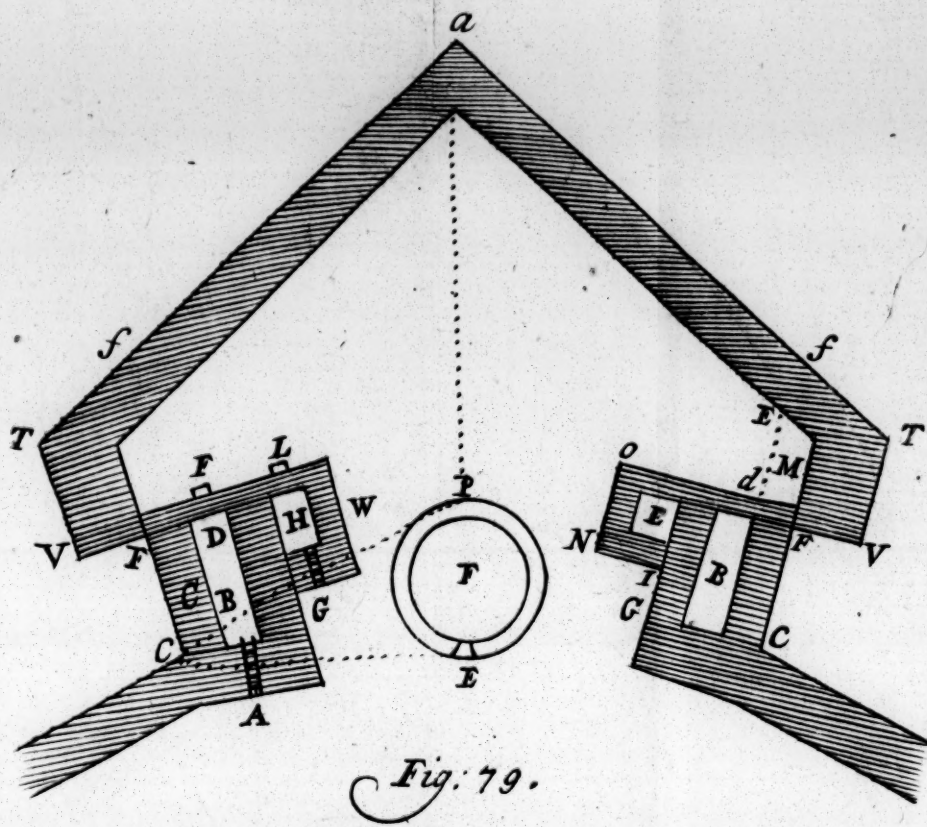


Fig: 79.

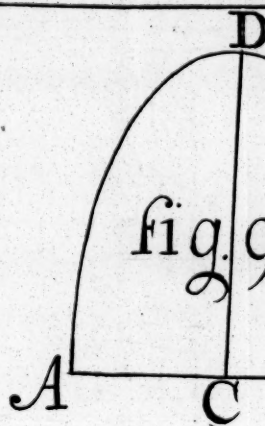
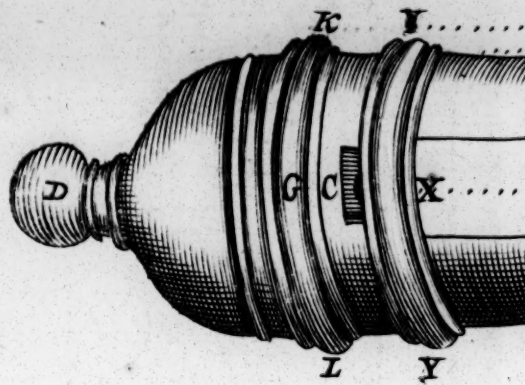


Figure. 80.

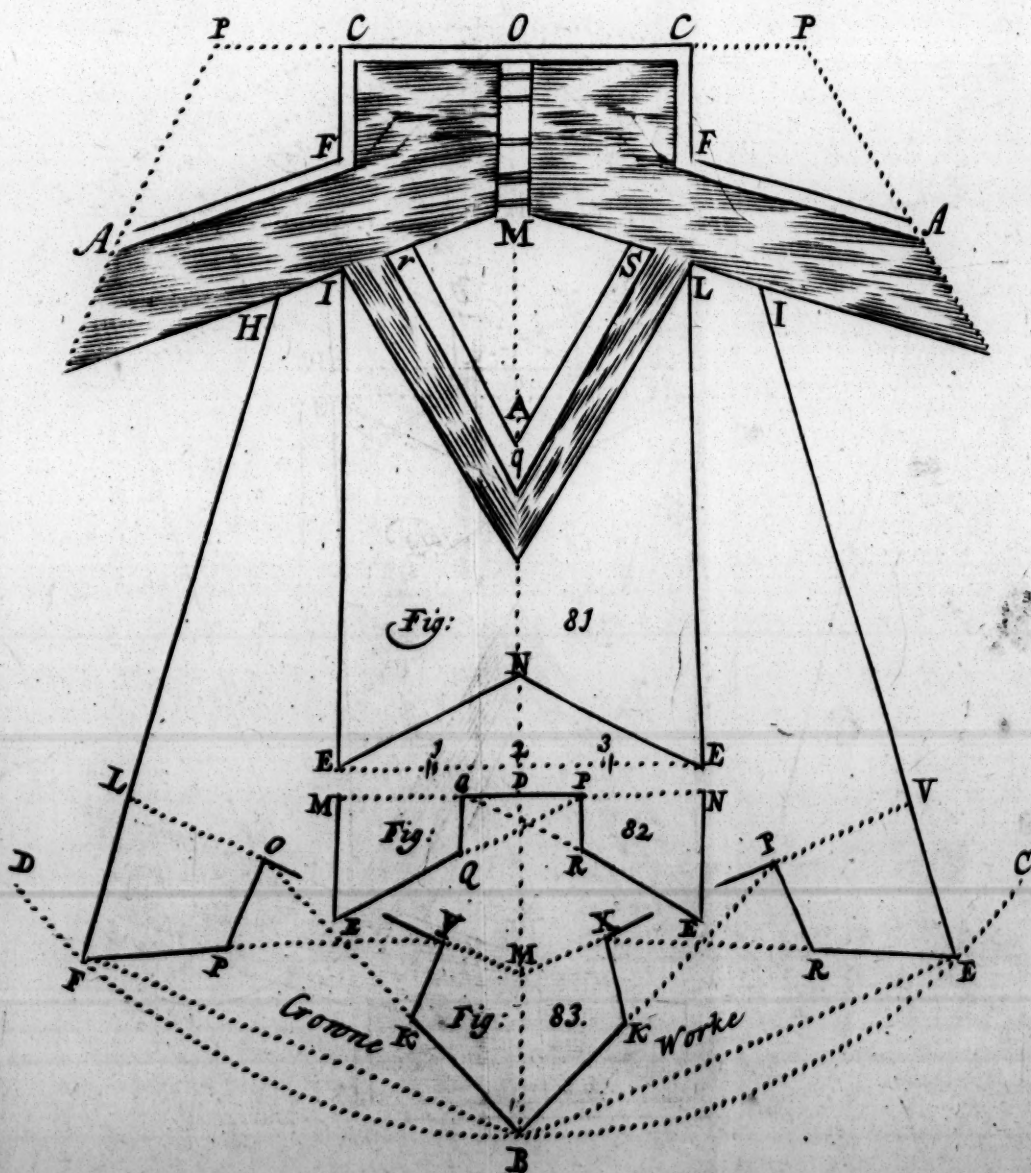
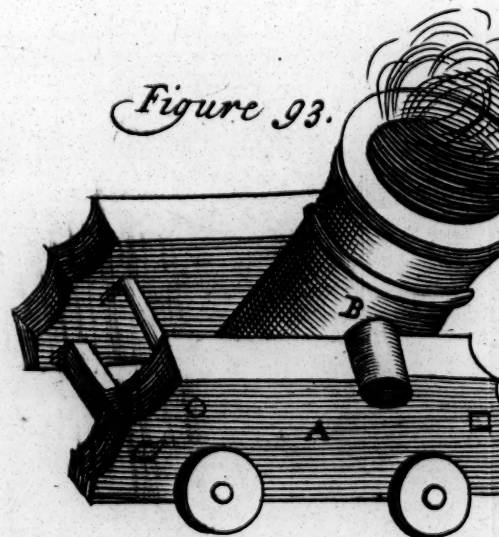


Figure 93.



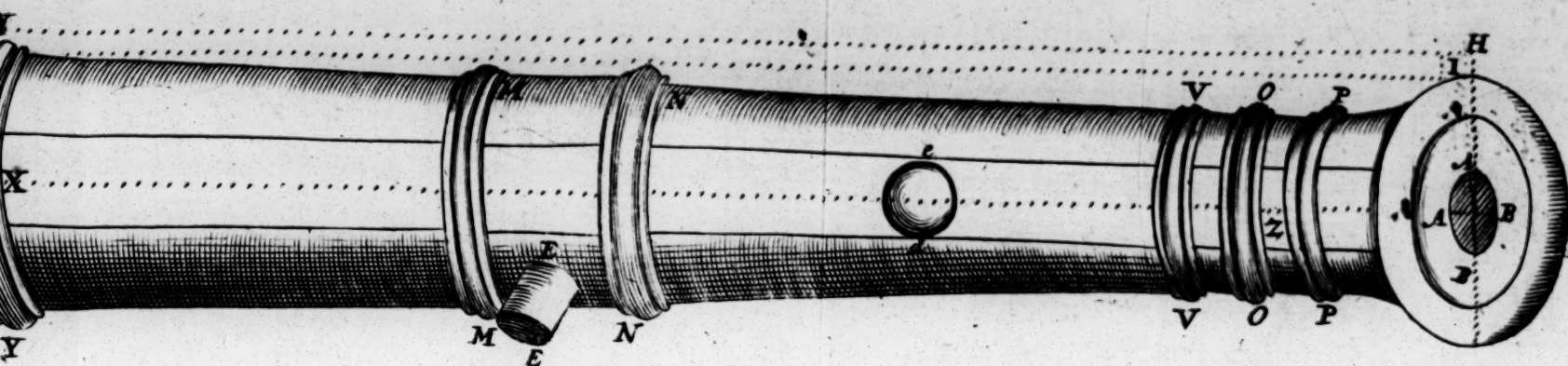


Figure 91

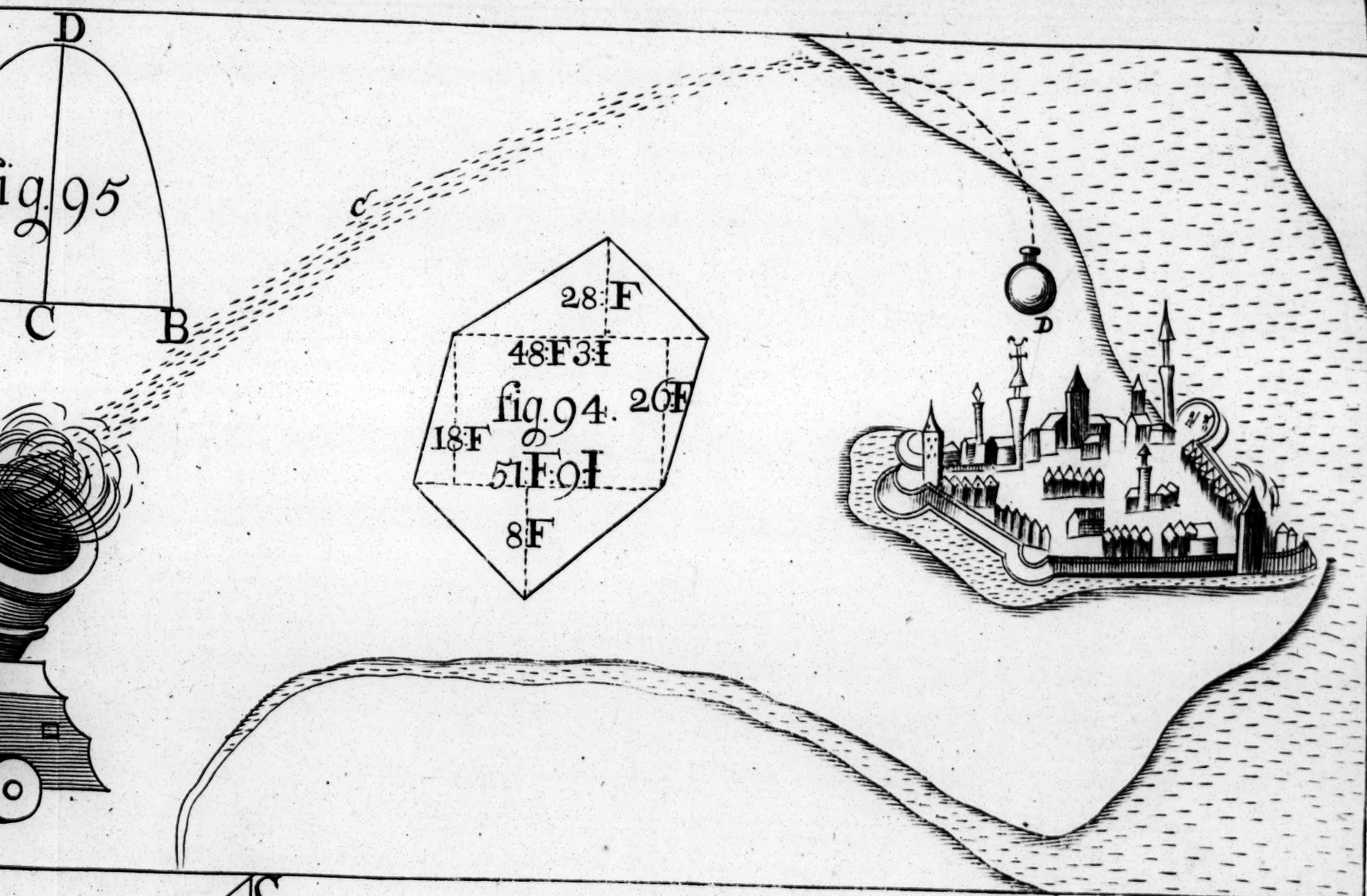


Fig. 95

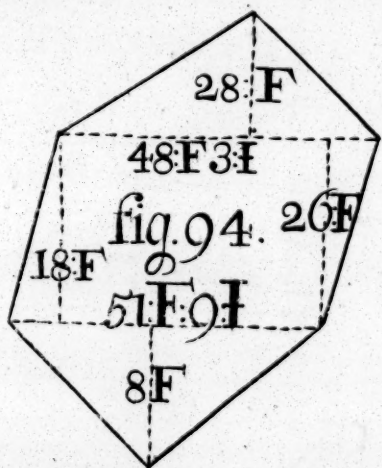


fig. 94.

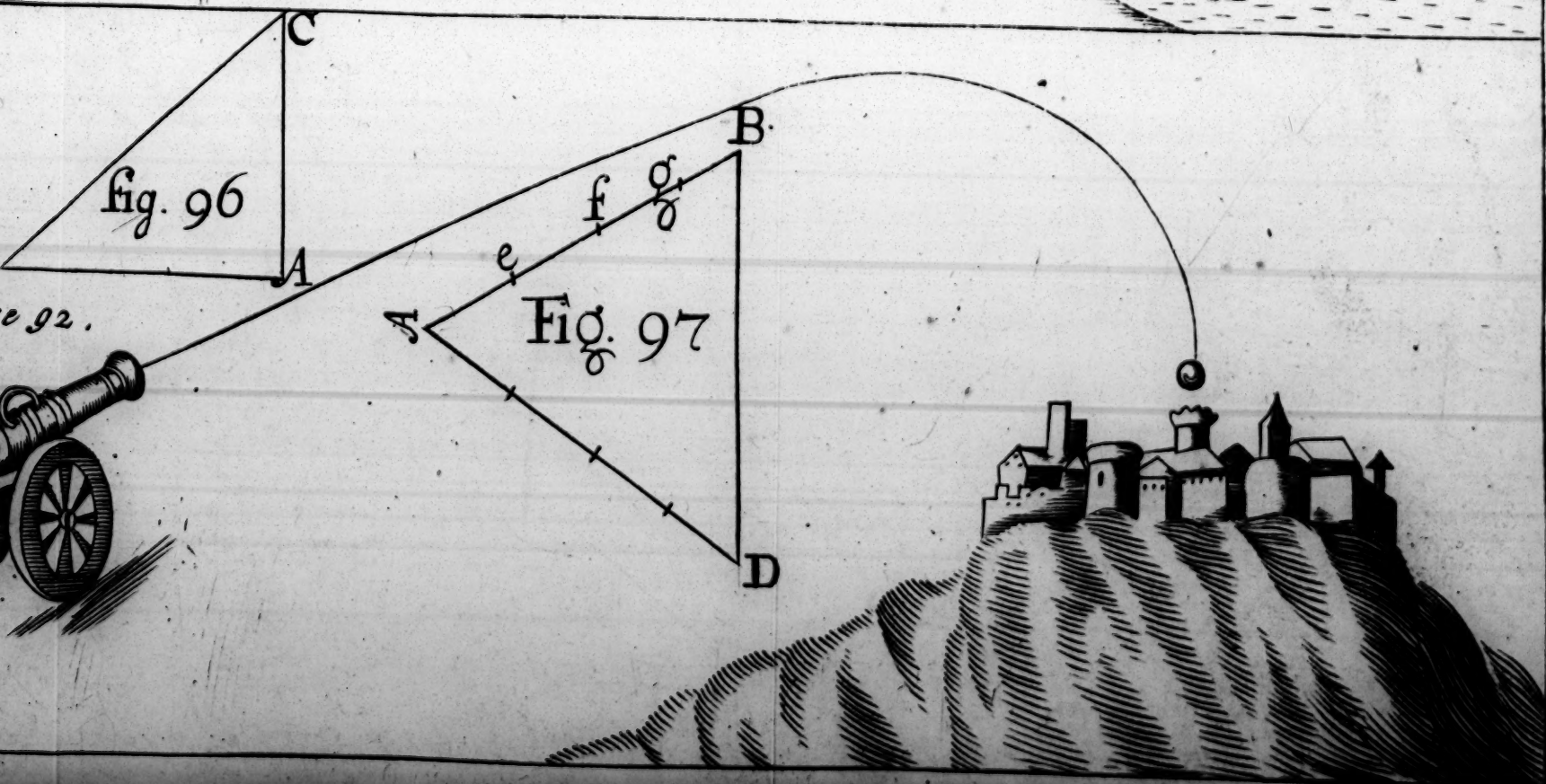


Fig. 96

Fig. 97

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Logarithm Numbers,

From One to Ten Thousand :

Whereby the

LOGARITHM

O F A N Y

N U M B E R

Under Four Hundred Thousand
may be readily discovered.

L O N D O N,

Printed by *J. Leake*, for W. FREEMAN,
at the *Bible* against the *Middle-Temple-*
Gate in *Fleetstreet*. MDCCVII.

T A B L E

OF

CONTENTS

OF THE

PROCEEDINGS

OF THE

ANNUAL MEETING

OF THE

AMERICAN MEDICAL ASSOCIATION

HELD AT

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N	Log.	N	Log.	N	Log.
1	0. 000000	34	1.531479	67	1. 826075
2	0. 301030	35	1.544068	68	1. 832509
3	0. 477121	36	1.556303	69	1. 838849
4	0. 602060	37	1.568202	70	1. 845098
5	0. 698970	38	1.579783	71	1. 851258
6	0. 778151	39	1.591064	72	1. 857332
7	0. 845098	40	1.602060	73	1. 863323
8	0. 903090	41	1.612784	74	1. 869232
9	0. 954242	42	1.623249	75	1. 875061
10	1. 000000	43	1.633468	76	1. 880813
11	1. 041393	44	1.643452	77	1. 886491
12	1. 079181	45	1.653212	78	1. 892094
13	1. 113943	46	1.662758	79	1. 897627
14	1. 146128	47	1.672098	80	1. 903090
15	1. 176091	48	1.681241	81	1. 908485
16	1. 204120	49	1.690156	82	1. 913814
17	1. 230449	50	1.698970	83	1. 919078
18	1. 255272	51	1.707570	84	1. 924279
19	1. 278753	52	1.716003	85	1. 929419
20	1. 301030	53	1.724276	86	1. 934498
21	1. 322219	54	1.732394	87	1. 939519
22	1. 342422	55	1.740362	88	1. 944482
23	1. 361728	56	1.748188	89	1. 949390
24	1. 380211	57	1.755875	90	1. 954242
25	1. 397940	58	1.763428	91	1. 959041
26	1. 414973	59	1.770852	92	1. 963788
27	1. 431364	60	1.778151	93	1. 968483
28	1. 447158	61	1.785330	94	1. 973128
29	1. 462398	62	1.792391	95	1. 977723
30	1. 477121	63	1.799340	96	1. 982271
31	1. 491361	64	1.806180	97	1. 986772
32	1. 505150	65	1.812913	98	1. 991226
33	1. 518514	66	1.819544	99	1. 995635

The Table of Logarithms.

N	O	I	2	3	4
100	000000	000434	000868	001301	001734
101	004321	004751	005181	005609	006038
102	008600	009026	009451	009876	010299
103	012837	013259	013679	014100	014521
104	017033	017451	017898	018284	018700
105	021189	021603	022016	022428	022841
106	025306	025715	026125	026533	026942
107	029384	029789	030195	030599	031004
108	033424	033826	034227	034628	035029
109	037426	037825	038223	038620	039017
110	041393	041787	042182	042576	042969
111	045323	045714	046105	046495	046885
112	049218	049603	049993	050379	050766
113	053078	053463	053846	054229	054613
114	056905	057286	057666	058046	058426
115	060698	061075	061452	061829	062206
116	064458	064832	065206	065579	065953
117	068186	068557	068928	069298	069668
118	071882	072249	072617	072985	073352
119	075547	075912	076276	076640	077004
120	079181	079543	079904	080266	080626
121	082785	083144	083503	083861	084219
122	086359	086716	087071	087426	087781
123	089905	090258	090610	090963	091315
124	093422	093772	094122	094471	094820
125	096910	097257	097604	097951	098298
126	100371	100715	101059	101403	101747
127	103804	104146	104487	104828	105169
128	107209	107549	107888	108227	108565
129	109586	109926	110263	110599	110934

The Table of Logarithms.

5	6	7	8	9	D
002166	002598	003029	003461	003891	432
006466	006894	007321	007748	008174	428
010724	011147	011570	011993	012415	421
014940	015359	015779	016197	016616	416
019116	019532	019947	020361	020775	416
023252	023664	024075	024486	024896	412
027349	027757	028164	028571	028978	408
031408	031812	032216	032619	033021	404
035429	035829	036229	036629	037028	400
039414	039811	040207	040602	040998	396
043362	043755	044148	044539	044932	393
047275	047664	048053	048442	048830	389
051153	051538	051924	052309	052694	386
054996	055378	055760	056142	056524	382
058805	059185	059563	059942	060320	379
062582	062958	063333	063709	064083	376
066326	066699	067071	067443	067815	372
070038	070407	070776	071145	071514	369
073718	074085	074451	074816	075182	366
077368	077731	078094	078457	078819	363
080987	081347	081707	082067	082426	360
084576	084934	085291	085647	086004	357
088136	088490	088845	089198	089552	355
091667	092018	092369	092721	093071	351
095169	095518	095866	096215	096562	349
098644	098989	099335	099681	100026	346
102091	102434	102777	103119	103462	343
105510	105851	106191	106531	106871	340
108903	109241	109579	109916	110253	338
112269	112605	112939	113275	113609	335

The Table of Logarithms.

N	O	I	2	3	4
130	113943	114277	114611	114944	115278
131	117271	117603	117934	118265	118595
132	120574	120903	121231	121559	121888
133	123852	124178	124504	124830	125156
134	127105	127429	127753	128076	128399
135	130334	130655	130977	131298	131619
136	133539	133858	134177	134496	134814
137	136721	137037	137354	137671	137987
138	139879	140194	140508	140822	141136
139	143015	143327	143639	143951	144263
140	146128	146438	146748	147058	147367
141	149219	149527	149835	150142	150449
142	152288	152594	152899	153205	153509
143	155336	155639	155943	156246	156549
144	158362	158664	158965	159266	159567
145	161368	161667	161967	162266	162564
146	164353	164650	164947	165244	165541
147	167317	167613	167908	168203	168497
148	170262	170555	170848	171141	171434
149	173186	173478	173769	174059	174351
150	176091	176381	176669	176959	177248
151	178977	179264	179552	179839	180126
152	181844	182129	182415	182699	182985
153	184691	184975	185259	185542	185825
154	187521	187803	188084	188316	188647
155	190332	190612	190892	191171	191451
156	193125	193403	193681	193959	194237
157	195899	196176	196453	196729	197005
158	198657	198932	199206	199481	199755
159	201397	201670	201943	202216	202488

The Table of Logarithms.

5	6	7	8	9	D
115611	115943	116276	116608	116939	333
118926	119256	119586	119915	120245	330
122216	122544	122871	123198	123525	328
125481	125806	126121	126456	126781	325
128722	129045	129368	129689	130012	323
131939	132259	132579	132899	133219	321
135133	135451	135769	136086	136403	318
138303	138618	138934	139249	139564	315
141449	141763	142076	142389	142702	314
144574	144885	145196	145507	145818	311
147676	147985	148294	148603	148911	309
150756	151063	151369	151676	151982	307
153815	154119	154423	154728	155032	305
156852	157154	157457	157759	158061	303
159868	160168	160469	160769	161068	301
162863	163161	163459	163758	164055	299
165838	166134	166430	166726	167022	297
168792	169086	169380	169674	169968	295
171726	172019	172311	172603	172895	293
174641	174932	175222	175512	175802	291
177536	177825	178113	178401	178689	289
180413	180699	180986	181272	181558	287
183269	183555	183839	184123	184407	285
186108	186391	186674	186956	187239	283
188928	189209	189490	189771	190051	281
191730	192009	192289	192567	192846	279
194514	194792	195069	195346	195623	278
197281	197556	197832	198107	198382	276
200029	200303	200577	200850	201124	274
202761	203033	203303	203577	203848	272

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N	O	I	2	3	4
160	204119	204391	204663	204924	205204
161	206826	207096	207365	207634	207904
162	209515	209783	210051	210319	210586
163	212187	212454	212720	212986	213252
164	214844	215109	215373	215638	215902
165	217484	217747	218010	218273	218536
166	220108	220369	220631	220892	221153
167	222716	222676	222936	223196	223455
168	225309	225568	225827	226084	226342
169	227887	228142	228400	228657	228913
170	230449	230704	230959	231215	231469
171	232996	233250	233504	233752	234011
172	235528	235781	236033	236285	236537
173	238046	238297	238548	238799	239049
174	240549	240799	241048	241297	241546
175	243028	243286	243534	243782	244029
176	245513	245759	246006	246252	246499
177	247973	248219	248464	248709	248954
178	250420	250664	250908	251151	251395
179	252853	253096	253334	253580	253822
180	255273	255514	255755	255996	256237
181	257679	257918	258158	258398	258637
182	260071	260309	260548	260787	261025
183	262451	262688	262925	263162	263399
184	264818	265054	265289	265525	265761
185	267172	267406	267641	267875	268109
186	269513	269746	269979	270213	270446
187	271842	272074	272306	272538	272769
188	274158	274389	274619	274850	275081
189	276462	276692	276921	277151	277379

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5	6	7	8	9	D
205475	205746	206016	206286	206556	271
208173	208441	208710	208978	209247	260
210853	211121	211388	211654	211921	267
213518	213783	214049	214314	214579	266
216166	216429	216694	216957	217221	264
218798	219060	219323	219585	219846	262
221414	221675	221936	222196	222456	261
224015	224274	224533	224791	225051	259
226599	226858	227115	227372	227629	258
229169	229426	229682	229938	230193	256
231724	231979	232234	232488	232742	254
234264	234517	234770	235023	235276	253
236789	237041	237292	237544	237795	252
239299	239549	239799	240049	240299	250
241795	242044	242293	242541	242789	249
244177	244525	244772	245019	245266	248
246745	246991	247237	247482	247728	246
249198	249443	249687	249932	250176	245
251638	251881	252125	252368	252610	243
254064	254306	254548	254789	255031	242
256477	256718	256958	257198	257438	241
258877	259116	259355	259594	259833	239
261263	261501	261739	261976	262214	238
263636	263873	264109	264346	264582	237
265996	266232	266467	266702	266937	235
268344	268578	268812	269046	269279	234
270679	270912	271144	271377	271609	233
273001	273233	273464	273696	273927	232
275311	275542	275772	276002	276232	230
277609	277838	278067	278296	278525	229

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N	O	I	2	3	4
190	278754	278982	279211	279439	276667
191	281033	281261	281488	281714	281942
192	283301	283527	283753	283979	284205
193	285557	285782	286007	286232	286456
194	287802	288026	288249	288473	288696
195	290035	290257	290479	290702	290925
196	292256	292478	292699	292920	293141
197	294466	294687	294907	295127	295347
198	296665	296884	297104	297323	297542
199	298853	299071	299289	299507	299725
200	301030	301247	301464	301681	301898
201	303156	303412	303628	303844	304059
202	305351	305566	305781	305996	306211
203	307496	307709	307924	308137	308351
204	309630	309843	310056	310268	310481
205	311754	311966	312177	312389	312600
206	313867	314078	314289	314499	314709
207	315970	316180	316389	316599	316809
208	318063	318272	318481	318689	318898
209	320146	320354	320562	320769	320977
210	322219	322426	322633	322839	323046
211	324282	324488	324694	324899	325105
212	326336	326541	326745	326949	327155
213	328379	328583	328787	328991	329194
214	330414	330617	330819	331022	331225
215	332438	332640	332842	333044	333246
216	334454	334655	334856	335057	335257
217	336459	336659	336859	337059	337259
218	338456	338656	338856	339054	339253
219	340444	340642	340841	341039	341237

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5	6	7	8	9	D
279895	280123	280351	280578	280806	228
282169	282396	282622	282849	283075	227
284431	284656	284882	285107	285332	226
286681	286905	287129	287354	287578	225
288919	289143	289366	289589	289812	223
291147	291369	291591	291813	292034	222
293362	293584	293804	294025	294246	221
295567	295787	296007	296226	296446	220
297761	297979	298198	298416	298635	219
299943	200161	300378	200595	300813	218
302114	302331	302547	302764	302979	217
304275	304491	304706	304921	305136	216
306425	306639	306854	307068	307282	215
308564	308778	308991	309204	309417	213
310693	310906	311118	311329	311542	212
312812	313023	313234	313445	313656	211
314920	315130	315340	315551	315760	210
317018	317227	317436	317646	317854	209
319106	319314	319522	319730	319938	208
321184	321391	321598	321805	322012	207
323252	323458	323665	323871	324077	206
325310	325516	325721	325926	326131	205
327359	327563	327767	327972	328176	204
329398	329601	329805	330008	330211	203
331427	331629	331832	332034	332236	202
333447	333649	333859	334051	334253	202
335458	335658	335859	336059	336259	201
337459	337659	337859	338058	338257	200
339453	339650	339849	340047	340246	199
341435	341632	341830	342028	342225	198

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N	O	I	2	3	4
220	342227	342620	342817	343014	343212
221	344392	344589	344785	344981	345178
222	346353	346549	346744	346939	347135
223	348305	348499	348694	348889	349083
224	350248	350442	350636	350829	351023
225	352183	352375	352568	352761	352954
226	354108	354301	354493	354685	354876
227	356026	356217	356408	356599	356790
228	357935	358125	358316	358506	358696
229	359835	360025	360215	360404	360593
230	361728	361917	362105	362294	362482
231	363612	363799	363988	364176	364363
232	365488	365675	365862	366049	366236
233	367356	367542	367729	367915	368101
234	369216	369401	369587	369772	369958
235	371068	371253	371437	371622	371806
236	372912	373096	373279	373464	373647
237	374748	374932	375115	375298	375481
238	376577	376159	376942	377124	377306
239	378398	378579	378761	378943	379124
240	380211	380392	380573	380754	380934
241	382017	382197	382377	382557	382737
242	383815	383995	384174	384353	384533
243	385606	385785	385964	386142	386321
244	387389	387568	387746	387923	388101
245	389166	389343	389520	389698	389875
246	390935	391112	391288	391464	391641
247	392697	392873	393048	393224	393399
248	394452	394627	394802	394977	395152
249	396199	396374	396548	396722	396896

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5	6	7	8	9	D
343409	343606	343802	343999	344196	197
345373	345569	345766	345962	346157	196
347330	347525	347720	347915	348110	195
349278	349472	349659	349860	350054	194
351216	351409	351603	351796	351989	193
353147	353339	353532	353724	353916	193
355068	355239	355452	355643	355834	192
356981	357172	357363	357554	357744	191
358886	359076	359266	359456	359646	190
360783	360972	361161	361350	361539	189
362671	362859	363048	363236	363424	188
364551	364739	364926	365113	365301	188
366423	366609	366796	366983	367169	187
368287	368473	368659	368845	369030	186
370143	370328	370513	370698	370882	185
371991	372175	372359	372544	372728	184
373831	374015	374198	374382	374565	184
375664	375846	376029	376212	376394	183
377488	377670	377852	378034	378216	182
379306	379487	379668	379849	380030	181
381115	381296	381476	381656	381837	181
382917	383097	383277	383456	383636	180
384712	384891	385069	385249	385428	179
389499	386677	386856	387034	387212	178
388279	388456	388634	388811	388989	178
390051	390228	390405	390582	390759	177
391817	391993	392169	392345	392521	176
393575	393751	393926	394101	394277	176
395326	395501	395676	395850	396025	175
397071	397245	397419	397592	397766	174

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N	O	I	2	3	4
250	397940	398114	398287	398461	398634
251	399674	399847	400019	400192	400365
252	401401	401573	401745	401917	402089
253	403121	403292	403464	403635	403807
254	404834	405005	405176	405346	405517
255	406540	406710	406881	407051	407221
256	408239	408409	408579	408749	408918
257	409933	410102	410271	410439	410609
258	411619	411788	411956	412124	412293
259	413299	413467	413635	413803	413969
260	414973	415140	415307	415474	415641
261	416641	416807	416973	417139	417306
262	418301	418467	418633	418798	418964
263	419956	420121	420289	420451	420616
264	421604	421768	421933	422097	422261
265	423246	423409	423574	423737	423901
266	424882	425045	425208	425371	425534
267	426511	426674	426836	426999	427161
268	428135	428297	428459	428621	428783
269	429752	429914	430075	430236	430398
270	431369	431525	431685	431846	432007
271	432969	433129	433289	433449	433609
272	434569	434729	434888	435048	435207
273	436163	436322	436481	436639	436799
274	437751	437909	438067	438226	438384
275	439333	439491	439648	439806	439964
276	440909	441066	441224	441381	441538
277	442479	442637	442793	442949	443106
278	444045	444201	444357	444513	444669
279	445604	445759	445915	446071	446226

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5	6	7	8	9	D
398808	398981	399154	399328	399501	173
400538	400711	400883	401056	401228	173
402261	402433	402605	402777	402949	172
403978	404149	404320	404492	404663	171
405688	405858	406029	406199	406369	171
407391	407561	407731	407901	408070	170
409087	409257	409426	409595	409764	169
410777	410946	411114	411283	411451	169
412461	412629	412796	412964	413132	168
414137	414305	414472	414639	414806	167
415808	415974	416141	416308	416474	167
417472	417638	417804	417969	418135	166
419129	419295	419460	419625	419791	165
420781	420945	421110	421275	421439	165
422426	422589	422754	422918	423082	164
424065	424228	424392	424555	424718	164
425697	425860	426023	426186	426349	163
427324	427486	427648	427811	427973	162
428944	429106	429268	429429	429591	162
430559	430719	430881	431042	431203	161
432167	432328	432488	432649	432809	161
433769	433929	434089	434249	434409	160
435366	435526	435685	435844	436004	159
436957	437116	437275	437433	437592	159
438542	438701	438859	439175	439165	158
440122	440279	440437	440594	440752	158
441695	441852	442009	442166	442323	157
443263	443419	443576	443732	443889	157
444825	444981	445137	445293	445449	156
446382	446537	446692	446848	447003	155

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N	O	I	2	3	4
280	447158	447313	447468	447623	447778
281	448706	448861	449015	449169	449324
282	452040	450403	450557	450711	450865
283	451786	451939	452093	452247	452399
284	453318	453471	453624	453777	453929
285	454845	454997	455149	455302	455454
286	456266	456518	456669	456821	456973
287	457889	458033	458184	458336	458487
288	459292	459543	459694	459845	459995
289	460898	461048	461198	461348	461499
290	462398	462548	462647	462847	462997
291	463893	464042	464191	464340	464489
292	465283	465532	465680	465829	465977
293	466868	467016	467164	467312	467460
294	468347	468495	468643	468790	468938
295	469822	469969	470116	470263	470410
296	471292	471438	471585	471732	471878
297	472756	472903	473049	473195	473341
298	474216	474362	474508	474653	474799
299	475671	475816	475962	476107	476252
300	477121	477266	477411	477555	477699
301	478566	478711	478855	478999	479143
302	480007	480151	480294	480438	480582
303	481443	481586	481729	481872	482016
304	482874	483016	483159	483302	483445
305	484299	484442	484585	484727	484869
306	485721	485863	486005	486147	486289
307	487138	487279	487421	487563	487704
308	488551	488692	488833	488974	489114
309	489958	490099	490239	490379	490520

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5	6	7	8	9	D
447932	448088	448242	448397	448552	155
449478	449633	449787	459941	450095	154
451018	451172	451326	451479	451633	154
452553	452706	452859	453012	453165	153
454082	454235	454387	454539	454692	153
455606	455758	455910	456062	456214	152
457125	457276	457428	457579	457731	152
458638	458789	458939	459091	459242	151
460146	460296	460447	460597	460748	151
461649	461799	461948	462098	462248	150
463146	463296	463445	463594	463744	150
464639	464788	464936	465085	465234	149
466126	466274	466423	466571	466719	149
467608	467756	467904	468052	468199	148
469085	469233	469380	469527	469675	147
470557	470704	470851	470998	471145	147
472025	472171	472318	472464	472610	146
473487	473633	473779	473925	474071	146
474944	475089	475235	475381	475526	145
476397	476542	476687	476832	476976	145
477844	477989	478133	478278	478422	145
479287	479431	479575	479719	479863	144
480725	480869	481012	481156	481299	144
482159	482302	482445	482588	482731	143
483587	483729	483872	484015	484157	143
485011	485153	485295	485437	485579	142
486430	486572	486714	486855	486997	142
487845	487986	488127	488269	488409	141
489255	489396	489537	489677	489818	141
490661	490801	490941	491081	491222	140

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N	O	I	2	3	4
310	491362	491502	491642	491782	491922
311	492760	492900	493039	493179	493319
312	494155	494294	494433	494572	494711
313	495544	495683	495822	495960	496099
314	496929	497068	497206	497344	497483
315	498311	498448	498586	498724	498862
316	499687	499824	499962	500099	500236
317	501059	501196	501333	501470	501607
318	502427	502564	502700	502837	502973
319	503791	503927	504063	504199	504335
320	505149	505286	505421	505557	505693
321	506505	506640	506776	506911	507046
322	507856	507991	508126	508260	508395
323	509203	509337	509471	509606	509740
324	510545	510679	510813	510947	511081
325	511883	512017	512151	512284	512418
326	513218	513351	513485	513617	513750
327	514548	514681	514813	514946	515079
328	515874	516006	516139	516271	516403
329	517196	517328	517459	517592	517724
330	518514	518646	518777	518909	519040
331	519828	519959	520090	520221	520353
332	521138	521269	521399	521530	521661
333	522454	522575	522705	522835	522966
334	523746	523876	524006	524136	524266
335	525045	525174	525304	525434	525563
336	526339	526469	526598	526727	526856
337	527629	527759	527888	528016	528145
338	528916	529045	529174	529302	529430
339	530199	530328	530456	530584	530712

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5	6	7	8	9	D
492062	492201	492341	492481	492621	140
493458	493597	493737	493876	494015	139
494850	494989	495128	495267	495406	139
496238	496376	496515	496653	496791	139
497921	497759	497897	498035	498173	138
498999	491137	499275	499412	499549	138
500374	500510	500648	500785	500922	137
501744	501880	502017	502154	502291	137
503109	513246	503382	503518	503655	136
504471	504607	504743	504878	505014	136
505828	505964	506099	506234	506369	136
507181	507316	507451	507586	507721	135
508529	508664	508799	508934	509068	135
509894	510009	510143	510277	510411	134
511215	511349	511482	511616	511749	134
512551	512684	512818	512951	513084	133
513883	514016	514149	514282	514415	133
515211	515344	515476	515609	515741	133
516535	516668	516796	516932	517064	132
517855	517987	518119	518251	518382	132
519171	519303	519434	519566	519697	131
520484	520615	520745	520876	521007	131
521792	521922	522053	522183	522314	131
523096	523226	523356	523486	523616	130
524396	524526	524656	524785	524915	130
525693	525822	525951	526081	526210	129
526985	627114	527243	527372	527501	129
528274	528402	528531	528659	528788	129
525559	529687	529815	529943	530072	128
530839	530968	331096	531223	531351	128

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N	O	I	2	3	4
340	531479	531607	531734	531862	531989
341	532754	532882	533009	533136	533264
342	534026	534153	534280	534407	534534
343	535294	535421	535547	535674	535800
344	536558	536685	536811	536937	537063
345	537819	537945	538071	538197	538322
346	539076	539202	539327	539452	539578
347	440329	540455	540579	540705	540829
348	541579	541704	541829	541953	542078
349	542825	542949	543074	543199	543323
350	545008	544192	544316	544440	544564
351	545307	545431	545555	545678	545802
352	546543	546666	546789	546913	547036
353	547775	547898	548021	548144	548267
354	549003	549126	549249	549371	549494
355	550228	550351	550473	550595	550717
356	551449	551572	551694	551816	551938
357	552668	552789	552911	553033	553155
358	553883	554004	554126	554247	554368
359	555094	555215	555336	555457	555578
360	556303	556423	556544	556664	556785
361	557057	557177	557298	557418	557538
362	558270	558390	558510	558630	558750
363	559487	559607	559727	559847	559967
364	561101	561221	561341	561461	561581
365	562293	562412	562531	562651	562771
366	563481	563599	563718	563837	563957
367	564666	564784	564903	565021	565141
368	565848	565966	566084	566202	566321
369	567026	567144	567262	567381	567500

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5	6	7	8	9	D
532117	532245	532372	532499	532627	128
533391	533518	533645	533742	533899	127
534661	534787	534914	535041	535167	127
535927	536053	536179	536304	536432	126
537189	537315	537441	537567	537693	126
538448	538574	538699	538825	538951	126
539703	539829	539954	540079	540204	125
540955	541079	541205	541329	541454	125
542203	542327	542452	542576	542701	125
543447	543571	543696	543819	543944	124
544688	544812	544934	545059	545183	124
545925	546049	546172	546296	546419	124
547159	547282	547405	547529	547652	123
548389	548512	548635	548758	548881	123
549616	549739	549861	549984	550106	123
550839	550962	551084	551206	551328	122
552059	552181	552303	552425	552547	122
553276	553398	553519	553640	553762	121
554489	554610	554731	554852	554973	121
555699	555819	555940	556061	556182	121
556905	557026	557146	557267	557387	120
558108	558228	558349	558469	558589	120
559308	559428	559548	559667	559787	120
560504	560624	560743	560863	560982	119
561698	561817	561936	562055	562179	119
562887	563006	563125	563244	563362	119
564074	564192	564311	564429	564548	119
565257	565376	565494	565612	565729	118
566437	566555	566673	566791	566909	118
567614	567732	567849	567967	568084	118

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N	O	I	2	3	4
370	568202	568319	568436	568554	568671
371	569374	569491	569608	569725	569842
372	570543	570659	570776	570893	571009
373	571709	571825	571942	572058	572274
374	572872	572988	573104	573219	573336
375	574031	574147	574263	574379	574494
376	575188	575303	575419	575534	575649
377	576341	576457	576572	576687	576802
378	577492	577607	577722	577836	577951
379	578639	578754	578868	578983	579097
380	579784	579898	580012	580126	580241
381	580925	581039	581153	581267	581381
382	582063	582177	582291	582404	582518
383	583199	583312	583426	583539	583652
384	584331	584444	584557	584670	584783
385	585461	585574	585686	585799	585912
386	586587	586699	586812	586925	587037
387	587712	587823	587935	588047	588159
388	588832	588944	589056	589167	589279
389	589949	590061	590173	590284	590396
390	591065	591176	591287	591399	591509
391	592177	592288	592399	592509	592621
392	593286	593397	593508	593618	593729
393	594393	594503	594614	594724	594834
394	595496	595606	595717	595827	595937
395	596597	596707	596817	596927	597037
396	597695	597805	597914	598024	598134
397	598790	598899	599009	599119	599228
398	599883	599992	600101	600210	600319
399	600973	601082	601191	601299	601408

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5	6	7	8	9	D
568788	568095	569023	566139	569257	117
569959	570076	570193	570309	570426	117
571126	571243	571359	571476	571592	117
572291	572407	572523	572639	572755	116
573452	573568	573684	573799	573915	116
574609	574726	574841	574957	575072	116
575765	575880	575996	576111	576226	115
576917	577032	577147	577262	577377	115
578066	578181	578295	578409	578525	115
579212	579326	579441	579555	579669	114
580355	580469	580583	580697	580811	114
581495	581608	581722	581836	581949	114
582631	582745	582858	582972	583085	114
583765	583879	583992	584105	584218	113
584896	585009	585122	585235	585348	113
586024	586137	586249	586362	586475	113
587149	587262	587374	587486	587599	112
588272	588384	588496	588608	588719	112
589391	589503	589615	589726	589838	112
590507	590619	590730	590842	590953	112
591621	591732	591843	591955	592066	111
592732	592843	592954	593064	593175	111
593839	593950	594061	594171	594282	111
594945	595055	595165	595276	595386	110
596047	596157	596267	596377	596487	110
597146	597256	597366	597476	597586	110
598243	598353	598462	598572	598681	110
599337	599446	599556	599665	599774	109
600428	600537	600646	600755	600864	109
601517	601625	601734	601843	601951	109

The Table of Logarithms.

N	O	I	2	3	4
400	602059	602169	602277	602386	602494
401	603144	603253	603361	603469	603577
402	604226	604334	604442	604550	604658
403	605305	605413	605521	605628	605736
404	606381	606489	606596	606704	606811
405	607455	607562	607669	607777	607884
406	608526	608633	608739	608847	608954
407	609594	609701	609808	609914	610021
408	610660	610767	610873	610979	611086
409	611723	611829	611936	612042	612148
410	612784	612889	612996	613102	613207
411	613842	613947	614053	614159	614264
412	614897	615003	615108	615213	615319
413	615950	616055	616160	616265	616370
414	617000	617105	617210	617315	617419
415	618048	618153	618257	618362	618466
416	619093	619198	619302	619406	619511
417	620136	620240	620344	620448	620552
418	621176	621280	621384	621448	621592
419	622214	622318	622421	622525	622628
420	623249	623353	623456	623559	623663
421	624282	624385	624488	624591	624695
422	625312	625415	625518	625621	625724
423	626240	626443	626546	626648	626751
424	627366	627468	627571	627673	627775
425	628389	628491	628593	628695	628797
426	629409	629512	629613	629715	629817
427	630428	630529	630631	630733	630835
428	631444	631545	631647	631746	631849
429	632457	632559	632659	632761	632862

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5	6	7	8	9	D
602603	602817	602819	602928	603036	108
603636	603794	603902	604009	604118	108
604766	604874	604982	605089	605197	108
605844	605951	606059	606166	606274	108
606919	607026	607133	607241	607348	107
607991	608098	608205	608312	608419	107
609061	609167	609274	609381	609488	107
610128	610234	610341	610447	610555	107
611192	611298	611405	611511	611617	106
612254	612359	612466	612572	612678	106
613313	613419	613525	613630	613736	106
614369	614475	614581	614686	614792	106
615424	615529	615634	615739	615845	105
616476	616581	616686	616790	616895	105
617525	617629	617734	617839	617943	105
618571	618676	618780	618889	618989	105
619615	619719	619824	619928	620032	104
620656	620760	620864	620968	621072	104
621695	621799	621902	622007	622110	104
622722	622835	622939	623042	623146	104
623766	623869	623973	624076	624179	103
624798	624901	625004	625107	625209	103
625827	625929	626032	626135	626237	103
626853	626956	627058	627161	627263	103
627878	627979	628082	628185	628287	102
628899	629002	629104	629206	629308	102
629919	620021	630123	630224	630326	102
630936	631038	631139	631241	631342	102
631951	632051	632153	632255	632356	101
632963	633064	633165	633266	633367	101

The Table of Logarithms.

N	O	I	2	3	4
430	933468	633569	633670	633771	633872
431	634477	634578	634679	634779	634880
432	635484	635584	635685	635785	635886
433	636488	636588	636688	636789	636889
434	637489	637589	637689	637789	637889
435	638489	638589	638689	638789	638888
436	639486	639586	639686	639785	639885
437	640481	640581	640680	640779	640879
438	641475	641573	641672	641771	641871
439	642465	642563	642662	642761	642860
440	643453	643551	643650	643749	643847
441	644439	644537	644636	644734	644833
442	645422	645521	645619	645717	645815
443	646404	646502	646599	646698	646796
444	647383	647481	647579	647676	647774
445	648360	648458	648555	648653	648750
446	649335	649432	649529	649627	649724
447	650308	650405	650502	650599	650696
448	651278	651375	651472	651569	651666
449	652246	652343	652439	652536	652633
450	653213	653309	653405	653502	653598
451	654177	654273	654369	654465	654562
452	655138	655235	655331	655427	655523
453	656098	656194	656289	656386	656482
454	657056	657152	657247	657343	657438
455	658011	658107	658202	658298	658393
456	658965	659060	659155	659250	659346
457	659916	660011	660106	660201	660296
458	660865	660960	661055	661149	661245
459	661813	661907	662002	662096	662191

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5	6	7	8	9	D
633973	634075	634075	634276	634376	100
634981	635081	635182	635283	635383	100
635986	636087	636187	636288	636388	100
636989	637089	637189	637289	637389	100
637989	638089	638189	638289	638389	99
638988	639088	639188	639287	639387	99
639984	640084	640183	640283	640382	99
640978	641077	641177	641276	641375	99
641969	642069	642168	642267	642366	99
642959	643058	643156	643255	643354	99
643946	644044	644143	644242	644340	98
644931	645029	645127	645226	645324	98
645913	646011	646109	646208	646306	98
646894	646992	647089	647187	647285	98
647872	647969	648067	648165	648262	98
648848	648945	649043	649140	649237	97
649821	649919	650016	650113	650210	97
650793	650890	650987	651084	651181	97
651762	651859	651956	652053	652149	97
652729	652826	652923	653019	653116	97
653695	653791	653888	653984	654080	96
654658	654754	654850	654946	655042	96
655619	655715	655810	655906	656002	96
656577	656673	656769	656864	656960	96
657534	657629	657725	657820	657916	96
658488	658584	658679	658774	658869	95
659441	659536	659630	659726	659821	95
660391	660486	660581	660676	660771	95
661339	661434	661529	661623	661718	95
662289	662380	662475	662569	662663	95

The Table of Logarithms.

N	0	1	2	3	4
460	662758	662852	662947	663041	663135
461	663701	663795	663889	663983	664078
462	664642	664736	664829	664924	665018
463	665581	665675	665769	665862	665956
464	666518	666611	666705	666799	666892
465	667453	667546	667639	667733	667826
466	668386	668479	668572	668665	668759
467	669317	669409	669503	669596	669689
468	670246	670339	670431	670524	670617
469	671173	671265	671358	671451	671543
470	672098	672190	672283	672375	672467
471	673021	673113	673205	673297	673389
472	673942	674034	674126	674218	674309
473	674861	674953	675045	675137	675228
474	675778	675869	675962	676053	676145
475	676694	676785	676876	676968	677059
476	677607	677698	677789	677881	677972
477	678518	678609	678700	678791	678882
478	679428	679519	679609	679700	679791
479	680336	680426	680517	680607	680698
480	681241	681332	681422	681513	681603
481	682145	682235	682326	682416	682506
482	683047	683137	683227	683317	683407
483	683947	684037	684127	684217	684307
484	684845	684935	685025	685114	685204
485	685742	685831	685921	686010	686099
486	686636	686726	686815	686904	686994
487	687529	687618	687707	687796	687885
488	688419	688509	688598	688687	688776
489	689309	689398	689486	689575	689664

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5	6	7	8	9	D
663229	663324	663418	663512	663607	94
664172	664266	664359	664454	664548	94
665112	665206	665299	665393	665487	94
666049	666143	666237	666331	666424	94
666986	667079	667173	667266	667359	94
667919	668013	668106	668199	668293	93
668852	668945	669038	669131	669224	93
669782	669875	669967	670060	670153	93
670709	670802	670895	670988	671080	93
671636	671728	671821	671913	672005	93
672559	672652	672744	672836	672929	92
673482	673574	673666	673758	673849	92
674402	674494	674586	674677	674769	92
675319	675412	675503	675595	675687	92
676236	676328	676419	676511	676602	92
677151	677242	677333	677424	677516	91
678063	678154	678245	678335	678427	91
678973	679064	679155	679246	679337	91
679882	679972	680063	680154	680245	91
680789	680879	680969	681060	681151	91
681693	681784	681874	681964	682055	90
682596	682686	682777	682867	682957	90
683497	683587	683677	683767	683857	90
684396	684486	684576	684666	684756	90
685294	685383	685473	685563	685652	90
686189	686279	686368	686458	686547	89
687083	687172	687211	687351	687439	89
687975	688054	688153	688242	688331	89
688865	688953	689042	689131	689220	89
689753	689841	689930	690019	690107	89

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N	O	I	2	3	4
490	690196	690285	690373	690462	690550
491	691081	691169	691258	691347	691435
492	691965	692053	692142	692229	692318
493	692847	692935	693023	693111	693199
494	693727	693815	693903	693991	694078
495	694605	694693	694781	694868	694956
496	695482	695569	695657	695744	695832
497	696356	696444	696531	696618	696706
498	697229	697317	697404	697491	697578
499	698101	698188	698275	698362	698449
500	698970	699057	699144	699281	699317
501	699838	699924	700011	700098	700184
502	700704	700790	700877	700963	701049
503	701568	701654	701741	701827	701913
504	702430	702517	702603	702689	702775
505	703291	703377	703463	703549	703635
506	704151	704236	704322	704408	704494
507	705008	705094	705179	705265	705350
508	705862	705949	706035	706130	706206
509	706718	706803	706888	706974	707059
510	707570	707655	707740	707826	707911
511	708421	708506	708591	708676	708761
512	709269	709355	709439	709524	709609
513	710117	710202	710287	710371	710456
514	710963	711048	711132	711217	711301
515	711807	711891	711976	712060	712144
516	712649	712734	712818	712902	712986
517	713491	713575	713659	713742	713826
518	714329	714414	714497	714581	714665
519	715167	715251	715335	715418	715501

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5	6	7	8	9	D
690639	690728	690816	690905	690993	89
691524	691612	691700	691789	691877	88
692406	692494	692583	692671	692759	88
693287	693375	693463	693551	693639	88
694166	694254	694342	694429	694517	83
695044	695131	695219	695307	695394	88
695919	696007	696094	696182	696269	87
696793	696880	696968	697055	697142	87
697665	697752	697839	697926	698014	87
698535	698622	698709	698796	698883	87
699404	699491	699528	699664	699751	87
700271	700358	700444	700531	700617	87
701136	701222	701309	701395	701482	86
701999	702086	702172	702258	702344	86
702861	702947	703033	703119	703205	86
703721	703807	703893	703979	704065	86
704579	704665	704757	704837	704922	86
705436	705522	705607	705693	705778	86
706291	706376	706462	706547	706632	85
707144	707219	707315	707399	707485	85
707956	708081	708166	708251	708336	85
708846	708931	709015	709100	709185	85
709694	709789	709863	709948	710033	85
710540	710625	710709	710794	710879	85
711385	711469	711554	711639	711723	84
712229	712313	712397	712481	712566	84
713070	713154	713238	713223	713407	84
713910	713994	714078	714162	714246	84
714749	714833	714916	714999	715084	84
715586	715669	715753	715836	715919	84

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N	O	I	2	3	4
520	716003	716087	716170	716254	716337
521	716838	716921	717004	717088	717171
522	717671	717754	717837	717920	718003
523	718502	718585	718668	718751	718834
524	719331	719414	719497	719579	719663
525	720159	720242	720325	720407	720490
526	720986	721068	721151	721233	721316
527	721811	721893	721975	722058	722140
528	722634	722716	722798	722881	722963
529	723456	723538	723619	723702	723784
530	724276	724358	724439	724522	724604
531	725095	725176	725258	725339	725422
532	725912	725993	726075	726156	726238
533	726727	726809	726890	726972	727053
534	727541	727623	727704	727785	727866
535	728354	728435	728516	728597	728678
536	729165	729246	729327	729408	729489
537	729974	730055	730136	730217	730298
538	730782	730863	730944	731024	731105
539	731589	731669	731749	731830	731911
540	732394	732474	732555	732635	732715
541	733197	733278	733358	733438	733518
542	733999	734079	734159	734239	734319
543	734799	734879	734959	735039	735119
544	735599	735679	735759	735838	735918
545	736397	736476	736556	736635	736715
546	737191	737272	737352	737431	737511
547	737987	738067	738146	738225	738305
548	738781	738859	738939	739018	739097
549	739572	739651	739731	739809	739889

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5	6	7	8	9	D
716421	716504	716588	716671	716754	83
717254	717338	717421	717509	717587	83
718086	718169	718253	718336	718419	83
718917	718999	719083	719165	719248	83
719745	719828	719911	719994	720077	83
720573	720655	720738	720821	720903	83
721398	721481	721563	721646	721728	82
722222	722305	722387	722469	722552	82
723045	723127	723209	723291	723374	82
723866	723948	724029	724112	724194	82
724685	724767	724849	724931	725013	82
725503	725585	725667	725748	725829	82
726319	726401	726483	726564	726646	82
727134	727216	727297	727379	727459	81
727948	728029	728110	728191	728273	81
728759	728841	728922	729003	729084	81
729569	729651	729732	729813	729893	81
730378	730459	730540	730621	730702	81
731186	731266	731347	731423	731508	81
731991	732072	732152	732233	732313	81
732796	732876	732956	733037	733117	80
733598	733679	733759	733839	733919	80
734399	734479	734559	734639	734719	80
735199	735279	735359	735439	735519	80
735998	736078	736157	736237	736317	80
736795	736874	736954	737034	737113	80
737590	737669	737749	737829	737908	79
738384	738463	738543	738622	738701	79
739177	739259	739335	739414	739493	79
739968	740047	740126	740205	740284	79

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N	O	I	2	3	4
550	740363	740442	740521	740599	740678
551	741152	741230	741309	741388	741467
552	741939	742018	742096	742175	742254
553	742725	742802	742882	742961	743039
554	743509	743588	743667	743745	743823
555	744293	744371	744449	744528	744606
556	745075	745153	745231	745309	745387
557	745855	745933	746011	746089	746167
558	746634	746712	746789	746868	746945
559	747412	747489	747567	747645	747722
560	748188	748266	748343	748421	748498
561	748963	749040	749118	749195	749272
562	749736	749814	749891	749968	750045
563	750508	750586	750663	750739	750817
564	751279	751356	751433	751510	751587
565	752048	752125	752202	752279	752356
566	752816	752893	752969	753047	753123
567	753583	753659	753736	753813	753889
568	754348	754425	754501	754578	754654
569	755112	755189	755265	755341	755417
570	755875	755951	756027	756103	756179
571	756636	756712	756788	756864	756940
572	757396	757472	757548	757627	757699
573	758155	758230	758306	758382	758458
574	758912	758988	759063	759139	759214
575	759668	759743	759819	759894	759969
576	760422	760498	760573	760649	760723
577	761176	761251	761326	761402	761477
578	761928	762003	762078	762153	762228
579	762679	762754	762829	762904	762978

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5	6	7	8	9	D
740757	740836	740915	740994	741073	79
741546	741624	741703	741782	741860	79
742332	742411	742489	742568	742647	79
743118	743196	743275	743353	743431	78
743902	743979	744058	744136	744215	78
744684	744764	744840	744919	744997	78
745465	745543	745621	745699	745777	78
746245	746323	746401	746479	746556	78
747023	747101	747179	747256	747334	78
747800	747878	747955	748033	748110	78
748576	748653	748731	748808	748885	77
749349	749427	749504	749582	749659	77
750123	750199	750277	750354	750431	77
750894	750971	751048	751125	751202	77
751664	751741	751818	751895	751972	77
752433	752509	752586	752663	752739	77
753199	753277	753353	753429	753506	77
753966	754042	754119	754195	754272	77
754730	754800	754883	754959	755036	76
755494	755569	755646	755722	755799	76
756256	756332	756408	756484	756560	76
757016	757092	757167	757244	757320	76
757775	757851	757927	758003	758079	76
758533	758609	758685	758761	758836	76
759290	759366	759441	759517	759592	76
760045	760121	760196	760272	760347	75
760799	760875	760949	761025	761101	75
761552	761627	761702	761778	761853	75
762303	762378	762453	762529	762604	75
763053	763128	763203	763279	763353	75

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N	0	1	2	3	4
580	763428	763503	763578	763653	763727
581	764176	764251	764326	764400	764475
582	764923	764998	765072	765147	765221
583	765669	765743	765818	765892	765966
584	766413	766487	766562	766636	766710
585	767156	767230	767304	767379	767453
586	767898	767972	768046	768119	768194
587	768638	768712	768786	768860	768934
588	769377	769451	769525	769599	769673
589	770115	770189	770263	770336	770410
590	770852	770926	770999	771073	771146
591	771587	771661	771734	771808	771881
592	772322	772395	772468	772542	772615
593	773055	773128	773201	773274	773348
594	773786	773859	773933	774006	774079
595	774517	774589	774663	774736	774809
596	775246	775319	775392	775465	775538
597	775974	776047	776119	776193	776265
598	776701	776774	776846	776919	776992
599	777427	777499	777572	777644	777717
600	778151	778224	778296	778368	778441
601	778874	778947	779019	779091	779163
602	779596	779669	779741	779813	779884
603	780317	780389	780461	780533	780605
604	781037	781109	781181	781253	781324
605	781755	781827	781899	781971	782042
606	782473	782544	782616	782688	782759
607	783189	783260	783332	783403	783475
608	783904	783975	784046	784118	784189
609	784617	784689	784759	784831	784902

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5	6	7	8	9	D
763802	763877	763952	764027	764101	75
764549	764624	764699	764774	764848	75
765296	765370	765445	765519	765594	75
766041	766115	766189	766264	766338	74
766785	766859	766933	767007	767082	74
767527	767601	767675	767749	767823	74
768268	768342	768416	768490	768564	74
769008	769082	769156	769229	769303	74
769746	769820	769894	769968	770042	74
770484	770557	770631	770705	770778	74
771219	771293	771367	771440	771514	74
771955	772028	772102	772175	772248	73
772688	772762	772835	772908	772981	73
773421	773494	773567	773640	773713	73
774152	774225	774298	774371	774444	73
774882	774955	775028	775100	775173	73
775610	775683	775756	775829	775902	73
776338	776411	776483	776556	776629	73
777064	777137	777209	777282	777354	73
777789	777862	777934	778006	778079	72
778513	778585	778658	778729	778802	72
779236	779308	779380	779452	779524	72
779957	780029	780101	780173	780245	72
780677	780749	780821	780893	780965	72
781396	781468	781539	781612	781684	72
782114	782186	782258	782329	782401	72
782831	782902	782974	783046	783117	72
783546	783618	783689	783761	783832	71
784261	784332	784403	784472	784546	71
784974	785045	785116	785187	785259	71

The Table of Logarithms.

N	O	I	2	3	4
610	785329	785401	785472	785543	785615
611	786041	786112	786183	786254	786325
612	786751	786822	786893	786964	787035
613	787460	787531	787602	787673	787744
614	788164	788239	788309	788381	788451
615	788875	788946	789016	789087	789157
616	789581	789651	789722	789792	789863
617	790285	790356	790426	790496	790567
618	790988	791059	791129	791199	791269
619	791691	791761	791831	791901	791971
620	792392	792462	792532	792602	792672
621	793092	793162	793231	793301	793371
622	793791	793860	793930	793999	794069
623	794488	794558	794627	794697	794767
624	795185	795254	795324	795393	795463
625	795880	795949	796019	796088	796158
626	796574	796644	796713	796782	796852
627	797268	797337	797406	797475	797545
628	797959	798029	798098	798167	798236
629	798651	798719	798789	798858	798927
630	799341	799409	799478	799547	799616
631	800029	800098	800167	800236	800305
632	800717	800786	800854	800923	800992
633	801404	801472	801541	801609	801678
634	802089	802158	802226	802295	802363
635	802774	802842	802910	802979	803047
636	803457	803525	803594	803662	803730
637	804139	804208	804276	804344	804412
638	804821	804889	804957	805025	805093
639	805501	805569	805637	805705	805773

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5	6	7	8	9	D
785686	785757	785828	785899	785970	71
786396	786467	786538	786609	786680	71
787106	787177	787248	787319	787389	71
787815	787885	787956	788027	788098	71
788522	788593	788663	788734	788804	71
789228	789299	789369	789439	789510	71
789933	790004	790074	790144	790215	70
790637	790707	790778	790848	790918	70
791339	791409	791480	791550	791620	70
792041	792111	792181	792252	792322	70
792742	792812	792882	792952	793022	70
793441	793511	793581	793651	793721	70
794139	794209	794279	794349	794418	70
794836	794906	794976	795045	795115	70
795532	795602	795672	795741	795810	70
796227	796297	796366	796436	796505	69
796921	796990	797059	797129	797198	69
797614	797683	797752	797821	797890	69
798305	798374	798443	798513	798582	69
798996	799065	799134	799203	799272	69
799685	799754	799823	799892	799961	69
800373	800442	800511	800579	800648	69
801061	801129	801198	801266	801335	69
801747	801815	801884	801952	802021	69
802432	802500	802568	802637	802705	69
803116	803184	803252	803321	803389	68
803798	803867	803935	804003	804071	68
804480	804548	804616	804685	804753	68
805161	805229	805297	805365	805433	68
805841	805908	805976	806044	806112	68

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	0	1	2	3	4
640	806170	806248	806316	806384	806451
641	806358	806426	806494	806561	806629
642	806535	806602	806670	806738	806806
643	806711	806779	806846	806914	806981
644	806886	806953	807021	807088	807156
645	807059	807127	807194	807262	807329
646	807233	807300	807367	807434	807501
647	807404	807471	807539	807606	807673
648	807645	807712	807779	807846	807913
649	807984	808051	808118	808185	808252
650	808319	808386	808453	808520	808587
651	808654	808721	808788	808855	808922
652	808990	809057	809124	809191	809258
653	809324	809391	809458	809525	809592
654	809659	809726	809793	809860	809927
655	809994	810061	810128	810195	810262
656	810329	810396	810463	810530	810597
657	810664	810731	810798	810865	810932
658	810999	811066	811133	811200	811267
659	811334	811401	811468	811535	811602
660	811669	811736	811803	811870	811937
661	811994	812061	812128	812195	812262
662	812329	812396	812463	812530	812597
663	812664	812731	812798	812865	812932
664	812999	813066	813133	813200	813267
665	813334	813401	813468	813535	813602
666	813669	813736	813803	813870	813937
667	813994	814061	814128	814195	814262
668	814329	814396	814463	814530	814597
669	814664	814731	814798	814865	814932

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5	6	7	8	9	D
806519	806587	806655	806723	806790	68
807197	807264	807332	807399	807467	68
807873	807941	808008	808076	808143	68
808549	808616	808684	808751	808818	67
809223	809290	809358	809425	809492	67
809866	809934	810001	810068	810135	67
810569	810636	810703	810770	810837	67
811239	811307	811374	811441	811508	67
811909	811977	812044	812111	812178	67
812579	812646	812713	812779	812847	67
813247	813314	813381	813448	813514	67
813914	813981	814048	814114	814181	67
814581	814647	814714	814780	814847	67
815246	815312	815378	815445	815511	66
815909	815976	816042	816109	816175	66
816573	816639	816705	816771	816838	66
817235	817301	817367	817433	817499	66
817896	817962	818028	818094	818159	66
818556	818622	818688	818754	818819	66
819215	819281	819346	819412	819478	66
819873	819939	820004	820070	820136	66
820529	820595	820661	820727	820792	66
821186	821251	821317	821382	821448	66
821841	821906	821972	822037	822103	65
822495	822560	822626	822691	822756	65
823148	823213	823279	823344	823409	65
823800	823865	823930	823996	824061	65
824451	824516	824581	824646	824711	65
825101	825166	825231	825296	825361	65
825751	825815	825880	825945	826009	65

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N	O	I	2	3	4
670	826075	826139	826204	826269	826334
671	826723	826787	826852	826917	826981
672	827369	827434	827499	827563	827628
973	828015	828079	828144	828209	828273
674	828659	828724	828789	828853	828918
675	829304	829368	829432	829497	829561
676	829947	830011	830075	830139	830204
677	830589	830653	830717	830781	830845
678	831229	831294	831358	831422	831486
679	831869	831934	831998	832062	832126
680	832509	832573	832637	832700	832764
681	833147	833211	833275	833338	833402
682	833784	833848	833912	833975	834039
683	834421	834484	834548	834611	834675
684	835056	835119	835183	835247	835310
685	835691	835754	835817	835881	835944
686	836324	836387	836451	836514	836577
687	836957	837019	837085	837146	837209
688	837588	837652	837715	837777	837841
689	838219	838282	838345	838408	838471
690	838849	838912	838975	839038	839101
691	839478	839541	839604	839667	839729
692	840106	840169	840232	840294	840357
693	840733	840796	840859	840921	840984
694	841359	841423	841485	841547	841609
695	841985	842047	842109	842172	842235
696	842609	842672	842734	842796	842859
697	843233	843295	843357	843419	843482
698	843855	843918	843979	844042	844104
699	844477	844539	844601	844664	844726

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5	6	7	8	9	D
826399	826464	826528	826593	826658	65
827046	827111	827175	827239	827305	65
827692	827757	827822	827886	827951	65
828338	828402	828467	828531	828595	64
828982	829046	829111	829175	629239	64
829625	829689	829754	829818	829882	64
830268	830332	830396	830460	830525	64
830909	830973	831037	831102	831166	64
831549	831614	831678	831742	831806	64
833189	832253	832317	832381	832445	64
832828	832892	832956	833019	833083	64
833466	833529	833593	833657	833721	64
834103	834166	834229	834294	834357	64
834739	834802	834866	834929	834993	64
835373	835437	835500	835564	835627	63
836007	836071	836134	836197	836261	63
836641	836704	836767	836830	836894	63
837273	837339	837399	837462	837525	63
837904	837967	838030	838093	838156	63
838534	838597	838660	838723	838786	63
839164	839227	839289	839352	839415	63
839792	839855	839918	839981	840043	63
840419	840482	840545	840608	840671	63
841049	841109	841172	841234	841297	63
841672	841735	841797	841859	841922	63
842297	842359	842432	842484	842547	62
842921	842983	843046	843108	843170	62
843544	843606	843669	843731	843793	62
844166	844229	844291	844353	844415	62
844788	844849	844912	844974	845036	62

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N	O	I	2	3	4
700	845098	845160	845222	845284	845346
701	845718	845779	845842	845904	845966
702	846337	846399	846461	846523	846585
703	846955	847017	847079	847141	847202
704	847573	847634	847696	847758	847819
705	848189	848251	848312	848374	848435
706	848805	848866	848928	848989	849051
707	849419	849481	849542	849604	849665
708	850033	850095	850156	850217	850279
709	850646	850707	850769	850829	850891
710	851258	851319	851381	851442	851503
711	851869	851931	851992	852053	852114
712	852479	852541	852602	852663	852724
713	853089	853150	853211	853272	853333
714	853698	853759	853819	853881	853941
715	854306	854367	854428	854488	854549
716	854913	854974	855034	855095	855156
717	855519	855579	855640	855701	855761
718	856124	856185	856245	856306	856366
719	856729	856789	856849	856910	856970
720	857332	857393	857453	857513	857574
721	857935	857995	858056	858116	858176
722	858537	858597	858657	858718	858778
723	859138	859198	859258	859318	859379
724	859739	859799	859859	859918	859978
725	860338	860398	860458	860518	860578
726	860937	860996	861056	861116	861176
727	861534	861594	861654	861714	861773
728	862131	862191	862251	862310	862369
729	862728	862787	862847	862906	862966

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5	6	7	8	9	D
845408	845470	845532	845594	845656	62
846028	846089	846151	846213	846275	62
846646	846708	846769	846832	846894	62
847264	847326	847388	847449	847511	62
847881	847943	848004	848067	848128	62
848497	848559	848620	848682	848743	62
849112	849174	849235	849297	849358	61
849726	849788	849849	849911	849972	61
850339	850401	850462	850524	850585	61
850952	851014	851075	851136	851197	61
851564	851625	851686	851747	851809	61
852175	852236	852297	852358	852419	61
852785	852846	852907	852968	853029	61
853394	853455	853516	853577	853637	61
854002	854063	854124	854185	854245	61
854609	854670	854731	854792	854852	61
855216	855277	855337	855398	855459	61
855822	855882	855943	856003	856064	61
856427	856487	856548	856608	856668	60
857031	857091	857152	857212	857272	60
857634	857694	857755	857815	857875	60
858236	858297	858357	858417	858477	60
858838	858898	858958	859018	859078	60
859439	859499	859559	859619	859679	60
860038	860098	860158	860218	860278	60
860637	860697	860757	860817	860877	60
861236	861295	860355	861415	861475	60
861833	861893	861952	862012	862072	60
862429	862489	862549	862608	762668	60
863025	863085	863144	863204	863263	60

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N	O	I	2	3	4
730	863323	863382	863442	863501	863561
731	863917	863977	864036	864096	864155
732	864511	864570	864629	864689	864748
733	865104	865163	865222	865282	865341
734	865696	865755	865814	865874	865933
735	866187	866346	866405	866465	866524
736	866878	866937	866996	867055	867114
737	867467	867526	867585	867644	867703
738	868056	868115	868174	868233	868292
739	868643	868703	868762	868821	868879
740	869232	869290	869349	869408	869466
741	869818	869877	869935	869994	870053
742	870404	870462	870521	870579	870638
743	870989	871047	871106	871164	871223
744	871573	871631	871689	871748	871806
745	872156	872215	872273	872331	872389
746	872739	872797	872855	872913	872972
747	873321	873379	873437	873495	873553
748	873902	873959	874018	874076	874134
749	874482	874539	874598	874656	874714
750	875061	875119	875177	875235	875293
751	875639	875698	875756	875813	875871
752	876218	876276	876333	876391	876449
753	876795	876853	876910	876968	877026
754	877371	877429	877487	877544	877602
755	877947	878004	878062	878119	878177
756	878522	878579	878637	878694	878752
757	879096	879153	879211	879268	879325
758	879669	879726	879784	879841	879898
759	880242	880299	880356	880413	880471

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5	6	7	8	9	D
863620	863679	863739	863799	863858	59
864214	864274	864333	864392	864452	59
864808	764867	864926	864985	864045	59
865400	865459	865519	865578	865637	59
865992	866051	866110	866169	866228	59
866583	866642	866701	866759	866819	59
867173	867232	867291	867349	867409	59
867762	867821	867879	867939	867998	59
868350	868409	868468	868527	868586	59
868938	868997	869056	869114	869173	59
869525	869584	869642	869701	869759	59
870111	870169	870228	870287	870345	59
870696	870755	870813	870872	870930	59
871281	871339	871398	871456	871515	58
871865	871923	871981	871039	872098	58
872448	872506	872564	872622	872681	58
873029	873088	873146	873204	873262	58
873611	873669	873727	873785	873844	58
874192	874249	874308	874366	874424	58
874772	874829	874888	874945	875003	58
875351	875409	875466	875524	875582	58
875929	875987	876045	876102	876160	58
876507	876564	876622	876679	876737	58
877083	877141	877199	877256	877314	58
877659	877717	877774	877832	877889	58
878234	878292	878349	878407	878464	57
878808	878866	878924	878981	879039	57
879382	879459	879497	879555	879612	57
879955	880013	880070	880127	880185	57
880527	880585	880642	880699	880756	57

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N	O	I	2	3	4
760	880814	880871	880928	880985	881042
761	881385	881442	881499	881556	881613
762	881955	882012	882069	882126	882183
763	882525	882581	882638	882695	882752
764	883093	883050	883207	883264	883321
765	883661	883718	883775	883832	883888
766	884229	884285	884342	884399	884455
767	884795	884852	884909	884965	885022
768	885361	885418	885474	885531	885587
769	885926	885983	886039	886096	886152
770	886491	886547	886604	886659	886716
771	887054	887111	887167	887223	887279
772	887617	887674	887720	887786	887842
773	888179	888236	888292	888348	888404
774	888741	888797	888853	888909	888965
775	889302	889358	889414	889469	889523
776	889862	889918	889974	890029	890086
777	890421	890477	890533	890589	890645
778	890979	891035	891091	891147	891203
779	891537	891593	891649	891705	891760
780	892095	892150	892206	892262	892317
781	892651	892707	892762	892818	892873
782	893207	893262	893318	893373	893429
783	893762	893817	893873	893928	893984
784	894316	894371	894427	894482	894538
785	894869	894925	894980	895036	895091
786	895423	895478	895533	895588	895644
787	895975	896029	896085	896140	896195
788	896526	896581	896636	896692	896747
789	897077	897132	897184	897242	897297

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5	6	7	8	9	D
881099	881156	881213	881271	881328	57
881669	881727	881784	881841	881898	57
882239	882297	882354	882411	882468	57
882809	882866	882923	882979	883037	57
883377	883434	883491	883548	883605	57
883945	884002	884059	884115	884172	57
884512	884569	884625	884682	884739	57
885078	885135	885192	885248	885305	57
885644	885700	885757	885813	885869	57
886209	886265	886321	886378	886434	56
886773	886829	886885	886941	886998	56
887336	887392	887449	887505	887561	56
887898	887955	888011	888067	888123	56
888460	888516	888573	888629	888685	56
889021	889077	889134	889189	889246	56
889582	889638	889694	889749	889806	56
890141	890197	890253	890309	890365	56
890700	890756	890812	890868	890924	56
891259	891314	891370	891426	891482	56
891816	891872	891928	891983	892039	56
892373	892429	892484	892539	892595	56
892929	892985	893040	893096	893151	56
893484	893539	893595	893651	893706	56
894039	894094	894149	894205	894261	55
894593	894648	894704	894759	894814	55
895146	895201	895257	895312	895367	55
895699	895754	895809	895864	895919	55
896251	896306	896361	896416	896471	55
896802	896857	896912	896967	897022	55
897352	897407	897462	897517	897572	55

The Table of Logarithms.

N	O	I	2	3	4
790	897627	897682	897737	897792	897847
791	898176	898231	898286	898341	898396
792	898725	898780	898835	898889	898944
793	899273	899328	899383	899437	899492
794	899821	899875	899929	899985	900039
795	900367	900422	900476	900531	900586
796	900913	900968	901022	901077	901131
797	901458	901513	901567	901622	901676
798	902003	902057	902112	902166	902221
799	902547	902601	902655	902709	902764
800	903089	903144	903199	903253	903307
801	903633	903687	903741	903795	903849
802	904174	904229	904283	904337	904391
803	904716	904769	904824	904878	904932
804	905256	905310	905364	905418	905472
805	905796	905849	905904	905958	906012
806	906335	906389	906443	906497	906551
807	906874	906927	907981	907035	907089
808	907411	907465	907519	907573	907626
809	907949	908002	908056	908109	908163
810	908485	908539	908592	908646	908699
811	909021	909074	909128	909181	909235
812	909556	909609	909663	909716	909769
813	910091	910144	910197	910251	910304
814	910624	910678	910731	910784	910838
815	911158	911211	911263	911317	911371
816	911690	911743	911797	911849	911903
817	912222	912275	912323	912381	912435
818	912753	912806	912859	912913	912966
819	913284	913337	913380	913443	913496

The Table of Logarithms.

5	6	7	8	9	D
897902	897957	898012	898067	898122	55
898451	898506	898561	898615	898670	55
898999	899054	899109	899164	899218	55
899547	899602	899656	899711	899766	55
900094	900149	900203	900258	900312	55
900640	900695	900749	900804	900859	55
901186	901240	901295	901349	901404	55
901731	901785	901839	901894	901948	54
902275	902329	902384	902438	902492	54
902818	902873	902927	902981	903036	54
903361	903416	903469	903524	903578	54
903904	903956	904012	904066	904120	54
904445	904499	904553	904607	904661	54
904986	905039	905094	905148	905202	54
905526	905580	905634	905688	905742	54
906066	906119	906173	906227	906281	54
906604	906658	906712	906766	906819	54
907143	907196	907250	907304	907358	54
907680	907734	907787	907841	907895	54
908217	908270	908324	908378	908431	54
908753	908807	908860	908914	908967	54
909289	909341	909396	909449	909503	54
909823	909877	909930	909984	910037	53
910358	910411	910464	910518	910571	53
910891	910944	910998	911051	911104	53
911424	911477	911530	911584	911637	53
911956	912009	912063	912116	912169	53
912488	912541	912594	912647	912700	53
913019	913072	913125	913178	913231	53
913549	913602	913655	913708	913761	53

The Table of Logarithms.

N	O	I	2	3	4
820	913814	913867	913919	913973	914026
821	914343	914396	914449	914502	914555
822	914872	914925	914977	915030	915083
823	915399	915453	915505	915558	915611
824	915927	915979	916033	916085	916138
825	916454	916507	916559	916612	916664
826	916980	917033	917085	917138	917190
827	917506	917558	917611	917663	917716
828	918030	918083	918135	918188	918240
829	918555	918607	918659	918712	918764
830	919078	919130	919183	919235	919287
831	919601	919653	919706	919758	919810
832	920123	920176	920228	920279	920332
833	920645	920697	920749	920801	920853
834	921166	921218	921270	921322	921374
835	921686	921738	921790	921842	921894
836	922206	922258	922310	922362	922414
837	922725	922777	922829	922881	922933
838	623244	923296	923348	923399	923451
839	923762	923814	923865	923917	923969
840	924279	924331	924383	924434	924486
841	924796	924848	924899	924951	925003
842	925312	925364	925415	925461	925518
843	925828	925879	925931	925982	926034
844	926342	626394	926445	926497	926548
845	926857	926908	926959	927011	927062
846	927370	927422	927473	927524	927576
847	927883	927935	927986	928037	928088
848	928396	928447	928498	928549	928601
849	928908	928959	929009	929061	929112

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5	6	7	8	9	D
914079	914122	914184	914237	914290	53
914608	914660	914713	914766	914819	53
915136	915189	915241	915294	915347	53
915664	915716	915769	915822	915875	53
916191	916243	916296	916349	916401	53
916717	916769	916822	916875	916927	53
917243	917295	917348	917400	917453	53
917768	917820	917873	917925	917978	52
918293	918345	918397	918449	918502	52
918816	9188 9	918921	918973	919026	22
919339	919392	919444	919496	919549	52
919862	919914	919967	920019	920071	52
920384	920436	920489	920541	920593	52
920906	920958	921009	921062	921114	52
921426	921478	921530	921582	921634	52
921946	921998	922050	922102	922154	52
922466	922518	922569	922622	922674	52
922985	923037	923089	923140	923192	52
923503	923555	923607	923658	923710	52
924021	924072	924124	924176	924228	52
924538	924589	924641	924693	924744	52
925054	925106	925157	925209	925261	52
925569	925621	925672	925725	925776	52
926085	926137	926188	926239	926291	51
926599	926651	926702	926754	926805	51
927114	927165	927216	927268	927319	51
927627	927678	927729	927781	927832	51
928139	928191	928242	928293	928345	51
928652	928703	928754	928805	928857	51
929163	929215	929266	929317	929368	51

The Table of Logarithms.

N	O	I	2	3	4
850	929419	929470	929521	929572	929623
851	929929	929981	930032	930083	930134
852	930439	930491	930542	930592	930643
853	930949	930999	931051	931102	931153
854	931458	931509	931559	931610	931661
855	931966	932017	932068	932118	932169
856	932474	932524	932575	932626	932677
857	932981	933031	933082	933133	933183
858	933487	933538	933589	933639	933689
859	933993	934044	934094	934145	934195
860	934498	934549	934599	934649	934700
861	935003	935056	935104	935154	935205
862	935507	935558	935608	935658	935709
863	936011	936061	936111	936162	936212
864	936514	936564	936614	936665	936715
865	937016	937066	937117	937167	937217
866	937518	937568	937618	937668	937718
867	938019	938069	938119	938169	938219
868	938519	938569	938619	938669	938719
869	939019	939069	939119	939169	939219
870	939519	939569	939619	939669	939719
871	940018	940068	940118	940168	940218
872	940516	940566	940616	940666	940716
873	941014	941064	941114	941163	941213
874	941511	941561	941611	941660	941710
875	942008	942058	942107	942157	942207
876	942504	942554	942603	942653	942702
877	942999	943049	943099	943148	943198
878	943495	943544	943594	943643	943692
879	943989	944038	944088	944137	944186

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5	6	7	8	9	D
925674	929725	929776	929827	929879	51
930185	930236	930287	930338	930389	51
930694	930745	930796	930847	930898	51
931204	931254	931305	931356	931407	51
931712	931763	931814	931865	931915	51
932220	932271	932322	932372	932423	51
932727	932778	932829	932879	932930	51
933234	933284	933335	933386	933437	51
933740	933791	933841	933892	933943	51
934246	934296	934347	934397	934448	51
934751	934801	934852	934902	934953	50
935255	935306	935356	935406	935457	50
935759	935809	935859	935910	935960	50
936262	936313	936363	936413	936463	50
936765	936815	936865	936916	936966	50
937267	637317	937367	937418	937468	50
937769	937819	937869	937919	937969	50
938269	938319	938369	938419	938469	50
938769	938819	938869	938919	938969	50
939269	939319	939369	939419	939469	50
939769	939819	939869	939918	939968	50
940267	940317	940367	940417	940467	50
940765	940815	940865	940915	940964	50
941263	941313	941362	941412	941462	50
941759	941809	941859	941909	941958	50
942256	942306	942355	942405	942455	50
942752	942801	942851	942901	942950	50
943247	943297	943346	943396	943445	49
943742	943791	943841	943890	943939	49
944236	944285	944335	944384	944433	49

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N	O	I	2	3	4
880	944483	944532	944581	944631	944680
881	944976	945025	945074	945124	945173
882	945468	945518	945567	945616	945665
883	945961	946009	946059	946108	946157
884	946452	946501	946551	946599	946649
885	946943	946992	947041	947090	947139
886	947434	947483	947532	947581	947629
887	947924	947973	948022	948070	948119
888	948413	948462	948511	948559	948609
889	948902	948951	948999	949048	949097
890	949390	949439	949488	949536	949585
891	949878	949926	949975	950024	950073
892	950365	950414	950462	950511	950559
893	950851	950900	950949	950997	951046
894	951338	951386	951435	951483	951532
895	951822	951872	951920	951969	952017
896	952308	952356	952405	952453	952502
897	952792	952841	952889	952938	952986
898	953276	953325	953373	953421	953469
899	953759	953808	953856	953905	953953
900	954243	954292	954339	954387	954435
901	954725	954773	954821	954869	954918
902	955207	955255	955303	955351	955399
903	955688	955736	955784	955832	955880
904	956168	956216	956265	956313	956361
905	956649	956697	956745	956793	956840
906	957128	957176	957224	957272	957319
907	957607	957655	957703	957751	957799
908	958086	958134	958181	958229	958277
909	958564	958612	958659	958707	958755

The Table of Logarithms.

5	6	7	8	9	D
944729	944779	944828	944877	944927	49
945222	945272	945321	945370	945419	49
945715	945764	945813	945862	945912	49
946207	946256	946305	946354	946403	49
946698	946747	946796	946845	946894	49
947189	947232	947287	947336	947385	49
947679	947728	947777	947826	947875	49
948168	948217	948266	948315	948364	49
948657	948706	948755	948804	948853	49
949146	949195	949244	949292	949341	49
949633	949683	949731	949780	949829	49
950121	950170	950219	950267	950316	49
950608	950657	950706	950754	950803	49
951095	951143	951192	951240	951289	49
951580	951629	951677	951729	951775	49
952066	952114	952163	952210	952259	49
952550	952599	952647	952696	952744	48
953034	953083	953131	953179	953228	48
953518	953566	953615	953663	953711	48
954001	954049	954099	654146	954194	48
954484	954532	654580	954628	954677	48
954966	955014	955062	955110	955158	48
955447	955495	955543	955592	955639	48
955928	955976	956024	956075	956120	48
956409	956457	956505	956533	956601	48
956888	956936	956984	957032	957080	48
657368	957416	957464	957512	957559	48
957847	957894	957942	957900	958038	48
958325	958373	958421	658468	658516	48
958803	958850	958898	958946	958994	48

The Table of Logarithms.

N	O	I	2	3	4
910	959041	959089	959137	959185	959231
911	959518	959566	959614	959661	959709
912	959995	960042	960090	960138	960185
913	960471	960518	960566	960613	960661
914	960946	960994	961041	961089	961136
915	961421	961469	961516	961563	961611
916	961895	961943	961990	962038	962085
917	962369	962417	962464	962511	962559
918	962842	962886	962937	962985	963032
919	963315	963363	963410	963457	963504
920	963788	963835	963882	963929	963977
921	964259	964307	964354	964401	964448
922	964731	964778	964825	964872	964919
923	965202	965249	965296	965343	965389
924	965672	965719	965766	965813	965859
925	966142	966189	966239	966283	966329
926	966611	966658	966705	966752	966799
927	967079	967127	967173	967220	967267
928	967548	967595	967642	967688	967735
929	968016	968062	968109	968156	968202
930	968483	968529	968576	968623	968669
931	968949	968996	969043	969089	969136
932	969416	969463	969509	969556	969602
933	969882	969928	969975	970021	970068
934	970347	970393	970439	970486	970533
935	970812	970858	970904	970951	970997
936	971286	971322	971369	971415	971461
937	971739	971786	971832	971879	971925
938	972203	972249	972295	972342	972388
939	972666	972712	972758	972804	972851

The Table of Logarithms.

5	6	7	8	9	D
959279	959328	959375	959423	959471	48
959757	959804	959852	959899	959947	48
960233	960280	960328	960376	960423	48
960709	960756	960804	960851	960899	48
961184	961231	961279	961326	961374	47
961658	961706	961753	961801	961848	47
962132	962179	962227	962275	962322	47
962606	962653	962701	962748	962795	47
963079	963126	963174	963221	963268	47
963552	963599	963646	963693	963741	47
964024	964071	964118	964165	964212	47
964495	964542	964589	964637	964684	47
964966	965013	965061	965108	965155	47
965437	965484	965531	965578	965624	47
965906	965954	966001	966048	966095	47
966376	966423	966470	966517	966564	47
966845	966892	966939	966986	967033	47
967314	967361	967408	967454	967501	47
967782	967829	967875	967922	967969	47
968249	968296	968343	968389	968436	47
968716	968763	968809	968856	968902	47
969183	969229	969276	969323	969369	47
969649	969695	969741	969789	969835	47
970114	970161	970207	970254	970300	47
970579	970626	970672	970719	970765	46
971044	971090	971137	971183	971229	46
971508	971554	971601	971647	971693	46
971971	972018	972064	972110	972157	46
972434	972481	972527	972573	972619	46
972897	972943	972989	973035	973082	46

The Table of Logarithms.

N	O	I	2	3	4
940	973128	973174	973220	973266	973313
941	973589	973636	973682	973728	973774
942	974050	974097	974143	974189	974235
943	974512	974558	974604	974649	974695
944	974972	975018	975064	975109	975156
945	975432	975478	975524	975569	975616
946	975891	975937	975983	976029	976075
947	976349	976396	976442	976488	976533
948	976808	976854	976899	976946	976992
949	977266	977312	977358	977403	977449
950	977724	977769	977815	977861	977906
951	978181	978226	978272	978317	978363
952	978637	978683	978728	978774	978819
953	979093	979138	979184	979229	979275
954	979548	979594	979639	979685	979730
955	980003	980049	980094	980139	980185
956	980458	980503	980549	980594	980639
957	980912	980957	981003	981048	981093
958	981366	981411	981456	981501	981547
959	981819	981864	981909	981954	981999
960	982271	982316	982362	982407	982452
961	982723	982769	982814	982859	982904
962	983175	983220	983265	983310	983356
963	983626	983671	983716	983762	983807
964	984077	984122	984167	984212	984257
965	984527	984572	984617	984662	984707
966	984977	985022	985067	985112	985157
967	985426	985471	985516	985561	985606
968	985875	985920	985965	986009	986055
969	986324	986369	986413	986458	986504

The Table of Logarithms.

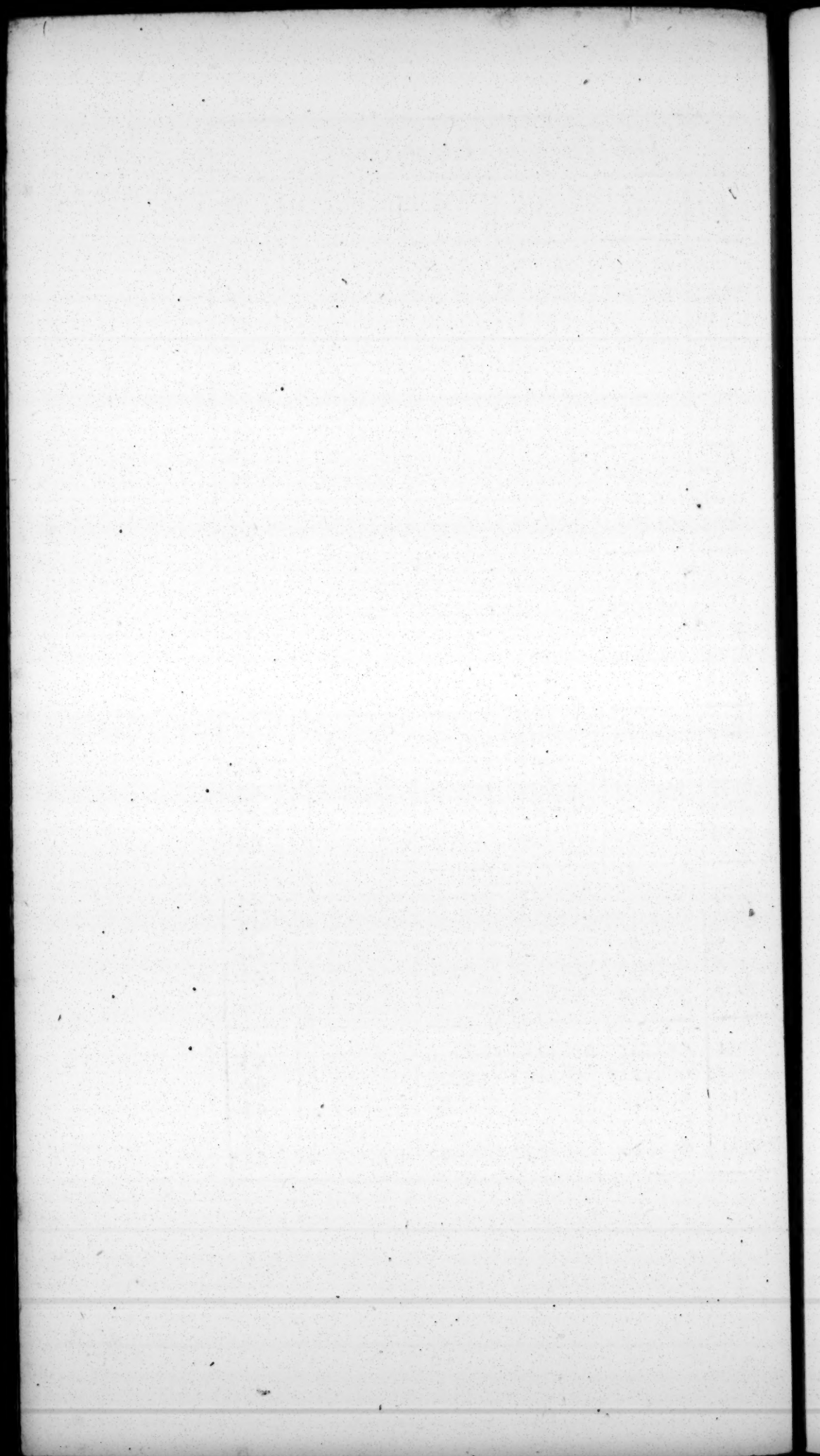
5	6	7	8	9	D
973359	973405	973451	973497	973543	46
973820	973866	973913	973959	974005	46
974281	974327	974374	974419	974466	46
974742	974788	974834	974819	974926	46
975202	975248	975294	975339	975386	46
975662	975707	975753	975799	975845	46
976121	976167	976212	976258	976304	46
976579	976625	976671	976717	976763	46
977037	977083	977129	977175	977220	46
977495	977541	977586	977632	977678	46
977952	977998	978042	978089	978135	46
978409	978454	978500	978546	978591	46
978865	978911	978956	979002	979047	46
979321	979366	979412	979457	979503	46
979776	979821	979867	979912	979958	46
980231	980276	980322	980367	980412	45
980685	980730	980776	980821	980867	45
981139	981184	981229	981275	981320	45
981592	981637	981683	981728	981773	45
982045	982090	982135	982181	982226	45
982497	982543	982588	982633	982678	45
982949	982994	983039	983085	983129	45
983401	983446	983490	983536	983581	45
983852	983897	983942	983987	984032	45
984302	984347	984392	984437	984482	45
984752	984797	984842	984887	984932	45
985202	985247	985292	985337	985382	45
985651	985696	985741	985786	985830	45
986099	986144	986189	986234	986279	45
986548	986593	986637	986682	986727	45

The Table of Logarithms.

N	O	I	2	3	4
970	986772	986817	986861	986906	986951
971	987219	987264	987309	987353	987398
972	987666	987711	987756	987800	987845
973	988113	988157	988202	988247	988291
974	988559	988604	988748	988693	988737
975	989005	989049	989094	989138	989183
976	989449	989494	989539	989584	989628
977	989895	989939	989983	990028	990072
978	990339	990383	990428	990472	990516
979	990783	990827	990871	990916	990960
980	991226	991270	991315	991359	991403
981	991669	991713	991758	991802	991846
982	992111	992156	992199	992244	992288
983	992554	992598	992642	992686	992730
984	992995	993039	993083	993127	993172
985	993436	993480	993524	993568	993613
986	993877	993921	993965	994009	994053
987	994317	994361	994405	994449	994493
988	994756	994801	994845	994889	994933
989	995196	995240	995284	995328	895372
990	995635	995679	995723	995764	995811
991	996074	996117	996161	996205	996249
992	996512	996555	996599	996643	996687
993	996949	996993	997037	997080	997124
994	997386	997430	997474	997517	997561
995	997823	997867	997910	997954	997998
996	998259	998303	998347	998390	998434
997	998695	998739	998783	998826	998869
998	999133	999174	999218	999261	999305
999	999565	999609	999652	999696	999739

The Table of Logarithms.

5	6	7	8	9	D
986956	987040	987085	987129	987175	45
987443	987488	987532	987577	987622	45
987889	987934	987979	988024	988068	45
988326	988381	988425	988469	988514	45
988782	988826	988871	988916	988960	45
989227	989272	989316	989361	989405	45
989672	989717	989761	989806	989850	44
990117	990161	990206	990150	990294	44
990561	990605	990649	990694	990738	44
991004	991049	991093	991137	991182	44
991448	991492	991536	991580	991625	44
991890	991935	991979	992023	992067	44
992333	992377	992421	992465	992509	44
992774	992819	992863	992907	992951	44
993216	993259	993304	993348	993392	44
993657	993701	993745	993789	993833	44
994097	994141	994185	994229	994273	44
994537	994581	994625	994669	994713	44
994977	995021	995065	995108	995152	44
995416	995459	995504	995547	995591	44
995854	995898	995942	995986	996029	44
996293	996337	996380	996424	996468	44
996731	996774	996818	996862	996906	44
997168	997212	997255	997299	997343	44
997605	997648	997692	997736	997779	44
998041	998085	998129	998170	998216	44
998477	998521	998564	998608	998652	44
998913	998956	998999	999043	999087	44
999348	999392	999435	999479	999522	44
999783	999826	999869	999913	999957	43



A

TABLE

OF

Proportional Parts.

D	1	2	3	4	5	6	7	8	9
43	4	8	12	17	21	25	30	34	38
44	4	8	13	17	22	26	30	35	39
45	4	9	13	18	22	27	31	36	40
46	4	9	13	18	23	27	32	36	41
47	4	9	14	18	23	28	32	37	42
48	4	9	14	19	24	28	33	38	43
49	4	9	14	19	24	29	34	39	44
50	5	10	15	20	25	30	35	40	45
51	5	10	15	20	25	30	35	40	45
52	5	10	15	20	26	31	36	41	46
53	5	10	15	21	26	31	37	42	47
54	5	10	16	21	27	32	37	43	48
55	5	11	16	22	27	33	38	44	49
56	5	11	16	22	28	33	39	44	50
57	5	11	17	22	28	34	39	45	51
58	5	11	17	23	29	34	40	46	52
59	5	11	17	23	29	35	41	47	53
60	6	12	18	24	30	36	42	48	54
61	6	12	18	24	30	36	42	48	54
62	6	12	18	24	31	37	43	49	55

E e

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
63	6	12	18	25	31	37	44	50	56
64	6	12	19	25	32	38	44	51	57
65	6	13	19	26	32	39	45	52	58
66	6	13	19	26	33	39	46	52	59
67	6	13	20	26	33	40	46	53	60
68	6	13	20	27	34	40	47	54	61
69	6	13	20	27	34	41	48	55	62
70	7	14	21	28	35	42	49	56	63
71	7	14	21	28	35	42	49	56	63
72	7	14	21	28	36	43	50	57	64
73	7	14	21	29	36	43	51	58	65
74	7	14	22	29	37	44	51	59	66
75	7	15	22	30	37	45	52	60	67
76	7	15	22	30	38	45	53	60	68
77	7	15	23	30	38	46	53	61	69
78	7	15	23	31	39	46	54	62	70
79	7	15	23	31	39	47	55	63	71
80	8	16	24	32	40	48	56	64	72
81	8	16	24	32	40	48	56	64	72
82	8	16	24	32	41	49	57	65	73
83	8	16	24	33	41	49	58	66	74
84	8	16	25	33	42	50	58	67	75
85	8	17	25	34	42	51	59	68	76
86	8	17	25	34	43	51	60	68	77
87	8	17	26	34	43	52	60	69	78
88	8	17	26	35	44	52	61	70	79
89	8	17	26	35	44	53	62	71	80
90	9	18	27	36	45	54	63	72	81
91	9	18	27	36	45	54	63	72	81
92	9	18	27	36	46	55	64	73	82

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
93	9	18	27	37	46	55	65	74	83
94	9	18	28	37	47	56	65	75	84
95	9	19	28	38	47	57	66	76	85
96	9	19	28	38	48	57	67	76	86
97	9	19	29	38	48	58	67	77	87
98	9	19	29	39	49	58	68	78	88
99	9	19	29	39	49	59	69	79	89
100	10	20	30	40	50	60	70	80	90
101	10	20	30	40	50	60	70	80	90
102	10	20	30	40	51	61	71	81	91
103	10	20	30	41	51	61	72	82	92
104	10	20	31	41	52	62	72	83	93
105	10	21	31	42	52	63	73	84	94
106	10	21	31	42	53	63	74	84	95
107	10	21	32	42	53	64	74	85	96
108	10	21	32	43	54	64	75	86	97
109	10	21	32	43	54	65	76	87	98
110	11	22	33	44	55	66	77	88	99
111	11	22	33	44	55	66	77	88	99
112	11	22	33	44	56	67	78	89	100
113	11	22	33	45	57	67	78	90	101
114	11	22	34	45	57	68	79	91	102
115	11	23	34	46	57	69	80	92	103
116	11	23	34	46	58	69	81	92	104
117	11	23	35	46	58	70	81	93	105
118	11	23	35	47	59	70	82	94	106
119	11	23	35	47	59	71	83	95	107
120	12	24	36	48	60	72	84	96	108
121	12	24	36	48	60	72	84	96	108
122	12	24	36	48	61	73	85	97	109

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
123	12	24	36	48	61	73	86	98	110
124	12	24	37	49	62	74	86	99	111
125	12	25	37	50	62	75	87	100	112
126	12	25	37	50	63	75	88	100	113
127	12	25	38	50	63	76	88	101	114
128	12	25	38	51	64	76	89	102	115
129	12	25	38	51	64	77	90	103	116
130	13	26	39	52	65	78	91	104	117
131	13	26	39	52	65	78	91	104	117
132	13	26	39	52	66	79	92	105	118
133	13	26	39	53	66	79	93	106	119
134	13	26	40	53	67	80	93	107	120
135	13	27	40	54	67	81	94	108	121
136	13	27	40	54	68	81	95	108	122
137	13	27	41	54	68	82	95	109	123
138	13	27	41	55	69	82	96	110	124
139	13	27	41	55	69	83	97	111	125
140	14	28	42	56	70	84	98	112	126
141	14	28	42	56	70	84	99	112	126
142	14	28	42	56	71	85	99	113	127
143	14	28	42	57	71	85	100	114	128
144	14	28	43	57	72	86	100	115	129
145	14	28	43	58	72	87	101	116	130
146	14	29	43	58	73	87	102	116	131
147	14	29	44	58	73	88	102	117	132
148	14	29	44	59	74	88	103	118	133
149	14	29	44	59	74	89	104	119	134
150	15	30	45	60	75	90	105	120	135
151	15	30	45	60	75	90	105	120	135
152	15	30	45	60	76	91	106	121	136

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
153	15	30	45	60	76	91	107	122	137
154	15	30	46	61	77	92	107	123	138
155	15	31	46	62	77	93	108	124	139
156	15	31	46	62	78	93	109	124	140
157	15	31	47	62	78	94	109	125	141
158	15	31	47	63	79	94	110	126	142
159	15	31	47	63	79	95	111	127	143
160	16	32	48	64	80	96	112	128	144
161	16	32	48	64	80	96	112	128	144
162	16	32	48	64	81	97	113	129	145
163	16	32	48	65	82	98	114	130	146
164	16	32	49	66	82	98	114	131	147
165	16	33	49	66	82	99	115	132	148
166	16	33	49	66	83	99	116	132	149
167	16	33	50	66	83	100	116	133	150
168	16	33	50	67	84	100	117	134	151
169	16	33	50	67	84	101	118	135	152
170	17	34	51	68	85	102	119	136	153
171	17	34	51	68	85	102	119	136	153
172	17	34	51	68	86	103	120	137	154
173	17	34	51	69	86	103	121	138	155
174	17	34	52	69	87	104	121	139	156
175	17	34	52	70	87	105	122	140	157
176	17	35	52	70	88	105	123	140	158
177	17	35	53	70	88	106	123	141	159
178	17	35	53	71	89	106	124	142	160
179	17	35	53	71	89	107	125	143	161
180	18	36	54	72	90	108	126	144	162
181	18	36	54	72	90	108	126	144	162
182	18	36	54	72	91	109	127	145	163

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
183	18	36	54	73	91	109	128	146	164
184	18	36	55	73	92	110	128	147	165
185	18	37	55	74	92	111	129	148	166
186	18	37	55	74	93	111	130	148	167
187	18	37	56	74	93	112	130	149	168
188	18	37	56	75	94	112	131	150	169
189	18	37	56	75	94	113	132	151	170
190	19	38	57	76	95	114	133	152	171
191	19	38	57	76	95	114	133	152	171
192	19	38	57	76	96	115	134	153	172
193	19	38	57	77	96	115	135	154	173
194	19	38	58	77	97	116	135	155	174
195	19	39	58	78	97	117	136	156	175
196	19	39	59	78	98	117	136	156	176
197	19	39	59	78	98	118	137	157	177
198	19	39	59	79	99	118	138	158	178
199	19	39	59	79	99	119	139	159	179
200	20	40	60	80	100	120	140	160	180
201	20	40	60	80	100	120	140	160	180
202	20	40	60	80	101	121	141	161	181
203	20	40	60	81	101	121	142	162	182
204	20	40	61	81	102	122	142	163	183
205	20	41	61	82	102	123	143	164	184
206	20	41	61	82	103	123	144	164	185
207	20	41	62	82	103	124	144	165	186
208	20	41	62	83	104	124	145	166	187
209	20	41	62	83	104	125	146	167	188
210	21	42	63	84	105	126	147	168	189
211	21	42	63	84	105	126	147	168	189
212	21	42	63	84	106	127	148	169	190

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
213	21	42	63	85	106	127	149	170	191
214	21	42	64	85	107	128	149	171	192
215	21	43	64	86	107	129	150	172	193
216	21	43	64	86	108	129	151	172	194
217	21	43	65	86	108	130	151	173	195
218	21	43	65	87	109	130	152	174	196
219	21	43	65	87	109	131	153	175	197
220	22	44	66	88	110	132	154	176	198
221	22	44	66	88	110	132	154	176	198
222	22	44	66	88	111	133	155	177	199
223	22	44	66	89	111	133	156	178	200
224	22	44	67	89	112	134	156	179	201
225	22	45	67	90	112	135	157	180	202
226	22	45	67	90	113	135	158	180	203
227	22	45	68	90	113	136	158	181	204
228	22	45	68	91	114	136	159	182	205
229	22	45	68	91	114	137	160	183	206
230	23	46	69	92	115	138	161	184	207
231	23	46	69	92	115	138	161	184	207
232	23	46	69	92	116	139	162	185	208
233	23	46	69	93	116	139	163	186	209
234	23	46	70	93	117	140	163	187	210
235	23	47	70	94	117	141	164	188	211
236	23	47	70	94	118	141	165	188	212
237	23	47	71	94	118	142	165	189	213
238	23	47	71	95	119	142	166	190	214
239	23	47	71	95	119	143	167	191	215
240	24	48	72	96	120	144	168	192	216
241	24	48	72	96	120	144	168	192	216
242	24	48	72	96	121	145	169	193	217

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
243	24	48	72	97	121	145	170	194	218
244	24	48	73	97	122	146	170	195	219
245	24	49	73	98	122	147	171	196	220
246	24	49	73	98	123	147	172	196	221
247	24	49	74	98	123	148	172	197	222
248	24	49	74	99	124	148	173	198	223
249	24	49	74	99	124	149	174	199	224
250	25	50	75	100	125	150	175	200	225
251	25	50	75	100	125	150	175	200	225
252	25	50	75	100	126	151	176	201	226
253	25	50	75	101	126	151	177	202	227
254	25	50	76	101	127	152	177	203	228
255	25	50	76	102	127	153	178	204	229
256	25	51	76	102	128	153	179	204	230
257	25	51	77	102	128	154	179	205	231
258	25	51	77	103	129	154	180	206	232
259	25	51	77	103	129	155	181	207	233
260	26	52	78	104	130	156	182	208	234
261	26	52	78	104	130	156	182	208	234
262	26	52	78	104	131	156	183	209	235
263	26	52	78	105	131	157	184	210	236
264	26	52	79	105	132	158	184	211	237
265	26	53	79	106	132	159	185	212	238
266	26	53	79	106	133	159	186	212	239
267	26	53	80	106	133	160	186	213	240
268	26	53	80	107	134	160	187	214	241
269	26	53	80	107	134	161	188	215	242
270	27	54	81	108	135	162	189	216	243
271	27	54	81	108	135	162	189	216	243
272	27	54	81	108	136	163	190	217	244

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
273	27	54	81	109	136	163	191	218	245
274	27	54	82	109	137	164	191	219	246
275	27	55	82	110	137	165	192	220	247
276	27	55	82	110	138	165	193	220	248
277	27	55	83	110	138	166	193	221	249
278	27	55	83	111	139	166	194	222	250
276	27	55	83	111	139	167	195	223	251
280	28	56	84	112	140	168	196	224	252
281	28	56	84	112	140	168	196	224	252
282	28	56	84	112	141	169	197	225	253
283	28	56	84	113	141	169	198	226	254
284	28	56	85	113	142	170	198	227	255
285	28	57	85	114	142	171	199	228	256
286	28	57	85	114	143	171	200	228	257
287	28	57	86	114	143	172	200	229	258
288	28	57	86	115	144	172	201	230	259
289	28	57	86	115	144	173	202	231	260
290	29	58	87	116	145	174	203	232	261
291	29	58	87	116	145	174	203	232	261
292	29	58	87	116	146	175	204	233	262
293	29	58	87	117	146	175	205	234	263
294	29	58	88	117	147	176	205	235	264
295	29	59	88	118	147	177	206	236	265
296	29	59	88	118	148	177	207	236	266
297	29	59	88	118	148	178	207	237	267
298	29	59	89	119	149	178	208	238	268
299	29	59	89	119	149	179	209	239	269
300	30	60	90	120	150	180	210	240	270
301	30	60	90	120	150	180	210	240	270
302	30	60	90	120	151	181	211	241	271

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
303	30	60	90	121	151	181	212	242	272
304	30	60	91	121	152	182	212	243	273
305	30	61	91	122	152	183	213	244	274
306	30	61	91	122	153	183	214	244	275
307	30	61	92	122	153	184	214	245	276
308	30	61	92	123	154	184	215	246	277
309	30	61	92	123	154	185	216	247	278
310	31	62	93	124	155	186	217	248	279
311	31	62	93	124	155	186	217	248	279
312	31	62	93	124	156	187	218	249	280
313	31	62	93	125	156	187	219	250	281
314	31	62	94	125	157	188	219	251	282
315	31	63	94	126	157	189	220	252	283
316	31	63	94	126	158	189	221	252	284
317	31	63	95	126	158	190	221	253	285
318	31	63	95	127	159	190	222	254	286
319	31	63	95	127	159	191	223	255	287
320	32	64	96	128	160	192	224	256	288
321	32	64	96	128	160	192	224	256	288
322	32	64	96	128	161	193	225	257	289
323	32	64	96	129	161	193	226	258	290
324	32	64	97	129	162	194	226	259	291
325	32	65	97	130	162	195	227	260	292
326	32	65	97	130	163	195	228	260	293
327	32	65	98	130	163	196	228	261	294
328	32	65	98	131	163	196	229	262	295
329	32	65	98	131	164	197	230	263	296
330	33	66	99	132	165	198	231	264	297
331	33	66	99	132	165	198	231	264	297
332	33	66	99	132	166	199	232	265	298

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
333	33	66	99	133	166	199	233	266	299
334	33	66	100	133	167	200	233	267	300
335	33	67	100	134	167	201	234	268	301
336	33	67	100	134	168	201	235	268	302
337	33	67	101	134	168	202	235	269	303
338	33	67	101	135	169	202	236	270	304
339	33	67	101	135	169	203	237	271	305
340	34	68	102	136	170	204	238	272	306
341	34	68	102	136	170	204	238	272	306
342	34	68	102	136	171	205	239	273	307
343	34	68	102	137	171	205	240	274	308
344	34	68	103	137	172	206	240	275	309
345	34	69	103	138	172	207	241	276	310
346	34	69	103	138	173	207	242	276	311
347	34	69	104	138	173	208	242	277	312
348	34	69	104	139	174	208	243	278	313
349	34	69	104	139	174	209	244	279	314
350	35	70	105	140	175	210	245	280	315
351	35	70	105	140	175	210	245	280	315
352	35	70	105	140	176	211	246	281	316
353	35	70	105	141	176	211	247	282	317
354	35	70	106	141	177	212	247	283	318
355	35	71	106	142	177	213	248	284	319
356	35	71	106	142	178	213	249	284	320
357	35	71	107	142	178	214	249	285	321
358	35	71	107	143	179	214	250	286	322
359	35	71	107	143	179	215	251	287	323
360	36	72	108	144	180	216	252	288	324
361	36	72	108	144	180	216	252	288	324
362	36	72	108	144	181	217	253	289	325

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
363	36	72	108	145	181	217	254	290	326
364	36	72	109	145	182	218	254	291	327
365	36	73	109	146	182	219	255	292	328
366	36	73	109	146	182	219	256	292	329
367	36	73	110	146	183	220	256	293	330
368	36	73	110	147	184	220	257	294	331
369	36	73	110	147	184	221	258	295	332
370	37	74	111	148	185	222	259	296	333
371	37	74	111	148	185	222	259	296	333
372	37	74	111	148	186	223	260	297	334
373	37	74	111	149	186	223	261	298	335
374	37	74	112	149	187	224	261	299	336
375	37	75	112	150	187	225	262	300	337
376	37	75	112	150	188	225	263	300	338
377	37	75	113	150	188	226	263	301	339
378	37	75	113	151	189	226	264	302	340
379	37	75	113	151	189	227	265	303	341
380	38	76	114	152	190	228	266	304	342
381	38	76	114	152	190	228	266	304	342
382	38	76	114	152	191	229	267	305	343
383	38	76	114	153	191	229	268	306	344
384	38	76	115	153	192	230	268	307	345
385	38	77	115	154	192	231	269	308	346
386	38	77	115	154	193	231	270	308	347
387	38	77	116	154	193	232	270	309	348
388	38	77	116	155	194	232	271	310	349
389	38	77	116	155	194	233	272	311	350
390	39	78	117	156	195	233	273	312	351
391	39	78	117	156	195	233	273	312	351
392	39	78	117	156	196	234	274	313	352

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
393	39	78	117	157	196	235	275	314	353
394	39	78	118	157	197	236	275	315	354
395	39	79	118	158	197	237	276	316	355
396	39	79	118	158	198	237	277	316	356
397	39	79	119	158	198	238	277	317	357
398	39	79	119	159	199	238	278	318	358
399	39	79	119	159	199	239	279	319	359
400	40	80	120	160	200	240	280	320	360
401	40	80	120	160	200	240	280	320	360
402	40	80	120	160	201	241	281	321	361
403	40	80	120	161	201	241	282	322	362
404	40	80	121	161	202	242	282	323	363
405	40	81	121	162	202	243	283	324	364
406	40	81	121	162	203	243	284	324	365
407	40	81	122	162	203	244	284	325	366
408	40	81	122	163	204	244	285	326	367
409	40	81	122	163	204	245	286	327	368
410	41	82	123	164	205	246	287	328	369
411	41	82	123	164	205	246	287	328	369
412	41	82	123	164	206	247	288	329	370
413	41	82	123	165	206	247	289	330	371
414	41	82	124	165	207	248	289	331	372
415	41	83	124	166	207	249	290	332	373
416	41	83	124	166	208	249	291	332	374
417	41	83	125	166	208	250	291	333	375
418	41	83	125	167	209	250	292	334	376
419	41	83	125	167	209	251	293	335	377
420	42	84	126	168	210	252	294	336	378
421	42	84	126	168	210	252	294	336	378
422	42	84	126	168	211	253	295	337	379

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
423	42	84	126	169	211	253	296	338	380
424	42	84	127	169	212	254	296	339	381
425	42	85	127	170	212	255	297	340	382
426	42	85	127	170	213	255	298	340	383
427	42	85	128	170	213	256	298	341	384
428	42	85	128	171	214	256	299	342	385
429	42	85	128	171	214	257	300	343	386
430	43	86	129	172	215	258	301	344	387
431	43	86	129	172	215	258	301	344	387
432	43	86	129	172	216	259	302	345	388
433	43	86	129	173	216	259	302	346	389
434	43	86	130	173	217	260	304	347	390
435	43	87	130	174	217	261	304	348	391

A Ta-

A
TABLE
OF
ARTIFICIAL SINES
AND
TANGENTS
To every
DEGREE and MINUTE
OF THE
QUADRANT.

L O N D O N,

Printed by *J. Leake*, for W. FREEMAN,
at the *Bible* against the *Middle-Temple-*
Gate in *Fleetstreet*. MDCCVII.

Degree 0.

M	Sine		Tangent		
0	0.000000	0.000000	0.000000	<i>Infinita.</i>	60
1	6.463726	9.999999	6.463726	13.536274	59
2	6.764756	9.999999	6.764756	13.235244	58
3	6.940847	9.999999	6.940847	13.059153	57
4	7.065786	9.999999	7.065786	12.934214	56
5	7.162656	9.999999	7.162656	12.837304	55
6	7.241877	9.999999	7.241878	12.758122	54
7	7.308824	9.999999	7.308825	12.691175	53
8	7.366816	9.999999	7.366817	12.633183	52
9	7.417568	9.999999	7.417970	12.582030	51
10	7.463726	9.999998	7.463727	12.536273	50
11	7.505118	9.999998	7.505120	12.494880	49
12	7.542906	9.999997	7.542909	12.457091	48
13	7.577668	9.999997	7.577272	12.422328	47
14	7.609853	9.999996	7.609857	12.390143	46
15	7.639816	9.999996	7.639826	12.360180	45
16	7.667844	9.999995	7.667849	12.332151	44
17	7.694173	9.999995	7.694179	12.305821	43
18	7.718977	9.999994	7.719003	12.281997	42
19	7.742477	9.999993	7.742484	12.257516	41
20	7.764754	9.999993	7.764761	12.235239	40
21	7.785943	9.999992	7.785951	12.214049	39
22	7.806146	9.999991	7.806145	12.193845	38
23	7.825451	9.999990	7.826460	12.174540	37
24	7.843034	9.999989	7.843944	12.156056	36
25	7.861662	9.999989	7.861674	12.138326	35
26	7.878695	9.999988	7.878708	12.121292	34
27	7.895085	9.999987	7.895099	12.104901	33
28	7.910879	9.999986	7.910894	12.089106	32
29	7.926119	9.999985	7.926124	12.073886	31
30	7.940842	9.999983	7.940858	12.059142	30
		Sine		Tangent	

Degree 89.

Degree 0.

M	Sine		Tangent		
30	7.940842	9.999983	7.940858	12.059142	30
31	7.955082	9.999982	7.955100	12.044900	29
32	7.968870	9.999981	7.968889	12.031111	28
33	7.982233	9.999980	7.982253	12.017747	27
34	7.995198	9.999978	7.995215	12.004781	26
35	8.007787	9.999978	8.007810	11.992191	25
36	8.020021	9.999976	8.020044	11.979956	24
37	8.031919	9.999975	8.031945	11.968055	23
38	8.043601	9.999973	8.043527	11.956473	22
39	8.054781	9.999972	8.054809	11.945181	21
40	8.065776	9.999971	8.065806	11.934194	20
41	8.076500	9.999969	8.076531	11.923469	19
42	8.086965	9.999968	8.086997	11.913003	18
43	8.097183	9.999966	8.097217	11.902783	17
44	8.107167	9.999964	8.107203	11.892797	16
45	8.116926	9.999963	8.116963	11.883037	15
46	8.126471	9.999961	8.126510	11.873490	14
47	8.135810	9.999959	8.135851	11.864149	13
48	8.144953	9.999958	8.144996	11.855004	12
49	8.153907	9.999956	8.153952	11.846048	11
50	8.162681	9.999954	8.162737	11.837273	10
51	8.171280	9.999952	8.171328	11.828672	9
52	8.179713	9.999950	8.179763	11.820237	8
53	8.187985	9.999948	8.188056	11.811964	7
54	8.196102	9.999946	8.196156	11.803844	6
55	8.204070	9.999944	8.204126	11.795674	5
56	8.211895	9.999942	8.211953	11.788047	4
57	8.219581	9.999940	8.219641	11.780359	3
58	8.227134	9.999938	8.227195	11.772805	2
59	8.234557	9.999936	8.234621	11.765379	1
60	8.241855	9.999934	8.241921	11.758079	0
	Sine		Tangent		M

Degree 89.

F f

Degree 1.

M	Sine		Tangent		
0	8.241855	9.999934	8.241921	11.758079	60
1	8.249033	9.999932	8.249102	11.750898	59
2	8.256094	9.999929	8.256165	11.743835	58
3	8.263042	9.999927	8.263115	11.736885	57
4	8.269831	9.999925	8.269956	11.730044	56
5	8.276614	9.999922	8.276691	11.723309	55
6	8.283243	9.999920	8.283323	11.716677	54
7	8.289773	9.999918	8.289856	11.710144	53
8	8.296207	9.999915	8.296292	11.703708	52
9	8.302546	9.999913	8.302634	11.697366	51
10	8.308794	9.999910	8.308884	11.691116	50
11	8.314954	9.999907	8.315046	11.684954	49
12	8.321027	9.999905	8.321122	11.678878	48
13	8.327016	9.999902	8.327114	11.672886	47
14	8.332924	9.999899	8.333025	11.666975	46
15	8.338753	9.999897	8.338856	11.661144	45
16	8.344504	9.999894	8.344610	11.655390	44
17	8.350180	9.999891	8.350289	11.649711	43
18	8.355783	9.999888	8.355895	11.644105	42
19	8.361315	9.999885	8.361430	11.638570	41
20	8.366777	9.999882	8.366895	11.633105	40
21	8.372171	9.999879	8.372292	11.627708	39
22	8.377499	9.999876	8.377622	11.622378	38
23	8.382762	9.999873	8.382889	11.617111	37
24	8.387962	9.999870	8.388092	11.611908	36
25	8.393101	9.999867	8.393234	11.606766	35
26	8.398179	9.999864	8.398315	11.601685	34
27	8.403199	9.999861	8.403338	11.596662	33
28	8.408161	9.999858	8.408304	11.591696	32
29	8.413068	9.999854	8.413213	11.586787	31
30	8.417919	9.999851	8.418068	11.581932	30
		Sine		Tangent	M

Degree 88.

Degree 1.

M	Sine		Tangent		
30	8.417919	9.999851	8.418068	11.581932	30
31	8.422717	9.999848	8.422869	11.577131	29
32	8.427462	9.999844	8.427618	11.572382	28
33	8.432156	9.999841	8.432315	11.567685	27
34	8.436800	9.999838	8.436962	11.563038	26
35	8.441394	9.999834	8.441560	11.558440	25
36	8.445941	9.999831	8.446110	11.553990	24
37	8.450440	9.999827	8.450613	11.549387	23
38	8.454893	9.999824	8.455070	11.544930	22
39	8.459301	9.999820	8.459481	11.540519	21
40	8.463665	9.999816	8.463849	11.536151	20
41	8.467985	9.999812	8.468172	11.531828	19
42	8.472263	9.999809	8.472454	11.527546	18
43	8.476498	9.999805	8.476693	11.523307	17
44	8.480693	9.999801	8.480892	11.519108	16
45	8.484848	9.999797	8.485050	11.514950	15
46	8.488963	9.999794	8.486170	11.510830	14
47	8.493040	9.999790	8.483250	11.506750	13
48	8.497078	9.999786	8.497293	11.502707	12
49	8.501080	9.999782	8.501298	11.498702	11
50	8.505045	9.999778	8.505267	11.494733	10
51	8.508974	9.999774	8.509200	11.490800	9
52	8.512867	9.999769	8.513098	11.486902	8
53	8.516726	9.999765	8.516961	11.483039	7
54	8.520551	9.999761	8.520790	11.479210	6
55	8.524343	9.999756	8.524586	11.475414	5
56	8.528102	9.999753	8.528349	11.471651	4
57	8.531828	9.999748	8.532080	11.467620	3
58	8.535523	9.999744	8.535779	11.464221	2
59	8.539186	9.999740	8.539447	11.460553	1
60	8.542819	9.999735	8.543084	11.456916	0
	Sine		Tangent		M

Degree 88.

Degree 2.

M	Sine		Tangent		
0	8.542819	9.999735	8.543084	11.456916	60
1	8.546422	9.999731	8.546691	11.453309	59
2	8.549995	9.999726	9.550268	11.449732	58
3	8.553558	9.999722	8.553817	11.446183	57
4	8.557054	9.999717	8.557336	11.442664	56
5	8.560540	9.999713	8.560827	11.439172	55
6	8.563999	9.999708	8.564291	11.435709	54
7	8.567431	9.999703	8.567727	11.432272	53
8	8.570836	9.999699	8.571137	11.428863	52
9	8.574214	9.999694	8.574520	11.425480	51
10	8.577566	9.999689	8.577877	11.422123	50
11	8.580892	9.999685	8.581208	11.418792	49
12	8.584193	9.999680	8.584514	11.415486	48
13	8.587469	9.999675	8.587795	11.412205	47
14	8.590721	9.999670	8.591051	11.408949	46
15	8.593948	9.999665	8.594283	11.405717	45
16	8.597152	9.999660	8.597492	11.402508	44
17	8.600332	9.999655	8.600677	11.399323	43
18	8.603488	9.999650	8.603838	11.396161	42
19	8.606622	9.999645	8.606978	11.393022	41
20	8.609734	9.999640	8.610094	11.389906	40
21	8.612823	9.999635	8.613189	11.386811	39
22	8.615891	9.999629	8.616262	11.383738	38
23	8.618937	9.999624	8.619313	11.380687	37
24	8.621967	9.999619	8.622343	11.377657	36
25	8.624965	9.999614	8.625352	11.374648	35
26	8.627948	9.999608	8.628340	11.371660	34
27	8.630911	9.999603	8.631308	11.368692	33
28	8.633854	9.999597	8.634456	11.365744	32
29	8.636776	9.999592	8.637184	11.362816	31
30	8.639679	9.999586	8.640093	11.359907	30
		Sine		Tangent	M

Degree 87.

Degree 2.

M	Sine		Tangent		
30	8.639679	9.999586	8.640093	11.350907	30
31	8.642563	9.999581	8.642982	11.357017	29
32	8.645428	9.999575	8.645853	11.354147	28
33	8.648274	9.999570	8.648704	11.351296	27
34	8.651102	9.999564	8.651538	11.348463	26
35	8.653911	9.999558	8.654352	11.345648	25
36	8.656702	9.999553	8.657149	11.342851	24
37	8.659475	9.999547	8.659928	11.340072	23
38	8.662230	9.999541	8.662689	11.337311	22
39	8.664968	9.999535	8.665433	11.334567	21
40	8.667689	9.999529	8.668160	11.331840	20
41	8.670393	9.999523	8.670869	11.329130	19
42	8.673080	9.999518	8.673563	11.326437	18
43	8.675751	9.999512	8.676239	11.323761	17
44	8.678405	9.999506	8.678899	11.321100	16
45	8.681043	9.999499	8.681544	11.318456	15
46	8.683665	9.999493	8.684172	11.315828	14
47	8.686272	9.999487	8.686784	11.313216	13
48	8.688892	9.999481	8.689381	11.310619	12
49	8.691438	9.999475	8.691963	11.308037	11
50	8.693998	9.999469	8.694529	11.305471	10
51	8.696543	9.999462	8.697081	11.302919	9
52	8.699073	9.999456	8.699617	11.300383	8
53	8.701589	9.999450	8.702139	11.297861	7
54	8.704090	9.956443	8.704646	11.295354	6
55	8.706576	9.999437	8.707130	11.292860	5
56	8.709049	9.999431	8.709618	11.290381	4
57	8.711507	9.999424	8.712083	11.287917	3
58	8.713952	9.999418	8.714534	11.285466	2
59	8.716383	9.999411	8.716972	11.283028	1
60	8.718800	9.999404	8.719396	11.280604	0
	Sine			Tangent	M

Degree 87.

F f 3

Degree 3.

M	Sine		Tangent		
0	8.718800	9.999404	8.719356	11.280604	60
1	8.721204	9.999398	8.721806	11.278194	59
2	8.723595	9.999391	8.724254	11.275796	58
3	8.725972	9.999384	8.726588	11.273412	57
4	8.728336	9.999378	8.728959	11.271041	56
5	8.730688	9.999371	8.731317	11.268683	55
6	8.733027	9.999364	8.733663	11.266337	54
7	8.735354	9.999357	8.735956	11.264004	53
8	8.737667	9.999350	8.738317	11.261682	52
9	8.739969	9.999343	8.740626	11.259374	51
10	8.742259	9.999336	8.742922	11.257078	50
11	8.744536	9.999329	8.745007	11.254793	49
12	8.746801	9.999322	8.747479	11.252521	48
13	8.745955	9.999315	8.749740	11.250240	47
14	8.751297	9.999308	8.751989	11.248011	46
15	8.753528	9.999301	8.754227	11.245773	45
16	8.755747	9.999294	8.756453	11.243542	44
17	8.757955	9.999286	8.758668	11.241332	43
18	8.760151	9.999279	8.760872	11.239128	42
19	8.762337	9.999272	8.763065	11.236935	41
20	8.764511	9.999265	8.765246	11.234754	40
21	8.766675	9.999257	8.767417	11.232583	39
22	8.768828	9.999250	8.769578	11.230422	38
23	8.770970	9.999242	8.771727	11.228273	37
24	8.773101	9.999235	8.773866	11.229134	36
25	8.775223	9.999227	8.775995	11.224005	35
26	8.777333	9.999220	8.778114	11.221886	34
27	8.779434	9.999212	8.783222	11.219778	33
28	8.781524	9.999204	8.782320	11.217680	32
29	8.783605	9.999197	8.784404	11.215592	31
30	8.785675	9.999189	8.786486	11.213514	30
	Sine		Tangent		M

Degree 86.

Degree 3.

M	Sine		Tangent		
30	8.785675	9.999189	8.786486	11.213514	30
31	8.787736	9.999181	8.788554	11.211446	29
32	8.789787	9.999174	8.790613	11.209387	28
33	8.791828	9.999166	8.792662	11.207338	27
34	8.793859	9.999158	8.794701	11.205299	26
35	8.795881	9.999150	8.796731	11.203269	25
36	8.797894	9.999142	8.798752	11.201243	24
37	8.799897	9.999134	8.800763	11.199237	23
38	8.801891	9.999126	8.802765	11.197235	22
39	8.803876	9.999118	8.804758	11.195242	21
40	8.805852	9.999110	8.806742	11.193258	20
41	8.807819	9.999102	8.808717	11.191283	19
42	8.809777	9.999094	8.812683	11.189317	18
43	8.811726	9.999086	8.812641	11.187359	17
44	8.813667	9.999077	8.814589	11.185411	16
45	8.815598	9.999069	8.816529	11.183471	15
46	8.817522	9.999061	8.818461	11.181539	14
47	8.819436	9.999052	8.820384	11.179616	13
48	8.821342	9.999044	8.822298	11.177702	12
49	8.823240	9.999036	8.824205	11.175795	11
50	8.825130	9.999027	8.826103	11.173897	10
51	8.827011	9.999019	8.827992	11.172008	9
52	8.828884	9.999010	8.829874	11.170126	8
53	8.830749	9.999002	8.831748	11.168252	7
54	8.832610	9.998993	8.833613	11.166387	6
55	8.834456	9.998984	8.835471	11.164529	5
56	8.836297	9.998976	8.837321	11.162679	4
57	8.838130	9.998967	8.839163	11.160837	3
58	8.839956	9.998958	8.840998	11.159002	2
59	8.841774	9.998940	8.842825	11.157175	1
60	8.843585	9.998941	8.844644	11.155356	0
		Sine		Tangent	M

Degree 86.

Degree 4.

M	Sine		Tangent		M
0	8.843584	9.998941	8.844644	11.155356	60
1	8.845387	9.998931	8.846455	11.153545	59
2	8.847183	9.998923	8.848240	11.151740	58
3	8.848971	9.998914	8.850507	11.149943	57
4	8.850751	9.998905	8.851846	11.148154	56
5	8.852525	9.998896	8.853628	11.146372	55
6	8.854291	9.998887	8.855403	11.144597	54
7	8.856049	9.998878	8.857171	11.142829	53
8	8.857801	9.998869	8.858932	11.141068	52
9	8.859546	9.998860	8.860686	11.139314	51
10	8.861283	9.998851	8.862433	11.137567	50
11	8.863014	9.998841	8.864173	11.135827	49
12	8.864738	9.998832	8.865906	11.134094	48
13	8.866454	9.998823	8.867632	11.132368	47
14	8.868165	9.998813	8.869351	11.130649	46
15	8.869868	9.998804	8.871064	11.128936	45
16	8.871565	9.998795	8.872750	11.127230	44
17	8.873255	9.998785	8.874469	11.125531	43
18	8.874938	9.998776	8.876162	11.123838	42
19	8.876615	9.998766	8.877849	11.122151	41
20	8.878285	9.998757	8.879529	11.120471	40
21	8.879949	9.998747	8.881202	11.118798	39
22	8.881607	9.998738	8.882869	11.117131	38
23	8.883258	9.998728	8.884530	11.115470	37
24	8.884903	9.998718	8.886185	11.113815	36
25	8.886542	9.998708	8.887833	11.112167	35
26	8.888174	9.998699	8.889476	11.110524	34
27	8.889801	9.998689	8.891112	11.108888	33
28	8.891421	9.998679	8.892742	11.107258	32
29	8.893035	9.998669	8.894366	11.105634	31
30	8.894643	9.998659	8.845984	11.104016	30
	Sine		Tangent		M

Degree 85.

Degree 4.

M	Sine		Tangent		
30	8.894643	9.998659	8.895984	11.104016	30
31	8.896246	9.998649	8.897596	11.102404	29
32	8.897842	9.998639	8.899203	11.100797	28
33	8.899432	9.998629	8.900803	11.099197	27
34	8.901017	9.998619	8.902398	11.097602	26
35	8.902596	9.998609	8.903987	11.096013	25
36	8.904169	9.998599	8.905570	11.094430	24
37	8.905736	9.998589	8.907147	11.092853	23
38	8.907297	9.998577	8.908719	11.091281	22
39	8.908853	9.998568	8.910285	11.089715	21
40	8.910404	9.998558	8.911846	11.088154	20
41	8.911949	9.998548	8.913401	11.086599	19
42	8.913488	9.998537	8.914951	11.085049	18
43	8.915022	9.998527	8.916495	11.083505	17
44	8.916550	9.998516	8.918034	11.081966	16
45	8.918073	9.998506	8.919563	11.080432	15
46	8.919591	9.998495	8.921096	11.078904	14
47	8.921103	9.998485	8.922619	11.077381	13
48	8.922610	9.998474	8.924136	11.075864	12
49	8.924112	9.998464	8.925649	11.074351	11
50	8.925609	9.998453	8.927156	11.072844	10
51	8.927100	9.998442	8.928658	11.071342	9
52	8.928587	9.998431	8.930155	11.069845	8
53	8.930068	9.998421	8.931647	11.068353	7
54	8.931544	9.998410	8.933134	11.066866	6
55	8.933015	9.998399	8.934616	11.065384	5
56	8.934481	9.998388	8.936093	11.063907	4
57	8.935942	9.998377	8.937565	11.062435	3
58	8.937398	9.998366	8.939032	11.060968	2
59	8.938850	9.998355	8.940494	11.059506	1
60	8.940296	9.998344	8.941952	11.058048	0
	Sine		Tangent		M

Degree 4.

Degree 5.

M	Sine		Tangent		
0	8.940296	9.998344	8.941952	11.058048	60
1	8.941738	9.998333	8.943404	11.056596	59
2	8.943174	9.998322	8.944852	11.055148	58
3	8.944606	9.998311	8.946295	11.053705	57
4	8.946034	9.998300	8.947734	11.052266	56
5	8.957456	9.998289	8.949168	11.050832	55
6	8.958814	9.998277	8.950597	11.049403	54
7	8.950287	9.998266	8.952021	11.047979	53
8	8.951656	9.998255	8.953441	11.046559	52
9	8.953099	9.998243	8.954856	11.045144	51
10	8.954499	9.998232	8.956267	11.043733	50
11	8.955894	9.998220	8.957674	11.042326	49
12	8.957284	9.998209	8.959075	11.040925	48
13	8.958670	9.998197	8.960473	11.039527	47
14	8.960052	9.998186	8.961866	11.038134	46
15	8.961429	9.998174	8.963254	11.036746	45
16	8.962801	9.998163	8.964639	11.035361	44
17	8.964170	9.998151	8.966019	11.033981	43
18	8.965534	9.998139	8.967394	11.032606	42
19	8.966893	9.998128	8.968766	11.031234	41
20	8.968249	9.998106	8.970133	11.029867	40
21	8.969600	9.998104	8.971495	11.028505	39
22	8.970947	9.998092	8.972855	11.027145	38
23	8.972289	9.998080	8.974209	11.025791	37
24	8.973626	9.998068	8.975560	11.024440	36
25	0.974962	9.998056	8.976906	11.023094	35
26	8.976293	9.998044	8.978248	11.021752	34
27	8.977619	9.998032	8.979586	11.020414	33
28	8.978941	9.998020	8.980921	11.019079	32
29	8.980259	9.998008	8.982251	11.017749	31
30	8.981573	9.997996	8.983577	11.016423	30
		Sine		Tangent	M

Degree 84.

Degree 5.

M	Sine		Tangent		
30	8.981572	9.997996	8.983577	11.016423	30
31	8.982883	9.997984	8.984899	11.015101	29
32	8.984189	9.997971	8.986217	11.013783	28
33	8.985491	9.997959	8.987532	11.012468	27
34	8.986789	9.997947	8.988842	11.011158	26
35	8.988083	9.997935	8.990149	11.009851	25
36	8.989374	9.997922	8.991451	11.008549	24
37	8.990660	9.997910	8.992750	11.007250	23
38	8.991943	9.997897	8.994045	11.005955	22
39	8.993228	9.997885	8.995337	11.004663	21
40	8.994497	9.997873	8.996624	11.003376	20
41	8.995768	9.997860	8.997908	11.002092	19
42	8.997036	9.997847	8.999188	11.000812	18
43	8.998299	9.997835	9.000465	10.999535	17
44	8.999560	9.997822	9.001738	10.998262	16
45	9.000816	9.997809	9.003007	10.996993	15
46	9.002069	9.997797	9.004272	10.995728	14
47	9.003318	9.997784	9.005534	10.994466	13
48	9.004563	9.997771	9.006792	10.993208	12
49	9.005805	9.997758	9.008047	10.991953	11
50	9.007044	9.997730	9.009298	10.990708	10
51	9.008278	9.997732	9.010546	10.989454	9
52	9.009510	9.997719	9.011790	10.988210	8
53	9.010737	9.997706	9.013031	10.986969	7
54	9.011962	9.997693	9.014268	10.985732	6
55	9.013182	9.997680	9.015502	10.984498	5
56	9.014399	9.997667	9.016732	10.983268	4
57	9.015613	9.997654	9.017959	10.982041	3
58	9.016824	9.997641	9.019183	10.980817	2
59	9.018031	9.997628	9.020403	10.979597	1
60	9.019235	9.997612	9.021620	10.978380	0
	Sine		Tangent		

Degree 84.

Degree 6.

M	Sine		Tangent		
0	9.019235	9.997614	9.021620	10.978380	60
1	9.020435	9.997601	9.022834	10.977166	59
2	9.021632	9.997588	9.024044	10.975956	58
3	9.022825	9.997574	9.025251	10.974749	57
4	9.024016	9.997561	9.026455	10.973545	56
5	9.025203	9.997548	9.027655	10.972345	55
6	9.026386	9.997534	9.028852	10.971148	54
7	9.027567	9.997520	9.030046	10.969954	53
8	9.028744	9.997507	9.031237	10.968763	52
9	9.029918	9.997493	9.032425	10.967575	51
10	9.031089	9.997480	9.033609	10.966391	50
11	9.032257	9.997466	9.034791	10.965209	49
12	9.033421	9.997452	9.035969	10.964031	48
13	9.034582	9.997439	9.036144	10.962856	47
14	9.035741	9.997425	9.038316	10.961684	46
15	9.036896	9.997411	9.039485	10.960515	45
16	9.038048	9.997397	9.040651	10.959349	44
17	9.039197	9.997383	9.041813	10.958187	43
18	9.040342	9.997369	9.042973	10.957027	42
19	9.041485	9.997355	9.044130	10.955870	41
20	9.042625	9.997341	9.045284	10.954716	40
21	9.043762	9.997327	9.046434	10.953566	39
22	9.044895	9.997313	9.047582	10.952418	38
23	9.046026	9.997299	9.048727	10.951273	37
24	9.047154	9.997285	9.049869	10.950131	36
25	9.049279	9.997271	9.051008	10.948992	35
26	9.049400	9.997256	9.052144	10.947856	34
27	9.050519	9.997242	9.043277	10.946723	33
28	9.051635	9.997228	9.054408	10.945592	32
29	9.052749	9.997214	9.055535	10.944465	31
30	9.053859	9.997199	9.056640	10.943340	30
	Sine		Tangent		M

Degree 83.

Degree 6.

M	Sine		Tangent		
30	9.053850	9.997199	9.056640	10.943340	30
31	9.054966	9.997185	9.057781	10.942219	29
32	9.056071	9.997170	9.058900	10.941100	28
33	9.057172	9.997156	9.060016	10.939984	27
34	9.058271	9.997141	9.061130	10.938870	26
35	9.059267	9.997127	9.062240	10.937760	25
36	9.060460	9.997112	9.063348	10.936652	24
37	9.061551	9.997098	9.064453	10.935547	23
38	9.062638	9.997083	9.065556	10.934444	22
39	9.063723	9.997068	9.066655	10.933345	21
40	9.064806	9.997052	9.067752	10.932248	20
41	9.065885	9.997039	9.068847	10.931153	19
42	9.066962	9.997024	9.069938	10.930062	18
43	9.068036	9.997009	9.071027	10.928973	17
44	9.069107	9.996994	9.072113	10.927887	16
45	9.070176	9.996979	9.073197	10.926803	15
46	9.071242	9.996964	9.074278	10.925722	14
47	9.072306	9.996949	9.075356	10.924644	13
48	9.073366	9.996934	9.076432	10.923568	12
49	9.074424	9.996919	9.077505	10.922495	11
50	9.075480	9.996904	9.078576	10.921424	10
51	9.076533	9.996889	9.079644	10.920356	9
52	9.077583	9.996874	9.080710	10.919290	8
53	9.078631	9.996858	9.081773	10.918227	7
54	9.079676	9.996843	9.082833	10.917167	6
55	9.080719	9.996828	9.083891	10.916109	5
56	9.081759	9.996812	9.084947	10.915053	4
57	9.082797	9.996797	9.085999	10.914100	3
58	9.083832	9.996782	9.087050	10.912950	2
59	9.084864	9.996766	9.088098	10.911902	1
60	9.085894	9.996751	9.089144	10.910856	0
M	Sine		Tangent		M

Degree 83.

Degree 7.

M	Sine		Tangent		
0	9.085894	9.996751	9.089144	10.910856	60
1	9.086922	9.996735	9.090187	10.909813	59
2	9.087947	9.996720	9.091228	10.908772	58
3	9.088970	9.996704	9.092266	10.907734	57
4	9.089990	9.996688	9.093302	10.906698	56
5	9.091088	9.996673	9.094336	10.905664	55
6	9.092024	9.996657	9.095367	10.904633	54
7	9.093037	9.996641	9.096395	10.903604	53
8	9.094047	9.996625	9.097422	10.902578	52
9	9.095056	9.996610	9.098446	10.901554	51
10	9.096062	9.996594	9.099468	10.900532	50
11	9.097065	9.996578	9.100487	10.899513	49
12	9.098066	9.996562	9.101504	10.898496	48
13	9.099065	9.996546	9.102519	10.897481	47
14	9.100062	9.996530	9.103532	10.896468	46
15	9.101056	9.996514	9.104542	10.895458	45
16	9.102048	9.996498	9.105550	10.894450	44
17	9.103037	9.996482	9.106556	10.893444	43
18	9.104025	9.996465	9.107559	10.892441	42
19	9.105010	9.996449	9.108560	10.891440	41
20	9.105992	9.996433	9.109559	10.890441	40
21	9.106973	9.996417	9.110556	10.889444	39
22	9.107951	9.996400	9.111551	10.888449	38
23	9.108927	9.996384	9.112543	10.887457	37
24	9.109901	9.996368	9.113533	10.886467	36
25	9.110873	9.996351	9.114521	10.885478	35
26	9.111842	9.996335	9.115507	10.884493	34
27	9.112809	9.996318	9.116491	10.883509	33
28	9.113774	9.996302	9.117472	10.882528	32
29	9.114737	9.996285	9.118452	10.881548	31
30	9.115698	9.996269	9.119429	10.880571	30
		Sine		Tangent	M

Degree 82.

Degree 7.

M	Sine		Tangent		
30	9.115698	9.996269	9.119429	10.880571	30
31	9.116656	9.996252	9.120404	10.879596	29
32	9.117612	9.996235	9.121377	10.878623	28
33	9.118567	9.996218	9.122348	10.877652	27
34	9.119519	9.996202	9.123317	10.876683	26
35	9.120469	9.996185	9.124284	10.875716	25
36	9.121417	9.996168	9.125248	10.874751	24
37	9.122362	9.996151	9.126211	10.873789	23
38	9.123306	9.996134	9.127172	10.872828	22
39	9.124248	9.996117	9.128130	10.871870	21
40	9.125187	9.996100	9.129087	10.870913	20
41	9.126125	9.996083	9.130041	10.869959	19
42	9.127060	9.996066	9.130994	10.869006	18
43	9.127993	9.996049	9.131944	10.868056	17
44	9.128925	9.996032	9.132893	10.867107	16
45	9.129854	9.996015	9.133839	10.866161	15
46	9.130781	9.995998	9.134784	10.865216	14
47	9.131706	9.995980	9.135726	10.864274	13
48	9.132630	9.995963	9.136666	10.863334	12
49	9.133551	9.995946	9.137605	10.862395	11
50	9.134470	9.995928	9.138542	10.861458	10
51	9.135387	9.995911	9.139476	10.860524	9
52	9.136303	9.995894	9.140409	10.859591	8
53	9.137216	9.995876	9.141340	10.858660	7
54	9.138127	9.995859	9.142269	10.857731	6
55	9.139037	9.995841	9.143196	10.856804	5
56	9.139944	9.995825	9.144121	10.855879	4
57	9.140850	9.995806	9.145044	10.854956	3
58	9.141754	9.995788	9.145965	10.854035	2
59	9.142655	9.995770	9.146885	10.853115	1
60	9.142555	9.995753	9.147803	10.852197	0
	Sine		Tangent		

Degree 82.

Degree 8.

M	Sine		Tangent		
0	9.143555	9.995753	9.147803	10.852197	60
1	9.144453	9.995735	9.148718	10.851282	59
2	9.145349	9.995717	9.149632	10.850368	58
3	9.146243	9.995699	9.150544	10.849456	57
4	9.147136	9.995681	9.151454	10.848546	56
5	9.148026	9.995664	9.152363	10.847637	55
6	9.148919	9.995646	9.153269	10.846731	54
7	9.149831	9.995628	9.154174	10.845825	53
8	9.150686	9.995610	9.155077	10.844923	52
9	9.151569	9.995591	9.155978	10.844022	51
10	9.152451	9.995573	9.156877	10.843123	50
11	9.153330	9.995555	9.157775	10.842225	49
12	9.154208	9.995537	9.158671	10.841329	48
13	9.155082	9.995519	9.159565	10.840435	47
14	9.155957	9.995501	9.160457	10.839543	46
15	9.156830	9.995482	9.161347	10.838653	45
16	9.157700	9.995464	9.162236	10.837764	44
17	9.158569	9.995446	9.163123	10.836877	43
18	9.159436	9.995427	9.164008	10.835992	42
19	9.160301	9.995409	9.164892	10.835108	41
20	9.161164	9.995390	9.165773	10.834226	40
21	9.162025	9.995372	9.166654	10.833346	39
22	9.162885	9.995353	9.167532	10.832468	38
23	9.163743	9.995334	9.168409	10.831591	37
24	9.164600	9.995316	9.169284	10.830716	36
25	9.165454	9.995297	9.160157	10.829843	35
26	9.166307	9.995278	9.171029	10.828971	34
27	9.167158	9.995260	9.171899	10.828101	33
28	9.168008	9.995241	9.172767	10.827233	32
29	9.168856	9.995222	9.173634	10.826366	31
30	9.169702	9.995203	9.174499	10.825501	30
		Sine		Tangent	M

Degree 81.

Degree 8.

M	Sine		Tangent		
30	9.169702	9.995203	9.174499	10.825501	30
31	9.170546	9.995184	9.175362	10.824638	29
32	9.171389	9.995165	9.176224	10.823776	28
33	9.172230	9.995146	9.177084	10.822916	27
34	9.173070	9.995127	9.177942	10.822057	26
35	9.173908	9.995108	9.178799	10.821201	25
36	9.174744	9.995089	9.179655	10.820345	24
37	9.175578	9.995070	9.180508	10.819492	23
38	9.176411	9.995061	9.181360	10.818640	22
39	9.177242	9.995032	9.182211	10.817789	21
40	9.178072	9.995012	9.183060	10.816940	20
41	9.178900	9.994993	9.183907	10.816093	19
42	9.179726	9.994974	9.184752	10.815248	18
43	9.180551	9.994955	9.185597	10.814403	17
44	9.181374	9.994935	9.186439	10.813561	16
45	9.182196	9.994916	9.187280	10.812720	15
46	9.183016	9.994896	9.188120	10.811880	14
47	9.183834	9.994876	9.188957	10.811042	13
48	9.184651	9.994857	9.189794	10.810206	12
49	9.185466	9.994838	9.190629	10.809371	11
50	9.186280	9.994818	9.191462	10.808538	10
51	9.187092	9.994798	9.192294	10.807706	9
52	9.187903	9.994779	9.193124	10.806876	8
53	9.188712	9.994759	9.193953	10.806047	7
54	9.189519	9.994739	9.194780	10.805220	6
55	9.190325	9.994719	9.195606	10.804394	5
56	9.191130	9.994699	9.196440	10.803569	4
57	9.191933	9.994680	9.197253	10.802747	3
58	9.192734	9.994660	9.198067	10.801926	2
59	9.193534	9.994640	9.198894	10.801106	1
60	9.194332	9.994620	9.199712	10.800287	0
	Sine		Tangent		M

Degree 81.

G g

Degree 9.

M	Sine		Tangent		
0	9.194332	9.994620	9.199712	10.800287	60
1	9.195129	9.994600	9.200529	10.799470	59
2	9.195925	9.994580	9.201345	10.798655	58
3	9.196718	9.994560	9.202159	10.797841	57
4	9.197511	9.994540	9.202971	10.797029	56
5	9.198302	9.994519	9.203782	10.796218	55
6	9.199091	9.994499	9.204592	10.795408	54
7	9.199879	9.994479	9.205400	10.794600	53
8	9.200666	9.994459	9.206207	10.793793	52
9	9.201451	9.994438	9.207013	10.792897	51
10	9.202234	9.994418	9.207817	10.792183	50
11	9.203017	9.994398	9.208619	10.791381	49
12	9.203797	9.994377	9.209420	10.790580	48
13	9.204577	9.994357	9.210220	10.789780	47
14	9.205354	9.994336	9.211018	10.788982	46
15	9.206131	9.994316	9.211815	10.788185	45
16	9.206906	9.994295	9.212611	10.787385	44
17	9.207679	9.994274	9.213405	10.786595	43
18	9.208452	9.994254	9.214198	10.785802	42
19	9.209222	9.994233	9.214989	10.785011	41
20	9.209992	9.994212	9.215780	10.784220	40
21	9.210760	9.994191	9.216568	10.783432	39
22	9.211526	9.994171	9.217356	10.782644	38
23	9.212291	9.994150	9.218142	10.781858	37
24	9.213055	9.994129	9.218926	10.781070	36
25	9.213818	9.994108	9.219710	10.780294	35
26	9.214579	9.994087	9.220491	10.779508	34
27	9.215338	9.994066	9.221272	10.778728	33
28	9.216097	9.994044	9.222052	10.777948	32
29	9.216854	9.994024	9.222830	10.777170	31
30	9.217609	9.994003	9.223607	10.776393	30
		Sine		Tangent	M

Degree 80.

Degree 9.

M	Sine		Tangent		
30	9.217609	9.994003	9.223607	10.716393	30
31	9.218363	9.993982	9.224382	10.775618	29
32	9.219116	9.993960	9.225156	10.774844	28
33	9.219868	9.993939	9.225929	10.774071	27
34	9.220618	9.993918	9.226704	10.773300	26
35	9.221367	9.993897	9.227471	10.772529	25
36	9.222115	9.993875	9.228240	10.771760	24
37	9.222861	9.993854	9.229007	10.770993	23
38	9.223606	9.993832	9.229774	10.770226	22
39	9.224349	9.993811	9.230539	10.769461	21
40	9.225092	9.993789	9.231302	10.768698	20
41	9.225833	9.993768	9.232065	10.767935	19
42	9.226573	9.993746	9.232826	10.767174	18
43	9.227311	9.993725	9.233586	10.766414	17
44	9.228048	9.993703	9.234345	10.765655	16
45	9.228784	9.993681	9.235103	10.764897	15
46	9.229518	9.993660	9.235859	10.764141	14
47	9.230252	9.993638	9.236614	10.763386	13
48	9.230984	9.993616	9.237368	10.762632	12
49	9.231715	9.993594	9.238120	10.761880	11
50	9.232444	9.993572	9.238872	10.761128	10
51	9.233172	9.993550	9.239622	10.760378	9
52	9.233899	9.993528	9.240371	10.759629	8
53	9.234625	9.993506	9.241118	10.758882	7
54	9.235349	9.993484	9.241865	10.758135	6
55	9.236073	9.993462	9.242610	10.757390	5
56	9.236795	9.993440	9.243354	10.756646	4
57	9.237515	9.993418	9.244097	10.755903	3
58	9.238235	9.993396	9.244839	10.755161	2
59	9.238952	9.993374	9.245579	10.754421	1
60	9.239670	9.993351	9.246319	10.753681	0
	Sine		Tangent		M

Degree 80.

Degree 10.

M	Sine		Tangent		
0	9.239670	9.993351	9.246319	10.753681	60
1	9.240386	9.993329	9.247057	10.752943	59
2	9.241101	9.993307	9.247794	10.752206	58
3	9.241814	9.993284	9.248530	10.751470	57
4	9.242526	9.993262	9.249264	10.750736	56
5	9.243237	9.993240	9.249998	10.750002	55
6	9.243947	9.993217	9.250730	10.749270	54
7	9.244656	9.993195	9.251461	10.748539	53
8	9.245363	9.993172	9.252191	10.747809	52
9	9.246070	9.993149	9.252920	10.747080	51
10	9.246775	9.993127	9.253648	10.746352	50
11	9.247478	9.993104	9.254374	10.745626	49
12	9.248181	9.993081	9.255100	10.744900	48
13	9.248883	9.993059	9.255824	10.744176	47
14	9.249583	9.993036	9.256547	10.743453	46
15	9.250282	9.993013	9.257269	10.742731	45
16	9.250980	9.992990	9.257990	10.742010	44
17	9.251677	9.992967	9.258710	10.741290	43
18	9.252373	9.992944	9.259429	10.740571	42
19	9.253067	9.992921	9.260146	10.739854	41
20	9.253761	9.992898	9.260863	10.739137	40
21	9.254453	9.992875	9.261578	10.738422	39
22	9.255144	9.992852	9.262292	10.737708	38
23	9.255834	9.992829	9.263005	10.736995	37
24	9.256523	9.992806	9.263717	10.736283	36
25	9.257211	9.992783	9.264428	10.735572	35
26	9.257898	9.992759	9.265138	10.734862	34
27	9.258583	9.992736	9.265847	10.734153	33
28	9.259268	9.992713	9.266555	10.733445	32
29	9.259951	9.992690	9.267261	10.732739	31
30	9.260633	9.992666	9.267967	10.732033	30
		Sine		Tangent	M

Degree 79.

Degree 10.

M	Sine		Tangent		
30	9.260633	9.992666	9.267967	10.732033	30
31	9.261314	9.992643	9.268671	10.731329	29
32	9.261994	9.992619	9.269375	10.730625	28
33	9.262673	9.992596	9.270078	10.729923	27
34	9.263351	9.992572	9.271479	10.729221	26
35	9.264027	9.992549	9.271470	10.728521	25
36	9.264703	9.992525	9.272178	10.727822	24
37	9.265378	9.992501	9.272876	10.727124	23
38	9.266051	9.992478	9.273573	10.726427	22
39	9.266723	9.992454	9.274269	10.725731	21
40	9.267395	9.992430	9.274964	10.725036	20
41	9.268065	9.992406	9.275658	10.724342	19
42	9.268734	9.992382	9.276351	10.723649	18
43	9.269402	9.992362	9.277043	10.722957	17
44	9.270069	9.992335	9.277734	10.722267	16
45	9.270735	9.992311	0.278424	10.721576	15
46	9.271400	9.992287	9.279113	10.720887	14
47	9.272063	9.992263	9.279801	10.720199	13
48	9.272726	9.992239	9.280488	10.719512	12
49	9.273388	9.992214	9.281174	10.718826	11
50	9.274049	9.992190	9.281858	10.718142	10
51	9.274708	9.992166	9.282542	10.717458	9
52	9.275367	9.992142	9.283225	10.716775	8
53	9.276025	9.992118	9.283907	10.716093	7
54	9.276681	9.992093	9.284588	10.715412	6
55	9.277337	9.992069	9.285268	10.714732	5
56	9.277991	9.992045	9.285946	10.714053	4
57	9.278685	9.992020	9.286624	10.713376	3
58	9.279297	9.991996	9.287301	10.712699	2
59	9.279948	9.991971	9.287977	10.712023	1
60	9.280599	9.991947	9.288652	10.711348	0
	Sine		Tangent		M

Degree 79.

Degree 11.

M	Sine		Tangent		
0	9.280599	9.991947	9.288652	10.711348	60
1	9.281229	9.991922	9.289326	10.710674	59
2	9.281897	9.991897	9.289999	10.710001	58
3	9.282544	9.991873	9.290671	10.709329	57
4	9.283190	9.991848	9.291342	10.708658	56
5	9.283836	9.991823	9.292013	10.707987	55
6	9.284480	9.991799	9.292682	10.707318	54
7	9.285124	9.991774	9.293350	10.706650	53
8	9.285766	9.991749	6.294017	10.705983	52
9	9.286408	9.991724	9.294684	10.705316	51
10	9.287048	9.991699	9.295349	10.704651	50
11	9.287688	9.991674	9.296013	10.703987	49
12	9.288326	9.991649	9.296677	10.703323	48
13	9.288964	9.991624	9.297339	10.702661	47
14	9.289600	9.991599	9.298001	10.701999	46
15	9.290236	8.991574	9.298662	10.701338	45
16	9.290870	9.991549	9.299322	10.700678	44
17	9.291504	9.991524	9.299980	10.700020	43
18	9.292137	9.991498	9.300638	10.699362	42
19	9.292768	9.991473	9.301295	10.698705	41
20	9.293399	9.991448	9.301951	10.698049	40
21	9.294029	9.991422	9.302607	10.697393	39
22	9.294658	9.991397	9.303261	10.696739	38
23	9.295286	9.991372	9.303914	10.696086	37
24	9.295913	9.991346	9.304567	10.695433	36
25	9.296539	9.991321	9.305218	10.694782	35
26	9.297164	9.991295	9.305867	10.694131	34
27	9.297788	9.991270	9.306519	10.693481	33
28	9.298412	9.991244	9.307168	10.692832	32
29	9.299034	9.991218	9.307816	10.692184	31
30	9.299655	9.991193	9.308463	10.691537	30
		Sine		Tangent	M

Degree 78.

Degree 11.

M	Sine		Tangent		
30	9.296655	9.991193	9.308463	10.691537	30
31	9.300276	9.991167	9.309109	10.690891	29
32	9.300895	9.991141	9.309754	10.690246	28
33	9.301514	9.991115	9.310399	10.689601	27
34	9.302132	9.991090	9.311042	10.688958	26
35	9.302749	9.991064	9.311685	10.688315	25
36	9.303364	9.991038	9.312327	10.687673	24
37	9.303979	9.991012	9.312968	10.687032	23
38	9.304593	9.990986	9.313608	10.686392	22
39	9.305207	9.990960	9.314247	10.685753	21
40	9.305819	9.990934	9.314885	10.685115	20
41	9.306430	9.990908	9.315523	10.684477	19
42	9.307041	9.990882	9.316159	10.683841	18
43	9.307650	9.990855	9.316795	10.683205	17
44	9.308259	9.990829	9.317430	10.682570	16
45	9.308867	9.990803	9.318064	10.681936	15
46	9.309474	9.990777	9.318647	10.681303	14
47	9.310080	9.990750	9.319230	10.680670	13
48	9.310685	9.990724	9.319961	10.680039	12
49	9.311289	9.990697	9.320592	10.679408	11
50	9.311899	9.990671	9.321222	10.678778	10
51	9.312495	9.990645	9.321851	10.678149	9
52	9.313097	9.990618	9.322479	10.677521	8
53	9.313698	9.990591	9.323106	10.676894	7
54	9.314297	9.990565	9.323733	10.676267	6
55	9.314897	9.990538	9.324358	10.675642	5
56	9.315495	9.990512	9.324983	10.675017	4
57	9.316092	9.990485	9.325607	10.674393	3
58	9.316689	9.990458	9.326231	10.673769	2
59	9.317284	9.990431	9.326853	10.673147	1
60	9.317879	9.990404	9.327475	10.672525	0
		Sine		Tangent	M

Degree 78.

Degree 12.

M	Sine		Tangent		
0	9.317879	9.990404	9.327475	10.672525	60
1	9.318473	9.990377	9.328095	10.671905	59
2	9.319066	9.990351	9.328715	10.671285	58
3	9.319658	9.990324	9.329334	10.670666	57
4	9.320250	9.990297	9.329953	10.670047	56
5	9.320840	9.990270	9.330570	10.669430	55
6	9.321430	9.990242	9.331187	10.668813	54
7	9.322019	9.990215	9.331803	10.668197	53
8	9.322607	9.990188	9.332418	10.667582	52
9	9.323194	9.990161	9.333033	10.666967	51
10	9.323780	9.990134	9.333646	10.666354	50
11	9.324366	9.990107	9.334259	10.665741	49
12	9.324950	9.990079	9.334871	10.665129	48
13	9.325534	9.990052	9.335482	10.664518	47
14	9.326117	9.990025	9.336093	10.663907	46
15	9.326699	9.989997	9.336700	10.663298	45
16	9.327281	9.989970	9.337311	10.662689	44
17	9.327862	9.989942	9.337919	10.662081	43
18	9.328441	9.989915	9.338527	10.661473	42
19	9.329020	9.989887	9.339133	10.660867	41
20	9.329599	9.989860	9.339739	10.660261	40
21	9.330176	9.989832	9.340344	10.659656	39
22	9.330753	9.989804	9.340948	10.659052	38
23	9.331328	9.989777	9.341552	10.658448	37
24	9.331903	9.989749	9.342155	10.657845	36
25	9.332478	9.989721	9.342757	10.657243	35
26	9.333051	9.989693	9.343358	10.656642	34
27	9.333624	9.989665	9.343958	10.656042	33
28	9.334195	9.989637	9.344558	10.655442	32
29	9.334766	9.989609	9.345157	10.654843	31
30	9.335337	9.989581	9.345755	10.654245	30
		Sine		Tangent	M

Degree 77.

Degree 12.

M	Sine		Tangent		
30	9.335337	9.989581	9.345755	10.654245	30
31	9.335906	9.989553	9.346353	10.653647	29
32	9.336475	9.989525	9.346949	10.653051	28
33	9.337043	9.989597	9.347545	10.652455	27
34	9.337610	9.989469	9.348141	10.651859	26
35	9.338176	9.989441	9.348735	10.651265	25
36	9.338742	9.989413	9.349329	10.650671	24
37	9.339306	9.989384	9.349922	10.650078	23
38	9.339870	9.989356	9.350514	10.649486	22
39	9.340434	9.989328	9.351106	10.648894	21
40	9.340996	9.989299	9.351697	10.648303	20
41	9.341558	9.989271	9.352287	10.647713	19
42	9.342119	9.989243	9.352876	10.647124	18
43	9.342679	9.989214	9.353465	10.646535	17
44	9.343239	9.989186	9.354053	10.645947	16
45	9.343797	9.989157	9.354640	10.645360	15
46	9.344355	9.989128	9.355227	10.644773	14
47	9.344912	9.989100	9.355812	10.644187	13
48	9.345469	9.989071	9.356398	10.643602	12
49	9.346024	9.989042	9.356982	10.643018	11
50	9.346579	9.989014	9.357566	10.642434	10
51	9.347134	9.988985	9.358149	10.641851	9
52	9.347687	9.988956	9.358731	10.641269	8
53	9.348240	9.988927	9.359313	10.640687	7
54	9.348792	9.988898	9.359893	10.640107	6
55	9.349343	9.988869	9.360474	10.639526	5
56	9.349893	9.988840	9.361053	10.638947	4
57	9.350443	9.988811	9.361632	10.638368	3
58	9.350992	9.988782	9.362210	10.637790	2
59	9.351540	9.988754	9.362787	10.637213	1
60	9.352088	9.988724	9.363364	10.636636	0
	Sine		Tangent		M

Degree 77.

Degree 13.

M	Sine		Tangent		
0	9.352088	9.988724	9.363364	10.636636	60
1	9.352635	9.988695	9.363940	10.636060	59
2	9.353181	9.988666	9.364515	10.635485	58
3	9.353726	9.988636	9.365090	10.634910	57
4	9.354271	9.988607	9.365664	10.634336	56
5	9.354815	9.988578	9.366237	10.633763	55
6	9.355358	9.988548	9.366810	10.633190	54
7	9.355901	9.988519	9.367382	10.632618	53
8	9.356443	9.988489	9.367953	10.632047	52
9	9.356984	9.988460	9.368524	10.631476	51
10	9.357524	9.988430	9.369094	10.630906	50
11	9.358064	9.988401	9.369663	10.630337	49
12	9.358603	9.988371	9.370232	10.629768	48
13	9.359141	9.988341	9.370799	10.629201	47
14	9.359679	9.988312	9.371367	10.628633	46
15	9.350215	9.988282	9.371933	10.628067	45
16	9.360752	9.988252	9.372499	10.627501	44
17	9.361287	9.988223	9.373064	10.626936	43
18	9.361822	9.988193	9.373629	10.626371	42
19	9.362356	9.988163	9.374193	10.625807	41
20	9.362889	9.988133	9.374756	10.625244	40
21	9.363422	9.988103	9.375319	10.624681	39
22	9.363954	9.988073	9.375881	10.624119	38
23	9.364485	9.988043	9.376442	10.623558	37
24	9.365016	9.988013	9.377003	10.622997	36
25	9.365546	9.987983	9.377563	10.622437	35
26	9.366075	9.987953	9.378122	10.621878	34
27	9.366604	9.987922	9.378681	10.621319	33
28	9.367132	9.987892	9.379239	10.620761	32
29	9.367659	9.987862	9.379797	10.620203	31
30	9.368185	9.987832	9.380354	10.619646	30
	Sine		Tangent	M	

Degree 76.

Degree 13.

M	Sine		Tangent		
30	9.363185	9.987832	9.380354	10.619646	30
31	9.363711	9.987801	9.380910	10.619090	29
32	9.369236	9.987771	9.381466	10.618534	28
33	9.369761	9.987740	9.382021	10.617980	27
34	9.370285	9.987710	9.382575	10.617425	26
35	9.370808	9.987679	9.383129	10.616871	25
36	9.371330	9.987649	9.383682	10.616318	24
37	9.371852	9.987618	9.384234	10.615766	23
38	9.372373	9.987588	9.384786	10.615214	22
39	9.372894	9.987557	9.385337	10.614663	21
40	9.373414	9.987526	9.385888	10.614112	20
41	9.373933	9.987496	9.386438	10.613562	19
42	9.374452	9.987465	9.386987	10.613013	18
43	9.374970	9.987434	9.387536	10.612464	17
44	9.375487	9.987403	9.388084	10.611916	16
45	9.376003	9.987372	9.388631	10.611369	15
46	9.376519	9.987341	8.389178	10.610822	14
47	9.377035	9.987310	9.389724	10.610276	13
48	9.377549	9.987279	9.390270	10.609730	12
49	9.378063	9.987248	9.390815	10.609185	11
50	9.378577	9.987217	9.391360	10.608640	10
51	9.379089	9.987186	9.391907	10.608097	9
52	9.379601	8.987155	9.392467	10.607553	8
53	9.380113	9.987124	9.392989	10.607011	7
54	9.380624	9.987092	9.393531	10.606469	6
55	9.381134	9.987061	9.394074	10.605927	5
56	9.381643	9.987030	9.394614	10.605386	4
57	9.382152	9.986998	9.395154	10.604846	3
58	9.382661	9.986967	9.395694	10.604306	2
59	9.383168	9.986936	9.396233	10.603767	1
60	9.383675	9.986904	9.396770	10.603229	0
	Sine		Tangent		M

Degree 76.

Degree 14.

M	Sine		Tangent		
0	9.383675	9.986904	9.396771	10.603229	60
1	9.384181	9.986873	9.397309	10.602694	59
2	9.384687	9.986841	9.397846	10.602154	58
3	9.385192	9.986809	9.398383	10.601617	57
4	9.385697	9.986778	9.398919	10.601081	56
5	9.386201	9.986746	9.399455	10.600545	55
6	9.386704	9.986714	9.399990	10.600010	54
7	9.387207	9.986683	9.400524	10.599476	53
8	9.387709	9.986651	9.401058	10.598942	52
9	9.388210	9.986619	9.401591	10.598409	51
10	9.388711	9.986587	9.402124	10.597876	50
11	9.389211	9.986555	9.402656	10.597344	49
12	9.389711	9.986523	9.403187	10.596813	48
13	9.390210	9.986491	9.403718	10.596282	47
14	9.390708	9.986459	9.404249	10.595751	46
15	9.391206	9.986427	9.404778	10.595222	45
16	9.391703	9.986395	9.405306	10.594692	44
17	9.392199	9.986363	9.405836	10.594164	43
18	9.392695	9.986331	9.406364	10.593636	42
19	9.393190	9.986299	9.406892	10.593108	41
20	9.393685	9.986266	9.407419	10.592581	40
21	9.394179	9.986234	9.407945	10.592055	39
22	9.394673	9.986201	9.408471	10.591529	38
23	9.395166	9.986169	9.408996	10.591001	37
24	9.395654	9.986137	9.409521	10.590479	36
25	9.396150	9.986104	9.410045	10.589954	35
26	9.396641	9.986072	9.410569	10.589431	34
27	9.397131	9.986039	9.411097	10.588908	33
28	9.397621	9.986007	9.411615	10.588385	32
29	9.398111	9.985974	9.412137	10.587863	31
30	9.398600	9.985942	9.412658	10.587342	30
		Sine		Tangent	M

Degree 75.

Degree 14.

M	Sine		Tangent		
30	9.398600	9.985942	9.412658	10.587342	30
31	9.399087	9.985909	9.413179	10.586821	29
32	9.399575	9.985876	9.413699	10.586301	28
33	9.400062	9.985843	9.414219	10.585781	27
34	9.400549	9.985811	9.414738	10.585262	26
35	9.401035	9.985778	9.415257	10.584742	25
36	9.401520	9.985745	9.415775	10.584225	24
37	9.402005	9.985712	9.416293	10.583707	23
38	9.402489	9.985679	9.416810	10.583190	22
39	9.402972	9.985646	9.417326	10.582674	21
40	9.403455	9.985613	9.417842	10.582157	20
41	9.403938	9.985580	9.418357	10.581642	19
42	9.404420	9.985547	9.418873	10.581127	18
43	9.404901	9.985513	9.419387	10.580613	17
44	9.405382	9.985480	9.419901	10.580099	16
45	9.405862	9.985447	9.420415	10.579585	15
46	9.406341	9.985414	9.420927	10.579072	14
47	9.406820	9.985380	9.421440	10.578560	13
48	9.407299	9.985347	9.421951	10.578048	12
49	9.407776	9.985314	9.422463	10.577537	11
50	9.408254	9.985280	9.422973	10.577026	10
51	9.408731	9.985247	9.423484	10.576516	9
52	9.409207	9.985213	9.423993	10.576007	8
53	9.409682	9.985180	9.424503	10.575497	7
54	9.410157	9.985146	9.425011	10.574989	6
55	9.410632	9.985112	9.425518	10.574480	5
56	9.411106	9.985079	9.426027	10.573973	4
57	9.411579	9.985045	9.426534	10.573466	3
58	9.412052	9.985011	9.427041	10.572959	2
59	9.412524	9.984977	9.427547	10.572453	1
60	9.412996	9.984943	9.428052	10.571947	0
		Sine		Tangent	M

Degree 75.

Degree 15,

M	Sine		Tangent		
0	9.412956	9.984944	9.428052	10.571947	60
1	9.413467	9.984910	9.428557	10.571442	59
2	9.413938	9.984876	9.429067	10.570938	58
3	9.414408	9.984842	9.429566	10.570434	57
4	9.414878	9.984808	9.430070	10.569930	56
5	9.415347	9.984774	9.430573	10.569427	55
6	9.415815	9.984740	9.431075	10.568925	54
7	9.416283	9.984706	9.431577	10.568423	53
8	9.416850	9.984672	9.432079	10.567921	52
9	9.417217	9.984637	9.432580	10.567420	51
10	9.417684	9.984603	9.433080	10.566920	50
11	9.418149	9.984569	9.433580	10.566419	49
12	9.418615	9.984535	9.434080	10.565920	48
13	9.419079	9.984500	9.434579	10.565421	47
14	9.419544	9.984466	9.435078	10.564922	46
15	9.420007	9.984431	9.435576	10.564424	45
16	9.420470	9.984397	9.436073	10.563927	44
17	9.420933	9.984363	9.436570	10.563430	43
18	9.421395	9.984328	9.437067	10.562933	42
19	9.421856	9.984293	9.437563	10.562437	41
20	9.422317	9.984259	9.438059	10.561941	40
21	9.422778	9.984224	9.438554	10.561446	39
22	9.423238	9.984189	9.439048	10.560952	38
23	9.423697	9.984155	9.439543	10.560457	37
24	9.424156	9.984120	9.440036	10.559964	36
25	9.424615	9.984085	9.440529	10.559471	35
26	9.425072	9.984050	9.441022	10.558978	34
27	9.425530	9.984015	9.441514	10.558486	33
28	9.425987	9.983980	9.442006	10.557994	32
29	9.426443	9.983945	9.442497	10.557503	31
30	9.426899	9.983910	9.442988	10.557011	30
		Sine		Tangent	M

Degree 74.

Degree 15.

M	Sine		Tangent		
30	9.426899	9.983910	9.442988	10.557011	30
31	9.427354	9.983875	9.443479	10.556521	29
32	9.427809	9.983840	9.443968	10.556031	28
33	9.428264	9.983805	9.444458	10.555542	27
34	9.428717	9.983770	9.444947	10.555035	26
35	9.429170	9.983735	9.445435	10.554565	25
36	9.429623	9.983699	9.445923	10.554077	24
37	9.430075	9.983664	9.446411	10.553589	23
38	9.430507	9.983629	9.446898	10.553102	22
39	9.430978	9.983593	9.447384	10.552616	21
40	9.431429	9.983558	9.447870	10.552129	20
41	9.431879	9.983523	9.448356	10.551644	19
42	9.432328	9.983487	9.448841	10.551159	18
43	9.432778	9.983452	9.449326	10.550674	17
44	9.433206	9.983416	9.449810	10.550181	16
45	9.433674	9.983380	9.450294	10.559706	15
46	9.434122	9.983345	9.450777	10.549223	14
47	9.434569	9.983309	9.451260	10.548740	13
48	9.435016	9.983273	9.451743	10.548257	12
49	9.435462	9.983238	9.452225	10.547775	11
50	9.435918	9.983202	9.452706	10.547294	10
51	9.436353	9.983166	9.453187	10.546813	9
52	9.436798	9.983130	9.453668	10.546332	8
53	9.437242	9.983094	9.454148	10.545852	7
54	9.437686	9.983058	9.454629	10.545372	6
55	9.438129	9.983022	9.455107	10.544893	5
56	9.438572	9.982986	9.455586	10.544414	4
57	9.439014	9.982950	9.456064	10.543936	3
58	9.439456	9.982914	9.456542	10.543458	2
59	9.439897	9.982878	9.457019	10.542980	1
60	9.440338	9.982842	9.457496	10.542503	0
	Sine		Tangent		M

Degree 74.

Degree 16.

M	Sine		Tangent		
0	9.440338	9.982842	9.457496	10.542503	60
1	9.440778	9.982805	9.457973	10.542027	59
2	9.441218	9.982769	9.458449	10.541551	58
3	9.441658	9.982733	9.458925	10.541075	57
4	9.442096	9.982696	9.459400	10.540600	56
5	9.442535	9.982660	9.459875	10.540125	55
6	9.442973	9.982623	9.460349	10.539651	54
7	9.443416	9.982587	9.460829	10.539177	53
8	9.443848	9.982550	9.461297	10.538703	52
9	9.444284	9.982514	9.461770	10.538230	51
10	9.444720	9.982477	9.462242	10.537758	50
11	9.445155	9.982441	9.462714	10.537285	49
12	9.445590	9.982404	9.463186	10.536814	48
13	9.446025	9.982367	9.463658	10.536342	47
14	9.446459	9.982330	9.464129	10.535871	46
15	9.446893	9.982294	9.464599	10.535401	45
16	9.447326	9.982257	9.465069	10.534931	44
17	9.447759	9.982220	9.465539	10.534461	43
18	9.448191	9.982183	9.466008	10.533992	42
19	9.448623	9.982146	9.466476	10.533523	41
20	9.449054	9.982109	9.466945	10.533055	40
21	9.449485	9.982072	9.467413	10.532587	39
22	9.449915	9.982035	9.467880	10.532120	38
23	9.450345	9.981998	9.468347	10.531653	37
24	9.450775	9.981961	9.468814	16.531186	36
25	9.451203	9.981923	9.469280	10.530720	35
26	9.451632	9.981886	9.469746	10.530254	34
27	9.452060	9.981849	9.470211	10.529789	33
28	9.452488	9.981812	9.470676	10.529324	32
29	9.452915	9.981774	9.471141	10.528859	31
30	9.453342	9.981737	9.471605	10.528395	30
		Sine		Tangent	M

Degree 73.

Degree 16.

M	Sine		Tangent		
30	9.453342	9.981737	9.471605	10.528395	30
31	9.253768	9.981699	9.472068	10.527931	29
32	9.254194	9.981662	9.472532	10.527468	28
33	9.454619	9.981624	9.472995	10.527005	27
34	9.455044	9.981587	9.473457	10.526543	26
35	9.455469	9.981549	9.473919	10.526081	25
36	9.455892	9.981512	9.474381	10.525619	24
37	9.456316	9.981474	9.474842	10.525158	23
38	9.456739	9.981436	9.475303	10.524695	22
39	9.457162	9.981398	9.475763	19.524237	21
40	9.457584	9.981361	9.476223	10.523777	20
41	9.458006	9.981323	9.476683	10.523317	19
42	9.458427	9.981285	9.477142	10.522858	18
43	9.458848	9.981247	9.477601	10.522399	17
44	9.459268	9.981209	9.478059	10.521941	16
45	9.459684	9.981171	9.478517	10.521483	15
46	9.460108	9.781133	9.478975	10.521025	14
47	9.460527	9.981095	9.479432	10.520168	13
48	9.460946	9.981057	9.479886	10.520111	12
49	9.461364	9.981019	9.480345	10.519655	11
50	9.461782	9.980980	9.480801	10.519199	10
51	9.462199	9.980942	9.481257	10.518743	9
52	9.462616	9.980904	9.481712	10.518288	8
53	9.463032	9.980866	9.482167	10.517833	7
54	9.463448	9.980827	9.482621	10.517379	6
55	9.463864	9.980789	9.483075	10.516925	5
56	9.464279	9.980750	9.483528	10.516471	4
57	9.464694	9.980712	9.483982	10.516018	3
58	9.465108	9.980672	9.484434	10.515585	2
59	9.465522	9.980635	9.484887	10.515113	1
60	9.465935	9.980596	9.485339	10.514661	0
	Sine		Tangent		M

Degree 73.

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Degree 17.

M	Sine		Tangent		
0	9.465935	9.980596	9.485339	10.514661	60
1	9.466348	9.980558	9.485791	10.514209	59
2	9.466761	9.980519	9.486272	10.513758	58
3	9.467173	9.980480	9.486693	10.513307	57
4	9.467585	9.980441	9.487143	10.512857	56
5	9.467996	9.980403	9.487593	10.512407	55
6	9.468407	9.980364	9.488043	10.511957	54
7	9.468817	9.980325	9.488493	10.511507	53
8	9.469227	9.980286	9.488941	10.511059	52
9	9.469637	9.980247	9.489390	10.510610	51
10	9.460446	9.980208	9.489838	10.510162	50
11	9.470455	9.990169	9.490286	10.509714	49
12	9.471863	9.980130	9.490733	10.509267	48
13	9.471071	9.980091	9.491180	10.508820	47
14	9.471678	9.980052	9.491627	10.508373	46
15	9.472086	9.980012	9.492073	10.507928	45
16	9.472492	9.979973	9.492519	10.507481	44
17	9.472898	9.979934	9.492964	10.507035	43
18	9.473304	9.979894	9.493410	10.506590	42
19	9.473710	9.979855	9.493854	10.506145	41
20	9.474115	9.979816	9.494299	10.505701	40
21	9.474519	9.979776	9.494743	10.505257	39
22	9.474923	9.979737	9.495186	10.504813	38
23	9.475327	9.979697	9.495630	10.504370	37
24	9.475730	9.979658	9.496073	10.503928	36
25	9.476133	9.979618	9.496515	10.503485	35
26	9.476539	9.979578	9.496957	10.503043	34
27	9.476938	9.979539	9.497399	10.502601	33
28	9.477340	9.979499	9.497840	10.502160	32
29	9.477741	9.979459	9.498282	10.510718	31
30	9.478142	9.979419	9.498722	10.501278	30
		Sine		Tangent	M

Degree 72.

Degree 17,

M	Sine		Tangent		M
30	9.478142	9.979419	9.498722	10.501278	30
31	9.478542	9.979380	9.499163	10.500837	29
32	9.478942	9.979340	9.499602	10.500398	28
33	9.479342	9.979300	9.500042	10.499958	27
34	9.479741	9.979260	9.500481	10.499519	26
35	9.480140	9.979220	9.500920	10.499080	25
36	9.480538	9.979180	9.501359	10.498641	24
37	9.480936	9.979140	9.501797	10.498203	23
38	9.481334	9.979099	9.502234	10.497765	22
39	9.481731	9.979059	9.502672	10.497328	21
40	9.482128	9.979019	9.503109	10.496891	20
41	9.482525	9.978980	9.503546	10.496454	19
42	9.482921	9.978939	9.503982	10.496018	18
43	9.483316	9.978898	9.504418	10.495582	17
44	9.483711	9.978858	9.504854	10.495146	16
45	9.484106	9.978817	9.505289	10.494711	15
46	9.484501	9.978777	9.505724	10.494216	14
47	9.484895	9.978736	9.506158	10.493841	13
48	9.485289	9.978696	9.506593	10.493407	12
49	9.485682	9.978655	9.507026	10.492973	11
50	9.486075	9.978615	9.507459	10.492540	10
51	9.486467	9.978574	9.507892	10.492107	9
52	9.486859	9.978533	9.508326	10.491674	8
53	9.487251	9.978493	9.508759	10.491241	7
54	9.487642	9.978452	9.509181	10.490809	6
55	9.488033	9.978411	9.509622	10.490377	5
56	9.488424	9.978370	9.510044	10.489916	4
57	9.488814	9.978329	9.510486	10.489515	3
58	9.489204	9.978288	9.510916	10.489084	2
59	9.489593	9.978247	9.511346	10.488654	1
60	9.489982	9.978206	9.511776	10.488225	0
		Sine		Tangent	M

Degree 72.

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Degree 18.

M	Sine		Tangent		
0	9.489982	9.978206	9.511776	10.488224	60
1	9.490371	9.978165	9.512206	10.487794	59
2	9.490759	9.978124	9.512635	10.487365	58
3	9.491147	9.978083	9.513064	10.486936	57
4	9.491534	9.978042	9.513493	10.486507	56
5	9.491922	9.978000	9.513921	10.486079	55
6	9.492308	9.977959	9.514349	10.485651	54
7	9.492695	9.977918	9.514777	10.485223	53
8	9.493080	9.977877	9.515204	10.484796	52
9	9.493466	9.977835	9.515631	10.484369	51
10	9.493851	9.977794	9.516057	10.483942	50
11	9.494236	9.977752	9.516484	10.483516	49
12	9.494620	9.977711	9.516910	10.483090	48
13	9.495005	9.977669	9.517335	10.482665	47
14	9.495388	9.977628	9.517761	10.482239	46
15	9.495771	9.977586	9.518185	10.481814	45
16	9.496154	9.977544	9.518610	10.481390	44
17	9.496537	9.977503	9.519034	10.480966	43
18	9.496919	9.977461	9.519458	10.480542	42
19	9.497301	9.977419	9.519882	10.480118	41
20	9.497682	9.977377	9.520305	10.479695	40
21	9.498063	9.977335	9.520728	10.479272	39
22	9.498444	9.977293	9.521151	10.478849	38
23	9.498824	9.977251	9.521573	10.478427	37
24	9.499204	9.977209	9.521995	10.478005	36
25	9.499584	9.977167	9.522417	10.477583	35
26	9.499963	9.977125	9.522838	10.477162	34
27	9.500342	9.977083	9.523259	10.476741	33
28	9.500720	9.977041	9.523679	10.476320	32
29	9.501099	9.976999	9.524109	10.475900	31
30	9.501476	9.976956	9.524520	10.475480	30
		Sine		Tangent	M

Degree 71.

Degree 18.

M	Sine		Tangent		
30	9.501476	9.976956	9.524520	10.475480	30
31	9.501854	9.976914	9.524939	10.475060	29
32	9.502231	9.976872	9.525359	10.474641	28
33	9.502607	9.976830	9.525778	10.474222	27
34	9.502984	9.976787	9.526197	10.473803	26
35	9.503360	9.976745	9.526615	10.473385	25
36	9.503735	9.976702	9.527033	10.472967	24
37	9.504110	9.976660	9.527451	10.472549	23
38	9.504485	9.976617	9.527868	10.472132	22
39	9.504840	9.976574	9.528285	10.471715	21
40	9.505234	9.976532	9.528702	10.471298	20
41	9.505608	9.976489	9.529118	10.470881	19
42	9.505981	9.976446	9.529535	10.470465	18
43	9.506354	9.976404	9.529950	10.470049	17
44	9.506727	9.976361	9.530366	10.469634	16
45	9.507099	9.976318	9.530781	10.469219	15
46	9.507471	9.976275	9.531196	10.468804	14
47	9.507843	9.976232	9.531611	10.468389	13
48	9.508214	9.976185	9.532025	10.467975	12
49	9.508585	9.976146	9.532436	10.467561	11
50	9.508955	9.976103	9.532852	10.467147	10
51	9.509326	9.976060	9.533266	10.466734	9
52	9.509696	9.976017	9.533679	10.466321	8
53	9.510065	9.975973	9.534092	10.465908	7
54	9.510434	9.975930	9.534504	10.465496	6
55	9.510803	9.975887	9.534916	10.465084	5
56	9.511171	9.975844	9.535328	10.464672	4
57	9.511540	9.975800	9.535739	10.464261	3
58	9.511907	9.975757	9.536150	10.463849	2
59	9.512275	9.975713	9.536561	10.463439	1
60	9.512642	9.975670	9.536972	10.463028	0
		Sine		Tangent	M

Degree 71.

Degree 19.

M	Sine		Tangent		
0	9.512642	9.975670	9.536972	10.463028	60
1	9.513009	9.975626	9.537382	10.462618	59
2	9.513375	9.975583	9.537792	10.462208	58
3	9.513741	9.975539	9.538202	10.461798	57
4	9.514107	9.975496	9.538610	10.461389	56
5	9.514472	9.975452	9.539020	10.460980	55
6	9.514837	9.975408	9.539429	10.460571	54
7	9.515202	9.975364	9.539837	10.460163	53
8	9.515566	9.975321	9.540245	10.459755	52
9	9.515930	9.975277	9.540653	10.459347	51
10	9.516294	9.975233	9.541061	10.458939	50
11	9.516657	9.975189	9.541468	10.458532	49
12	9.517020	9.975145	9.541875	10.458125	48
13	9.517382	9.975101	9.542281	10.457719	47
14	9.517745	9.975057	9.542688	10.457312	46
15	9.518107	9.975013	9.543094	10.456906	45
16	9.518468	9.974969	9.543499	10.456501	44
17	9.518829	9.974925	9.543905	10.456095	43
18	9.519190	9.974880	9.544310	10.455690	42
19	9.519551	9.974836	9.544715	10.455285	41
20	9.519911	9.974792	9.545119	10.454881	40
21	9.520271	9.974747	9.545524	10.454476	39
22	9.520631	9.974703	9.545927	10.454072	38
23	9.520990	9.974659	9.546331	10.453669	37
24	9.521349	9.974614	9.546735	10.453265	36
25	9.521707	9.974570	9.547138	10.452862	35
26	9.522065	9.974525	9.547540	10.452459	34
27	9.522423	9.974480	9.547943	10.452057	33
28	9.522781	9.974436	9.548345	10.451655	32
29	9.523138	9.974391	9.548747	10.451253	31
30	9.523495	9.974346	9.549149	10.450851	30
		Sine		Tangent	M

Degree 70.

Degree 19.

M	Sine		Tangent		
30	9.523495	9.974346	9.549149	10.450851	30
31	9.523851	9.974302	9.549550	10.450450	29
32	9.524208	9.974257	9.549951	10.450049	28
33	9.524564	9.974212	9.550352	10.449648	27
34	9.524920	9.974167	9.550752	10.449248	26
35	9.525275	9.974122	9.551152	10.448848	25
36	9.525630	9.974077	9.551552	10.448448	24
37	9.525984	9.974032	9.551952	10.448048	23
38	9.526339	9.973987	9.552351	10.447649	22
39	9.526693	9.973942	9.552750	10.447250	21
40	9.527046	9.973897	9.553149	10.446851	20
41	9.527400	9.973852	9.553548	10.446452	19
42	9.527753	9.973807	9.553946	10.446054	18
43	9.528105	9.973761	9.554344	10.445656	17
44	9.528458	9.973716	9.554741	10.445259	16
45	9.528810	9.973671	9.555139	10.444861	15
46	9.529161	9.973615	9.555536	10.444464	14
47	9.529513	9.973580	9.555932	10.444068	13
48	9.529864	9.973535	9.556329	10.443671	12
49	9.530214	9.973489	9.556727	10.443275	11
50	9.530565	9.973443	9.557121	10.442879	10
51	9.530915	9.973398	9.557517	10.442483	9
52	9.531265	9.973352	9.557912	10.442088	8
53	9.531614	9.973307	9.558308	10.441693	7
54	9.531963	9.973261	9.558702	10.441298	6
55	9.532312	9.973215	9.559097	10.440903	5
56	9.532661	9.973169	9.559491	10.440509	4
57	9.533009	9.973123	9.559885	10.440115	3
58	9.533357	9.973078	9.560279	10.439721	2
59	9.533704	9.973032	9.560673	10.439327	1
60	9.534052	9.972986	9.561066	10.438934	0
	Sine		Tangent		M

Degree 70.

Degree 20.

M	Sine		Tangent		
0	9.534052	9.972986	9.561066	10.438934	60
1	9.534399	9.972940	9.561459	10.438541	59
2	9.534746	9.972894	9.561851	10.438148	58
3	9.535091	9.972848	9.562244	10.437756	57
4	9.535437	9.972801	9.562636	10.437364	56
5	9.535782	9.972755	9.563028	10.436972	55
6	9.436129	9.972709	9.563419	10.436580	54
7	9.536474	9.972663	9.563811	10.436189	53
8	9.536818	9.972617	9.564202	10.435798	52
9	9.537163	9.972570	9.564592	10.435407	51
10	9.537507	9.972524	9.564983	10.435017	50
11	9.537851	9.972477	9.565373	10.434627	49
12	9.538194	9.972431	9.565763	10.434237	48
13	9.538537	9.972384	9.566153	10.433847	47
14	9.538880	9.972338	9.566542	10.433457	46
15	9.539222	9.972291	9.566932	10.433068	45
16	9.539565	9.972245	9.567320	10.432679	44
17	9.539907	9.972198	9.567709	10.432291	43
18	9.540249	9.972151	9.568097	10.431902	42
19	9.540590	9.972105	9.568486	10.431514	41
20	9.540931	9.972058	9.568873	10.431126	40
21	9.541272	9.972011	9.569261	10.430739	39
22	9.541612	9.971964	9.569648	10.430351	38
23	9.541953	9.971917	9.570035	10.429964	37
24	9.542292	9.971870	9.570422	10.429578	36
25	9.542632	9.971823	9.570809	10.429191	35
26	9.542971	9.971776	9.571195	10.428805	34
27	9.543310	9.971729	9.571581	10.428419	33
28	9.543649	9.971682	9.571967	10.428033	32
29	9.543997	9.971635	9.572352	10.427648	31
30	9.544325	9.971588	9.572738	10.427262	30
		Sine		Tangent	M

Degree 69.

Degree 20.

M	Sine		Tangent		
30	9.544325	9.971588	9.572738	10.427262	30
31	9.544663	9.971540	9.573123	10.426877	29
32	9.545000	9.971493	9.573507	10.426492	28
33	9.545338	9.971446	9.573892	10.426108	27
34	9.545674	9.971398	9.574276	10.425724	26
35	9.546011	9.971351	9.574660	10.425340	25
36	9.546347	9.971303	9.575044	10.424956	24
37	9.546683	9.971256	9.575427	10.424573	23
38	9.547019	9.971208	9.575810	10.424189	22
39	9.547354	9.971161	9.576193	10.423807	21
40	9.547689	9.971112	9.576576	10.423424	20
41	9.548024	9.971065	9.576958	10.423041	19
42	9.548358	9.971018	9.577341	10.422659	18
43	9.548693	9.970970	9.577723	10.422277	17
44	9.549026	9.970922	9.578104	10.421896	16
45	9.549360	9.970874	9.578486	10.421514	15
46	9.549693	9.970826	9.578867	10.421133	14
47	9.550026	9.970779	9.579248	10.420752	13
48	9.550359	9.970731	9.579628	10.420371	12
49	9.550692	9.970683	9.580009	10.419991	11
50	9.551024	9.970634	9.580389	10.419611	10
51	9.551355	9.970586	9.580769	10.419231	9
52	9.551687	9.970538	9.581149	10.418851	8
53	9.552018	9.970490	9.581528	10.418472	7
54	9.552349	9.970442	9.581907	10.418092	6
55	9.552680	9.970394	9.582286	10.417713	5
56	9.553010	9.970345	9.582665	10.417335	4
57	9.553340	9.970297	9.583043	10.416956	3
58	9.553670	9.970249	9.583422	10.416578	2
59	9.554000	9.970200	9.583800	10.416200	1
60	9.554329	9.970152	9.584177	10.415823	0
	Sine		Tangent		M

Degree 69.

Degree 21.

M	Sine		Tangent		
0	9.554329	9.970152	9.584177	10.415822	60
1	9.554658	9.970103	9.584555	10.415445	59
2	9.554987	9.970055	9.584932	10.415068	58
3	9.555315	9.970006	9.585308	10.414691	57
4	9.555643	9.969957	9.585686	10.414314	56
5	9.555971	9.969909	9.586062	10.413938	55
6	9.556299	9.969860	9.586439	10.413561	54
7	9.556626	9.969811	9.586815	10.413185	53
8	9.556953	9.969762	9.587190	10.412800	52
9	9.557279	9.969713	9.587566	10.412434	51
10	9.557606	9.969665	9.587941	10.412059	50
11	9.557932	9.969616	9.588316	10.411684	49
12	9.558258	9.969567	9.588691	10.411309	48
13	9.558583	9.969518	9.589066	10.410934	47
14	9.558909	9.969469	9.589440	10.410560	46
15	9.559234	9.969419	9.589814	10.410185	45
16	9.559558	9.969370	9.590188	10.409812	44
17	9.559883	9.969321	9.590561	10.409438	43
18	9.560207	9.969272	9.590935	10.409065	42
19	9.560531	9.969223	9.591308	10.408692	41
20	9.560855	9.969173	9.591681	10.408319	40
21	9.161178	9.969124	9.592054	10.407946	39
22	9.561501	9.969075	9.592426	10.407574	38
23	9.561824	9.969025	9.592798	10.407201	37
24	9.562146	9.968976	9.593170	10.406829	36
25	9.562468	9.968926	9.693542	10.406457	35
26	9.562790	9.968877	9.593914	10.406086	34
27	9.563112	9.968827	9.594285	10.405715	33
28	9.563433	9.968777	9.594656	10.405344	32
29	9.563754	9.968728	9.595027	10.405073	31
30	9.564075	9.968678	9.595397	10.404602	30
		Sine		Tangent	M

Degree 68.

Degree 21.

M	Sine		Tangent		
30	9.564075	9.968678	9.595397	10.404602	30
31	9.564396	9.968628	9.595768	10.404232	29
32	9.564716	9.968578	9.596138	10.403862	28
33	9.565036	9.968528	9.596508	10.403492	27
34	9.565356	9.968478	9.596878	10.403122	26
35	9.565675	9.968428	9.597247	10.402753	25
36	9.565995	9.968378	9.597616	10.402384	24
37	9.566314	9.968328	9.597985	10.402015	23
38	9.566632	9.968278	9.598354	10.401646	22
39	9.566951	9.968228	9.598722	10.401277	21
40	9.567269	9.968178	9.599091	10.400909	20
41	9.567587	9.968128	9.599459	10.400541	19
42	9.567904	9.968078	9.599827	10.400173	18
43	9.568222	9.968027	9.600194	10.399806	17
44	9.568539	9.967977	9.600562	10.399438	16
45	9.568855	9.967927	9.600929	10.399071	15
46	9.569172	9.967876	9.601296	10.398704	14
47	9.569488	9.967826	9.601662	10.398337	13
48	9.569804	9.967775	9.602029	10.397971	12
49	9.570120	9.967725	9.602395	10.397605	11
50	9.570435	9.967674	9.602761	10.397239	10
51	9.570751	9.967623	9.603127	10.396873	9
52	9.571065	9.967573	9.603493	10.296507	8
53	9.571380	9.967522	9.603858	10.396142	7
54	9.571695	9.967471	9.604223	10.395777	6
55	9.572009	9.967420	9.604588	10.395412	5
56	9.572322	9.967370	9.604953	10.395047	4
57	9.572636	9.967319	9.605317	10.394683	3
58	9.572949	9.967268	9.605681	10.394318	2
59	9.573263	9.967217	9.606046	10.393954	1
60	9.573575	9.967166	9.606409	10.393590	0
	Sine		Tangent		M

Degree 68.

Degree 22.

M	Sine		Tangent		
0	9.573575	9.567166	9.606409	10.393590	60
1	9.573888	9.567115	9.606773	10.393227	59
2	9.574200	9.567064	9.607136	10.392863	58
3	9.574512	9.567012	9.607500	10.392500	57
4	9.574824	9.566961	9.607862	10.392137	56
5	9.575135	9.566910	9.608225	10.391774	55
6	9.575447	9.566859	9.608588	10.391412	54
7	9.575758	9.566807	9.608950	10.391050	53
8	9.576068	9.566756	9.609312	10.390688	52
9	9.576379	9.566705	9.609674	10.390326	51
10	9.576689	9.566653	9.600036	10.389964	50
11	9.576999	9.566602	9.610397	10.389603	49
12	9.577309	9.566550	9.610758	10.389241	48
13	9.577618	9.566499	9.611119	10.388880	47
14	9.577927	9.566447	9.611480	10.388520	46
15	9.578236	9.566395	9.611841	10.388159	45
16	9.578545	9.566344	9.612201	10.387799	44
17	9.578853	9.566292	9.612561	10.387438	43
18	9.579161	9.566240	9.612921	10.387078	42
19	9.579469	9.566188	9.613281	10.386719	41
20	9.579777	9.566136	9.613641	10.386359	40
21	9.580084	9.566084	9.614000	10.386000	39
22	9.580392	9.566032	9.614359	10.385641	38
23	9.580698	9.565980	9.614718	10.385282	37
24	9.581005	9.565928	9.615077	10.384923	36
25	9.581311	9.565876	9.615435	10.384565	35
26	9.581618	9.565824	9.615793	10.384207	34
27	9.581923	9.565772	9.616151	10.383848	33
28	9.582229	9.565720	9.616509	10.383491	32
29	9.582534	9.565668	9.616867	10.383133	31
30	9.582840	9.565615	9.617224	10.382776	30
		Sine		Tangent	M

Degree 67.

Degree 22.

M	Sine		Tangent		
30	9.582840	9.965615	9.617224	10.382776	30
31	9.583144	9.965563	9.617581	10.382418	29
32	9.583449	9.965511	9.617938	10.382061	28
33	9.583753	9.965458	9.618295	10.381705	27
34	9.584058	9.965406	9.618652	10.381348	26
35	9.584361	9.965353	9.619008	10.380992	25
36	9.584665	9.965301	9.619364	10.380635	24
37	9.584968	9.965248	9.619720	10.380279	23
38	9.585271	9.965195	9.620076	10.379924	22
39	9.585574	9.965143	9.620432	10.379568	21
40	9.585877	9.965090	9.620787	10.379213	20
41	9.586179	9.995037	9.621142	10.378858	19
42	9.586481	9.964984	9.621497	10.378503	18
43	9.586783	9.964931	9.621852	10.378148	17
44	9.587085	9.964978	9.622206	10.377793	16
45	9.587386	9.964825	9.622561	10.377439	15
46	9.587687	9.964772	9.622915	10.377085	14
47	9.587988	9.964719	9.623269	10.376731	13
48	9.588289	9.964666	9.623623	10.376377	12
49	9.588589	9.964613	9.623976	10.376024	11
50	9.588890	9.964560	9.624330	10.375670	10
51	9.589190	9.964507	9.624683	10.375317	9
52	9.589489	9.964454	9.625036	10.374964	8
53	9.589789	9.964400	9.625388	10.374612	7
54	9.590088	9.964347	9.625741	10.374259	6
55	9.590387	9.964294	9.626093	10.373907	5
56	9.590686	9.964240	9.626445	10.373555	4
57	9.590984	9.964187	9.626797	10.373203	3
58	9.591282	9.964133	9.627149	10.372850	2
59	9.591580	9.964080	9.627501	10.372499	1
60	9.591878	9.964026	9.627852	10.372148	0
	Sine		Tangent		M

Degree 67.

Degree 23.

M	Sine		Tangent		
0	9.591878	9.964026	9.627852	10.372148	60
1	9.592175	9.963972	9.628203	10.371797	59
2	9.592473	9.963919	9.628554	10.371446	58
3	9.592770	9.963865	9.628905	10.371095	57
4	9.593067	9.963811	9.629255	10.370744	56
5	9.593363	9.963757	9.629606	10.370394	55
6	9.593659	9.963703	9.629956	10.370044	54
7	9.593955	9.963650	9.630306	10.369694	53
8	9.594251	9.963596	9.630655	10.369344	52
9	9.594547	9.963542	9.631005	10.368995	51
10	9.594842	9.963488	9.631354	10.368645	50
11	9.595137	9.963433	9.631704	10.368296	49
12	9.595432	9.963379	9.632053	10.367947	48
13	9.595727	9.963325	9.632401	10.367598	47
14	9.596021	9.963271	9.632750	10.367250	46
15	9.596315	9.963217	9.633098	10.366901	45
16	9.596610	9.963162	9.633447	10.366553	44
17	9.596903	9.963108	9.633795	10.366205	43
18	9.597196	9.963054	9.634143	10.365857	42
19	9.597490	9.962999	9.634490	10.365510	41
20	9.597783	9.962945	9.634838	10.365162	40
21	9.598075	9.962892	9.635185	10.364815	39
22	9.598368	9.962836	9.635530	10.364468	38
23	9.598660	9.962781	9.635879	10.364121	37
24	9.598952	9.962726	9.636226	10.363774	36
25	9.599244	9.962672	9.636571	10.363428	35
26	9.599536	9.962617	9.636918	10.363081	34
27	9.599827	9.962562	9.637265	10.362735	33
28	9.600118	9.962507	9.637610	10.362389	32
29	9.600409	9.962453	9.637956	10.362044	31
30	9.600700	9.962398	9.638302	10.361698	30
	Sine		Tangent		M

Degree 66.

Degree 23.

M	Sine		Tangent		
30	9.600700	9.962398	9.638302	10.361698	30
31	9.600990	9.962343	9.638647	10.361353	29
32	9.601280	9.962288	9.638992	10.361007	28
33	9.601570	9.962233	9.639337	10.360662	27
34	9.601860	9.962178	9.639682	10.360318	26
35	9.602149	9.962122	9.640027	10.359973	25
36	9.602439	9.962067	9.640371	10.359629	24
37	9.602728	9.962012	9.640716	10.359284	23
38	9.603017	9.961957	9.641060	10.358940	22
39	9.603305	9.961902	9.641404	10.358596	21
40	9.603594	9.961846	9.641747	10.358253	20
41	9.603882	9.961791	9.642091	10.357909	19
42	9.604170	9.961735	9.642434	10.357566	18
43	9.604457	9.961680	9.642777	10.357223	17
44	9.604745	9.961624	9.643120	10.356980	16
45	9.605032	9.961569	9.643463	10.356737	15
46	9.605319	9.961513	9.643806	10.356494	14
47	9.605606	9.961458	9.644148	10.356252	13
48	9.605892	9.961402	9.644490	10.356010	12
49	9.606179	9.961346	9.644832	10.355768	11
50	9.606465	9.961290	9.645174	10.355526	10
51	9.606750	9.961235	9.645516	10.355284	9
52	9.607036	9.961179	9.645857	10.355042	8
53	9.607322	9.961123	9.646199	10.354801	7
54	9.607607	9.961067	9.646540	10.354560	6
55	9.907892	9.961011	9.646881	10.354319	5
56	9.608176	9.960955	9.647222	10.354078	4
57	9.608461	9.960899	9.647562	10.353838	3
58	9.608745	9.960842	9.647903	10.353597	2
59	9.609029	9.960786	9.648243	10.353357	1
60	9.609313	9.960730	9.648583	10.353117	0
	Sine		Tangent		M

Degree 66.

Degree 24.

M	Sine		Tangent		
0	9.609313	9.960730	9.648583	10.351417	60
1	9.609597	9.960674	9.648923	10.351077	59
2	9.609880	9.960617	9.649263	10.350737	58
3	9.610163	9.960561	9.649602	10.350398	57
4	9.610446	9.960505	9.649942	10.350058	56
5	9.610729	9.960448	9.650281	10.349719	55
6	9.611012	9.960392	9.650620	10.349380	54
7	9.611294	9.960335	9.650959	10.349041	53
8	9.611576	9.960279	9.651297	10.348703	52
9	9.611858	9.960222	9.651636	10.348364	51
10	9.612140	9.960165	9.651974	10.348026	50
11	9.612421	9.960109	9.652312	10.347688	49
12	9.612702	9.960052	9.652650	10.347350	48
13	9.612983	9.959995	9.652988	10.347012	47
14	9.613264	9.959938	9.653326	10.346674	46
15	9.613545	9.959881	9.653663	10.346337	45
16	9.613825	9.959824	9.654000	10.345999	44
17	9.614105	9.959768	9.654337	10.345662	43
18	9.614385	9.959710	9.654674	10.345325	42
19	9.614665	9.959653	9.655011	10.344989	41
20	9.614944	9.959596	9.655348	10.344652	40
21	9.615223	9.959539	9.655684	10.344316	39
22	9.615502	9.959482	9.656020	10.343980	38
23	9.615781	9.959425	9.656356	10.343643	37
24	9.616060	9.959367	9.656692	10.343308	36
25	9.616338	9.959310	9.657028	10.342972	35
26	9.616616	9.959253	9.657363	10.342636	34
27	9.616894	9.959195	9.657699	10.342301	33
28	9.617172	9.959138	9.658034	10.341966	32
29	9.617450	9.959080	9.658369	10.341631	31
30	9.617727	9.959023	9.658704	10.341296	30
		Sine		Tangent	M

Degree 65.

Degree 24.

M	Sine		Tangent		
30	9.617727	9.959023	9.658704	10.341296	30
31	9.618004	9.958965	9.659039	10.340926	29
32	9.618281	9.958908	9.659373	10.340627	28
33	9.618558	9.958850	9.659708	10.340292	27
34	9.618834	9.958792	9.660042	10.339958	26
35	9.619110	9.958734	9.660376	10.339624	25
36	9.619386	9.958677	9.660710	10.339290	24
37	9.619662	9.958619	9.661043	10.338957	23
38	9.619938	9.958561	9.661377	10.338623	22
39	9.620213	9.958503	9.661710	10.338290	21
40	9.620488	9.958445	9.662043	10.337956	20
41	9.620763	9.958387	9.662376	10.337623	19
42	9.621038	9.958329	9.662709	10.337291	18
43	9.621313	9.958271	9.663042	10.336958	17
44	9.621587	9.958212	9.663374	10.336625	16
45	9.621861	9.958154	9.663707	10.336293	15
46	9.622135	9.958096	9.664039	10.335961	14
47	9.622409	9.958038	9.664371	10.335629	13
48	9.622682	9.957979	9.664703	10.335297	12
49	9.622956	9.957921	9.665035	10.334965	11
50	9.623219	9.957862	9.665366	10.334624	10
51	9.623502	9.957804	9.665697	10.334302	9
52	9.623776	9.957745	9.666029	10.333971	8
53	9.624047	9.957687	9.666360	10.333640	7
54	9.624319	9.957628	9.666691	10.333309	6
55	9.624591	9.957570	9.667021	10.332979	5
56	9.624863	9.957511	9.667352	10.332648	4
57	9.625134	9.957452	9.667682	10.332318	3
58	9.625406	9.957393	9.668012	10.331987	2
59	9.625677	9.957334	9.668343	10.331657	1
60	9.625948	9.957276	9.668672	10.331327	0
	Sine		Tangent		M

Degree 65.

Degree 25.

M	Sine		Tangent		
0	9.625948	9.957276	9.668672	10.331327	60
1	9.626219	9.957217	9.669002	10.330998	59
2	9.626490	9.957158	9.669332	10.330668	58
3	9.626760	9.957099	9.669661	10.330339	57
4	9.627030	9.957040	9.669990	10.330009	56
5	9.627300	9.956981	9.670320	10.329680	55
6	9.627570	9.956922	9.670649	10.329351	54
7	9.627840	9.956862	9.670977	10.329022	53
8	9.628109	9.956803	9.671306	10.328694	52
9	9.628378	9.956744	9.671634	10.328365	51
10	9.628647	9.956684	9.671963	10.328037	50
11	9.628916	9.956625	9.672291	10.327709	49
12	9.629184	9.956565	9.672619	10.327381	48
13	9.629453	9.956508	9.672947	10.327053	47
14	9.629721	9.956446	9.673274	10.326725	46
15	9.629989	9.956387	9.673603	10.326398	45
16	9.630257	9.956327	9.673929	10.326070	44
17	9.630524	9.956267	9.674256	10.325743	43
18	9.630792	9.956208	9.674584	10.325416	42
19	9.631059	9.956148	9.674910	10.325089	41
20	9.631326	9.956088	9.675237	10.324763	40
21	9.631592	9.956029	9.675564	10.324436	39
22	9.631859	9.955969	9.675890	10.324110	38
23	9.632125	9.955909	9.676216	10.323783	37
24	9.632392	9.955849	9.676543	10.323457	36
25	9.632657	9.955789	9.676869	10.323131	35
26	9.632923	9.955739	9.677194	10.322805	34
27	9.633189	9.955669	9.677520	10.322480	33
28	9.633454	9.955609	9.677845	10.322154	32
29	9.633719	9.955548	9.678171	10.321829	31
30	9.633984	9.955488	9.678496	10.321504	30
		Sine		Tangent	M

Degree 64.

Degree 25.

M	Sine		Tangent		
30	9.632984	9.955488	9.678496	10.321504	30
31	9.634249	9.955428	9.678821	10.321179	29
32	9.634514	9.955367	9.679146	10.320854	28
33	9.634778	9.955307	9.679471	10.320529	27
34	9.635042	9.955246	9.679795	10.320205	26
35	9.635306	9.955186	9.680120	10.319880	25
36	9.635570	9.955125	9.680444	10.319556	24
37	9.635833	9.955065	9.680768	10.319232	23
38	9.636097	9.955004	9.681092	10.318908	22
39	9.636360	9.954944	9.681416	10.318584	21
40	9.636623	9.954883	9.681740	10.318260	20
41	9.636886	9.954823	9.682063	10.317937	19
42	9.637148	9.954762	9.682386	10.317613	18
43	9.637411	9.954701	9.682710	10.317290	17
44	9.637673	9.954640	9.683033	10.316967	16
45	9.637935	9.954579	9.683356	10.316644	15
46	9.638197	9.954518	9.683678	10.316321	14
47	9.638458	9.954457	9.684001	10.315999	13
48	9.638720	9.954396	9.684324	10.315676	12
49	9.638981	9.954335	9.684646	10.315354	11
50	9.639242	9.954274	9.684968	10.315032	10
51	9.639503	9.954213	9.685290	10.314710	9
52	9.639764	9.954152	9.685612	10.313388	8
53	9.640024	9.954090	9.685934	10.314066	7
54	9.640284	9.954029	9.686255	10.313745	6
55	9.640544	9.954968	9.686577	10.313423	5
56	9.640804	9.953906	9.686898	10.313102	4
57	9.641064	9.953845	9.687219	10.312781	3
58	9.641323	9.953783	9.687540	10.312460	2
59	9.641583	9.953722	9.687861	10.312138	1
60	9.641842	9.953660	9.688182	10.311818	0
	Sine		Tangent		M

Degree 64.

Degree 26.

M	Sine		Tangent		
0	9.641842	9.953660	9.688182	10.311818	60
1	9.642101	9.953598	9.688502	10.311498	59
2	9.642360	9.953537	9.688823	10.311177	58
3	9.642618	9.953475	9.689143	10.310857	57
4	9.642876	9.953413	9.689463	10.310537	56
5	9.643135	9.953351	9.689783	10.310217	55
6	9.643393	9.953290	9.690103	10.309897	54
7	9.643650	9.953228	9.690423	10.309577	53
8	9.643908	9.953166	9.690742	10.309258	52
9	9.644165	9.953104	9.691063	10.308938	51
10	9.644423	9.953042	9.691381	10.308619	50
11	9.644680	9.952980	9.691700	10.308300	49
12	9.644936	9.952917	9.692019	10.307981	48
13	9.645193	9.952855	9.692338	10.307662	47
14	9.645449	9.952793	9.692656	10.307343	46
15	9.645706	9.952731	9.692975	10.307025	45
16	9.645962	9.952668	9.693293	10.306706	44
17	9.646218	9.952606	9.693612	10.306388	43
18	9.646473	9.952544	9.693930	10.306070	42
19	9.646729	9.952481	9.694248	10.305752	41
20	9.646984	9.952419	9.694566	10.305434	40
21	9.647239	9.952356	9.694883	10.305117	39
22	9.647494	9.952294	9.695201	10.304799	38
23	9.647749	9.952231	9.695518	10.304482	37
24	9.648004	9.952168	9.695835	10.304164	36
25	9.648258	9.952105	9.696153	10.303847	35
26	9.648512	9.952043	9.696470	10.303530	34
27	9.648766	9.951980	9.696786	10.303213	33
28	9.648020	9.951917	9.697103	10.302897	32
29	9.649274	9.951854	9.697420	10.302580	31
30	9.649527	9.951791	9.697738	10.302264	30
		Sine		Tangent	M

Degree 63.

Degree 26.

M	Sine		Tangent		
30	9.649527	9.951791	9.697738	10.202264	30
31	9.649781	9.951728	9.698052	10.201947	29
32	9.650034	9.951665	9.698369	10.201631	28
33	9.650287	9.951602	9.698685	10.201315	27
34	9.650519	9.951539	9.699001	10.200999	26
35	9.650798	9.951476	9.699316	10.200684	25
36	9.651044	9.951412	9.699632	10.200368	24
37	9.651296	9.951349	9.699947	10.200052	23
38	9.651648	9.951286	9.700263	10.299737	22
39	9.651800	9.951222	9.700578	10.299422	21
40	9.652052	9.951159	9.700893	10.299107	20
41	9.652303	9.951095	9.701208	10.298792	19
42	9.652555	9.951032	9.701522	10.298477	18
43	9.652806	9.950968	9.701837	10.298163	17
44	9.653057	9.950905	9.702152	10.297848	16
45	9.653307	9.950841	9.702466	10.297534	15
46	9.653558	9.950777	9.702780	10.297219	14
47	9.653808	9.950714	9.703095	10.296905	13
48	9.654059	9.950650	9.703409	10.296591	12
49	9.654309	9.950586	9.703722	10.296277	11
50	9.653558	9.950522	9.704036	10.295964	10
51	9.654808	9.950458	9.704350	10.295656	9
52	9.655057	9.950394	9.704663	10.295337	8
53	9.655307	9.950330	9.704976	10.295023	7
54	9.655556	9.950266	9.905290	10.294710	6
55	9.655805	9.950202	9.705603	10.294397	5
56	9.656053	9.950138	9.705915	10.294084	4
57	9.656302	9.950074	9.706228	10.293771	3
58	9.656550	9.950009	9.706541	10.293459	2
59	9.656799	9.949945	9.706853	10.293146	1
60	9.656947	9.949881	9.707166	10.292834	0
	Sine		Tangent		M

Degree 63.

Degree 27.

M	Sine		Tangent		
0	9.657047	9.949880	9.707166	10.292834	60
1	9.657295	9.949816	9.707478	10.292523	59
2	9.657542	9.949752	9.707790	10.292210	58
3	9.657790	9.949687	9.708102	10.291897	57
4	9.658037	9.949623	9.708414	10.291586	56
5	9.658284	9.949598	9.708726	10.291274	55
6	9.658531	9.949494	9.709037	10.290962	54
7	9.658777	9.949429	9.709349	10.290651	53
8	9.659024	9.949364	9.709660	10.290340	52
9	9.659271	9.949300	9.709971	10.290029	51
10	9.659517	9.949235	9.710282	10.289718	50
11	9.659763	9.949170	9.710593	10.289407	49
12	9.660009	9.949105	9.710904	10.289096	48
13	9.660255	9.949040	9.711214	10.288785	47
14	9.660500	9.948976	9.711525	10.288475	46
15	9.660746	9.948910	9.711836	10.288164	45
16	9.660991	9.948845	9.712146	10.287854	44
17	9.661036	9.948760	9.712456	10.287544	43
18	9.661481	9.948715	9.712766	10.287234	42
19	9.661726	9.948650	9.713076	10.286924	41
20	9.661970	9.948584	9.713386	10.286614	40
21	9.662214	9.948519	9.713695	10.286305	39
22	9.662459	9.948453	9.714005	10.285995	38
23	9.662702	9.948388	9.714314	10.285686	37
24	9.662947	9.948323	9.714624	10.285376	36
25	9.663190	9.948257	9.714933	10.285067	35
26	9.663433	9.948191	9.715241	10.284758	34
27	9.663677	9.948126	9.715550	10.284449	33
28	9.663920	9.948060	9.715859	10.284140	32
29	9.664163	9.947995	9.716168	10.283832	31
30	9.664406	9.947929	9.716477	10.283523	30
		Sine		Tangent	M

Degree 62.

Degree 27.

M	Sine		Tangent		
30	9.664406	9.947929	9.716477	10.283523	30
31	9.664648	9.947863	9.716785	10.283215	29
32	9.664891	9.947797	9.717093	10.282907	28
33	9.665133	9.947731	9.717401	10.282598	27
34	9.665375	9.947665	9.717709	10.282290	26
35	9.665617	9.947599	9.718017	10.281983	25
36	9.665858	9.947533	9.718325	10.281675	24
37	9.666100	9.947467	9.718633	10.281367	23
38	9.666341	9.947401	9.718940	10.281060	22
39	9.666583	9.947335	9.719248	10.280752	21
40	9.666824	9.947269	9.719555	10.280445	20
41	9.667065	9.947203	9.719862	10.280138	19
42	9.667305	9.947136	9.720169	10.279831	18
43	9.667546	9.947070	9.720476	10.279524	17
44	9.667786	9.947004	9.720783	10.279217	16
45	9.668026	9.946937	9.721089	10.278911	15
46	9.668266	9.946871	9.721395	10.278604	14
47	9.668506	9.946804	9.721702	10.278298	13
48	9.668746	9.946738	9.722008	10.277991	12
49	9.668986	9.946671	9.722315	10.277685	11
50	9.669225	9.946604	9.722621	10.277379	10
51	9.669464	9.946537	9.722927	10.277073	9
52	9.669703	9.946471	9.723232	10.276768	8
53	9.669942	9.946404	9.723538	10.276462	7
54	9.670181	9.946337	9.723843	10.276156	6
55	9.670419	9.946270	9.724149	10.275851	5
56	9.670657	9.946203	9.724454	10.275546	4
57	9.670896	9.946136	9.724759	10.275240	3
58	9.671134	9.946069	9.725065	10.274935	2
59	9.671372	9.946002	9.725369	10.274630	1
60	9.671609	9.945935	9.725674	10.274326	0
		Sine		Tangent	M

Degree 62.

Degree 28.

M	Sine		Tangent		
0	9.671609	9.945935	9.725674	10.274326	60
1	9.671847	9.945869	9.725979	10.274021	59
2	9.672084	9.945800	9.726284	10.273816	58
3	9.672321	9.945733	9.726588	10.273412	57
4	9.672558	9.945666	9.726892	10.273107	56
5	9.672795	9.945598	9.727197	10.272803	55
6	9.673032	9.945531	9.727501	10.272499	54
7	9.673268	9.945463	9.727805	10.272195	53
8	9.673505	9.945396	9.728109	10.271891	52
9	9.673741	9.945328	9.728412	10.271587	51
10	9.673977	9.945261	9.728716	10.271284	50
11	9.674213	9.945193	9.729020	10.270980	49
12	9.674448	9.945125	9.729323	10.270677	48
13	9.674684	9.945058	9.729626	10.270374	47
14	9.674919	9.944990	9.729929	10.270070	46
15	9.675154	9.944922	9.730232	10.269767	45
16	9.675389	9.944854	9.730535	10.269464	44
17	9.675623	9.944786	9.730838	10.269162	43
18	9.675859	9.944718	9.731141	10.268859	42
19	9.676094	9.944650	9.731443	10.268559	41
20	9.676328	9.944582	9.731746	10.268254	40
21	9.676562	9.944514	9.732048	10.267952	39
22	9.676796	9.944446	9.732351	10.267649	38
23	9.677030	9.944377	9.732653	10.267347	37
24	9.677264	9.944309	9.732955	10.267045	36
25	9.677497	9.944241	9.733257	10.266743	35
26	9.677731	9.944172	9.733558	10.266441	34
27	9.677964	9.944104	9.733860	10.266140	33
28	9.678197	9.944016	9.734162	10.265838	32
29	9.678430	9.943967	9.734463	10.265537	31
30	9.678663	9.943898	9.734764	10.265236	30
		Sine		Tangent	M

Degree 61.

Degree 28.

M	Sine		Tangent		
30	9.678663	9.943898	9.734764	10.265236	30
31	9.678895	9.943830	9.735666	10.264934	29
32	9.679128	9.943761	9.735362	10.264633	28
33	9.679360	9.943692	9.735668	10.264332	27
34	9.679592	9.943624	9.735963	10.264031	26
35	9.679824	9.943555	9.736269	10.263731	25
36	9.680056	9.943486	9.736570	10.263430	24
37	9.680288	9.943417	9.736870	10.263130	23
38	9.680519	9.943348	9.737171	10.262829	22
39	9.680750	9.943279	9.737471	10.262529	21
40	9.680982	9.943210	9.737771	10.262229	20
41	9.681213	9.943141	9.738071	10.261929	19
42	9.681443	9.943072	9.738371	10.261629	18
43	9.681674	9.943003	9.738671	10.261329	17
44	9.681904	9.942933	9.738971	10.261029	16
45	9.682135	9.942864	9.739271	10.260729	15
46	9.682365	9.942795	9.739570	10.260430	14
47	9.682595	9.942725	9.739870	10.260130	13
48	9.682825	9.942656	9.740169	10.259831	12
49	9.683055	9.942587	9.740468	10.259532	11
50	9.683284	9.942517	9.740767	10.259233	10
51	9.683514	9.942448	9.741066	10.258934	9
52	9.683743	9.942378	9.741365	10.258635	8
53	9.683972	9.942308	9.741664	10.258336	7
54	9.684201	9.942239	9.741962	10.258038	6
55	9.684430	9.942169	9.742261	10.257739	5
56	9.684658	9.942099	9.742559	10.257441	4
57	9.684887	9.942029	9.742858	10.257142	3
58	9.685115	9.941959	9.743156	10.256844	2
59	9.685343	9.941889	9.743454	10.256546	1
60	9.685571	9.941819	9.743751	10.256248	0
		Sine		Tangent	M

Degree 61.

Degree 29.

M	Sine		Tangent		
0	9.685571	9.941819	9.743752	10.256248	60
1	9.685799	9.941749	9.744050	10.255950	59
2	9.686027	9.941679	9.744348	10.255652	58
3	9.686254	9.941609	9.744645	10.255355	57
4	9.686482	9.941539	9.744943	10.255057	56
5	9.686709	9.941468	9.745240	10.254760	55
6	9.686936	9.941398	9.745538	10.254462	54
7	9.687163	9.941328	9.745835	10.254165	53
8	9.687389	9.941257	9.746132	10.253868	52
9	9.687616	9.941187	9.746429	10.253571	51
10	9.687842	9.941116	9.746726	10.253274	50
11	9.688069	9.941046	9.747023	10.252977	49
12	9.688295	9.940975	9.747319	10.252680	48
13	9.688523	9.940905	9.747616	10.252384	47
14	9.688747	9.940834	9.747912	10.252087	46
15	9.688972	9.940763	9.748209	10.251791	45
16	9.689198	9.940693	9.748505	10.251495	44
17	9.689421	9.940622	9.748801	10.251199	43
18	9.689648	9.940551	9.749097	10.250902	42
19	9.689873	9.940480	9.749393	10.250607	41
20	9.690098	9.940409	9.749689	10.250311	40
21	9.690323	9.940338	9.749985	10.250015	39
22	9.690548	9.940267	9.750281	10.249719	38
23	9.690772	9.940196	9.750576	10.249424	37
24	9.690996	9.940125	9.750872	10.249128	36
25	9.691220	9.940053	9.751167	10.248833	35
26	9.691444	9.939982	9.751462	10.248538	34
27	9.691668	9.939911	9.751757	10.248243	33
28	9.691892	9.939840	9.752052	10.247948	32
29	9.692115	9.939768	9.752347	10.247653	31
30	9.692339	9.939697	9.752642	10.247358	30
		Sine		Tangent	M

Degree 60.

Degree 29.

M	Sine		Tangent		
30	9.692339	9.939697	9.752642	10.247358	30
31	9.692562	9.939625	9.752937	10.247063	29
32	9.692785	9.939554	9.753231	10.246769	28
33	9.693008	9.939482	9.753526	10.246474	27
34	9.693231	9.939410	9.753820	10.246180	26
35	9.693453	9.939339	9.754115	10.245885	25
36	9.693676	9.939267	9.754409	10.245591	24
37	9.693898	9.939195	9.754703	10.245297	23
38	9.694120	9.939123	9.754997	10.245003	22
39	9.694342	9.939051	9.755291	10.244709	21
40	9.694564	9.938980	9.755584	10.244415	20
41	9.694786	9.938908	9.755878	10.244122	19
42	9.695007	9.938835	9.756172	10.243828	18
43	9.695229	9.938763	9.756465	10.243535	17
44	9.695450	9.938691	9.756759	10.243241	16
45	9.695671	9.938619	9.757052	10.242948	15
46	9.695892	9.938547	9.757345	10.242655	14
47	9.696113	9.938475	9.757638	10.242362	13
48	9.696334	9.938402	9.757931	10.242069	12
49	9.696554	9.938330	9.758224	10.241776	11
50	9.696774	9.938257	9.758517	10.241483	10
51	9.696995	9.938185	9.758810	10.241190	9
52	9.697215	9.938112	9.759102	10.240898	8
53	9.697435	9.938040	9.759395	10.240605	7
54	9.697654	9.937967	9.759687	10.240313	6
55	9.697874	9.937895	9.759979	10.240021	5
56	9.698093	9.937822	9.760271	10.239728	4
57	9.698313	9.937749	9.760564	10.239436	3
58	9.698532	9.937676	9.760856	10.239144	2
59	9.698751	9.937603	9.761147	10.238852	1
60	9.698970	9.937531	9.761439	10.238561	0
	Sine		Tangent		M

Degree 60.

Degree 30.

M	Sine		Tangent		
0	9.698970	9.937531	9.761439	10.238561	60
1	9.699189	9.937458	9.761731	10.238269	59
2	9.699407	9.937385	9.762023	10.237977	58
3	9.699626	9.937312	9.762314	10.237686	57
4	9.699844	9.937238	9.762606	10.237394	56
5	9.700062	9.937165	9.762897	10.237103	55
6	9.700280	9.937092	9.763188	10.236812	54
7	9.700498	9.937019	9.763479	10.236521	53
8	9.700716	9.936945	9.763770	10.236230	52
9	9.700933	9.936872	9.764061	10.235939	51
10	9.701151	9.936799	9.764352	10.235648	50
11	9.701368	9.936725	9.764643	10.235357	49
12	9.701585	9.936652	9.764933	10.235067	48
13	9.701802	9.936578	9.765224	10.234776	47
14	9.702019	9.936505	9.765514	10.234486	46
15	9.702236	9.936431	9.765805	10.234195	45
16	9.702452	9.936357	9.766095	10.233905	44
17	9.702669	9.936284	9.766385	10.233615	43
18	9.702885	9.936210	9.766675	10.233325	42
19	9.703101	9.936136	9.766965	10.233035	41
20	9.703317	9.936062	9.767255	10.232745	40
21	9.703533	9.935988	9.767545	10.232455	39
22	9.703748	9.935914	9.767834	10.232166	38
23	9.703964	9.935840	9.768124	10.231876	37
24	9.704179	9.935766	9.768413	10.231587	36
25	9.704395	9.935692	9.768703	10.231297	35
26	9.704610	9.935618	9.768992	10.231008	34
27	9.704820	9.935543	9.769281	10.230719	33
28	9.705040	9.935469	9.769570	10.230430	32
29	9.705254	9.935395	9.769859	10.230141	31
30	9.705469	9.935320	9.770148	10.229852	30
		Sine		Tangent	M

Degree 59.

Degree 30.

M	Sine		Tangent		
30	9.705469	9.935320	9.770148	10.229852	30
31	9.705683	9.935246	9.770437	10.229563	29
32	9.705897	9.935171	9.770726	10.229274	28
33	9.706112	9.935097	9.771015	10.258985	27
34	9.706327	9.935022	9.771303	10.228696	26
35	9.706539	9.934948	9.771592	10.228408	25
36	9.706753	9.934873	9.771880	10.228120	24
37	9.706967	9.934798	9.772168	10.227832	23
38	9.707180	9.934723	9.772456	10.227543	22
39	9.707393	9.934649	9.772745	10.227255	21
40	9.707606	9.934574	9.773033	10.226967	20
41	9.707819	9.934499	9.773321	10.226679	19
42	9.708032	9.934424	9.773608	10.226391	18
43	9.708245	9.934349	9.773896	10.226104	17
44	9.708457	9.934274	9.774184	10.225816	16
45	9.708670	9.934199	9.774471	10.225529	15
46	9.708882	9.934123	9.774759	10.225241	14
47	9.709094	9.934048	9.775046	10.224954	13
48	9.709306	9.933973	9.775333	10.224666	12
49	9.709518	9.933897	9.775621	10.224379	11
50	9.709730	9.933822	9.775908	10.224092	10
51	9.709941	9.933747	9.776195	10.223805	9
52	9.710153	9.933671	9.776482	10.223518	8
53	9.710364	9.933596	9.776768	10.223232	7
54	9.710575	9.933520	9.777055	10.222945	6
55	9.710786	9.933444	9.777342	10.222658	5
56	9.710997	9.933369	9.777628	10.222372	4
57	9.711208	9.933293	9.777915	10.222085	3
58	9.711418	9.933217	9.778201	10.221799	2
59	9.711629	9.933141	9.778487	10.221513	1
60	9.711839	9.933066	9.778774	10.221226	0
	Sine		Tangent		M

Degree 59.

Degree 31.

M	Sine		Tangent		
0	9.711839	9.933066	9.778774	10.221226	60
1	9.712049	9.932990	9.779060	10.220940	59
2	9.712259	9.932914	9.779346	10.220654	58
3	9.712469	9.932838	9.779632	10.220368	57
4	9.712679	9.932761	9.779918	10.220082	56
5	9.712889	9.932685	9.780203	10.219796	55
6	9.713098	9.932609	9.780489	10.219511	54
7	9.713308	9.932533	9.780775	10.219225	53
8	9.713517	9.932457	9.781060	10.218940	52
9	9.713726	9.932380	9.781346	10.218654	51
10	9.713935	9.932304	9.781631	10.218369	50
11	9.714144	9.932227	9.781916	10.218084	49
12	9.714352	9.932151	9.782202	10.217799	48
13	9.714561	9.932074	9.782480	10.217514	47
14	9.714769	9.931998	9.782771	10.217229	46
15	9.714977	9.931921	9.783056	10.216944	45
16	9.715186	9.931845	9.783341	10.216659	44
17	9.715394	9.931768	9.783626	10.216374	43
18	9.715601	9.931691	9.783910	10.216090	42
19	9.715809	9.931614	9.784195	10.215805	41
20	9.716017	9.931537	9.784479	10.215520	40
21	9.716224	9.931460	9.784764	10.215236	39
22	9.716431	9.931383	9.785048	10.214952	38
23	9.716639	9.931306	9.785332	10.214668	37
24	9.716846	9.931229	9.785616	10.214384	36
25	9.717053	9.931152	9.785900	10.214099	35
26	9.717259	9.931075	9.786184	10.213816	34
27	9.717466	9.930998	9.786468	10.213532	33
28	9.717672	9.930920	9.786752	10.213248	32
29	9.717879	9.930843	9.787036	10.212964	31
30	9.718085	9.930766	9.787319	10.212681	30
		Sine		Tangent	M

Degree 58.

Degree 31.

M	Sine		Tangent		
30	9.718085	9.930766	9.787319	10.212681	30
31	9.718291	9.930688	9.787603	10.212397	29
32	9.718497	9.930611	9.787886	10.212114	28
33	9.718703	9.930533	9.788170	10.211830	27
34	9.718909	9.930456	9.788453	10.211547	26
35	9.719114	9.930378	9.788736	10.211264	25
36	9.719320	9.930300	9.789019	10.210981	24
37	9.719525	9.930223	9.789302	10.210698	23
38	9.719730	9.930145	9.789585	10.210415	22
39	9.719935	9.930067	9.789868	10.210132	21
40	9.720140	9.929989	9.790151	10.209849	20
41	9.720345	9.929911	9.790433	10.209566	19
42	9.720549	9.929833	9.790716	10.209284	18
43	9.720754	9.929755	9.790999	10.209001	17
44	9.720958	9.929677	9.791281	10.208719	16
45	9.721162	9.929599	9.791563	10.208436	15
46	9.721366	9.929521	9.791846	10.208154	14
47	9.721570	9.919442	9.792128	10.207872	13
48	9.721774	9.929364	9.792410	10.207590	12
49	9.721978	9.929286	9.792692	10.207308	11
50	9.722181	9.929207	9.792974	10.207024	10
51	9.722385	9.929129	9.793256	10.206744	9
52	9.722588	9.929050	9.793538	10.206462	8
53	9.722791	9.928972	9.793819	10.206180	7
54	9.722994	9.928893	9.794101	10.205899	6
55	9.723197	9.928814	9.794383	10.205617	5
56	9.723400	9.928736	9.794664	10.205336	4
57	9.723603	9.928657	9.794945	10.205054	3
58	9.723805	9.928578	9.795227	10.204773	2
59	9.724007	9.928499	9.795508	10.204492	1
60	9.724210	9.928420	9.795789	10.204211	0
	Sine		Tangent		M

Degree 58.

Degree 32.

M	Sine		Tangent		
0	9.724210	9.928420	9.795789	10.204211	60
1	9.724412	9.928341	9.796070	10.203930	59
2	9.724614	9.928262	9.796351	10.203649	58
3	9.724816	9.928183	9.796632	10.203368	57
4	9.725017	9.928104	9.796913	10.203087	56
5	9.725219	9.928025	9.797194	10.202806	55
6	9.725420	9.927946	9.797474	10.202522	54
7	9.725622	9.927867	9.797755	10.202245	53
8	9.725823	9.927787	9.798036	10.201964	52
9	9.726024	9.927708	9.798316	10.201684	51
10	9.726225	9.927628	9.798596	10.201404	50
11	9.726426	9.927549	9.798877	10.201123	49
12	9.726626	9.927469	9.799157	10.200843	48
13	9.726827	9.927390	9.799437	10.200563	47
14	9.727027	9.927310	9.799717	10.200283	46
15	9.727228	9.927231	9.799997	10.200003	45
16	9.727428	9.927151	9.800277	10.199723	44
17	9.727628	9.927071	9.800557	10.199443	43
18	9.727828	9.926991	9.800836	10.199163	42
19	9.728027	9.926911	9.801116	10.198884	41
20	9.728227	9.926831	9.801396	10.198604	40
21	9.728427	9.926751	9.801675	10.198325	39
22	9.728626	9.926671	9.801955	10.198045	38
23	9.728825	9.926591	9.802234	10.197766	37
24	9.729024	9.926511	9.802513	10.197487	36
25	9.729223	9.926431	9.802792	10.197207	35
26	9.729422	9.926351	9.803072	10.196928	34
27	9.729621	9.926270	9.803351	10.196649	33
28	9.729820	9.926190	9.803630	10.196370	32
29	9.730018	9.926110	9.803908	10.196091	31
30	9.730216	9.926029	9.804187	10.195813	30
	Sine		Tangent		M

Degree 57.

Degree 32.

M	Sine		Tangent		
30	9.730216	9.926029	9.804187	10.195813	30
31	9.730415	9.925949	9.804466	10.195534	29
32	9.730613	9.925868	9.804745	10.195255	28
33	9.730811	9.925787	9.805023	10.194977	27
34	9.731009	9.925707	9.805302	10.194698	26
35	9.731206	9.925626	9.805580	10.194420	25
36	9.731404	9.925545	9.805859	10.194141	24
37	9.731601	9.925464	9.806137	10.193863	23
38	9.731799	9.925384	9.806415	10.193585	22
39	9.731996	9.925303	9.806693	10.193309	21
40	9.732193	9.925222	9.806971	10.193028	20
41	9.732390	9.925141	9.807249	10.192751	19
42	9.732587	9.925060	9.807527	10.192433	18
43	9.732784	9.924978	9.807805	10.192195	17
44	9.732980	9.924897	9.808083	10.191917	16
45	9.733177	9.924816	9.808361	10.191639	15
46	9.733373	9.924735	9.808638	10.191362	14
47	9.733569	9.924653	9.808916	10.191084	13
48	9.733765	9.924572	9.809193	10.190807	12
49	9.733961	9.924491	9.809471	10.190529	11
50	9.734157	9.924409	9.809748	10.190252	10
51	9.734353	9.924328	9.810025	10.189975	9
52	9.734548	9.924246	9.810302	10.189697	8
53	9.734744	9.924164	9.810580	10.189420	7
54	9.734939	9.924083	9.810857	10.189143	6
55	9.735134	9.924001	9.811134	10.188866	5
56	9.735330	9.923919	9.811410	10.188589	4
57	9.735525	9.923837	9.811687	10.188313	3
58	9.735719	9.923755	9.811964	10.188036	2
59	9.735914	9.923673	9.812241	10.187759	1
60	9.736109	9.923591	9.812517	10.187483	0
		Sine		Tangent	M

Degree 57.

K k

Degree 33.

M	Sine		Tangent		
0	9.736109	9.023591	9.812517	10.187483	60
1	9.736309	9.923509	9.812794	10.187206	59
2	9.736497	9.923427	9.813070	10.186930	58
3	9.736692	9.923345	9.813347	10.186653	57
4	9.736886	9.923263	9.813623	10.186377	56
5	9.737080	9.923180	9.813899	10.186101	55
6	9.737274	9.923098	9.814175	10.185824	54
7	9.737467	9.923016	9.814452	10.185548	53
8	9.737661	9.922933	9.814728	10.185272	52
9	9.737854	9.922851	9.815004	10.184996	51
10	9.738048	9.922768	9.815279	10.184720	50
11	9.738241	9.922686	9.815555	10.184445	49
12	9.738434	9.922603	9.815831	10.184169	48
13	9.738627	9.922520	9.816107	10.183893	47
14	9.738820	9.922438	9.816382	10.183617	46
15	9.739013	9.922355	9.816658	10.183342	45
16	9.739205	9.922272	9.816933	10.183066	44
17	9.739398	9.922189	9.817209	10.182791	43
18	9.739590	9.922106	9.817484	10.182516	42
19	9.739783	9.922023	9.817759	10.182240	41
20	9.739975	9.921940	9.818035	10.181965	40
21	9.740167	9.921857	9.818310	10.181690	39
22	9.740359	9.921774	9.818585	10.181415	38
23	9.740550	9.921691	9.818860	10.181140	37
24	9.740742	9.921607	9.819135	10.180865	36
25	9.740934	9.921524	9.819410	10.180590	35
26	9.741125	9.921441	9.819684	10.180315	34
27	9.741316	9.921357	9.819959	10.180041	33
28	9.741507	9.921274	9.820234	10.179766	32
29	9.741698	9.921190	9.820508	10.179492	31
30	9.741889	9.921107	9.820783	10.179217	30
		Sine		Tangent	M

Degree 56.

Degree 33.

M	Sine		Tangent		
30	9.741889	9.921107	9.820783	10.179217	30
31	9.742080	9.921023	9.821057	10.178943	29
32	9.742271	9.920939	9.821332	10.178668	28
33	9.742461	9.920855	9.821606	10.178394	27
34	9.742652	9.920772	9.821880	10.178120	26
35	9.742842	9.920688	9.822154	10.177846	25
36	9.743032	9.920604	9.822429	10.177571	24
37	9.743223	9.920520	9.822703	10.177297	23
38	9.743412	9.920436	9.822977	10.177023	22
39	9.743602	9.920352	9.823250	10.176739	21
40	9.743792	9.920268	9.823524	10.176476	20
41	9.743982	9.920184	9.823798	10.176202	19
42	9.744171	9.920099	9.824072	10.175928	18
43	9.744361	9.920015	9.824345	10.175655	17
44	9.744550	9.919931	9.824619	10.175381	16
45	9.744739	9.919846	9.824892	10.175108	15
46	9.744928	9.919762	9.825166	10.174834	14
47	9.745117	9.919677	9.825439	10.174560	13
48	9.745306	9.919593	9.825713	10.174287	12
49	9.745494	9.919508	9.825986	10.174014	11
50	9.745683	9.919424	9.826259	10.173741	10
51	9.745871	9.919339	9.826532	10.173468	9
52	9.746059	9.919254	9.826805	10.173195	8
53	9.746248	9.919169	9.827078	10.172922	7
54	9.746436	9.919084	9.827351	10.172649	6
55	9.746624	9.918999	9.827624	10.172376	5
56	9.746811	9.918915	9.827897	10.172103	4
57	9.746999	9.918830	9.828170	10.171830	3
58	9.747187	9.918744	9.828442	10.171558	2
59	9.747374	9.918659	9.828715	10.171285	1
60	9.747562	9.918574	9.828987	10.171012	0
		Sine		Tangent	M

Degree 56.

Degree 34.

M	Sine		Tangent		
0	9.747562	9.918574	9.828987	10.171012	60
1	9.747749	9.918489	9.829260	10.170740	59
2	9.747936	9.918404	9.829532	10.170468	58
3	9.748123	9.918318	9.829805	10.170195	57
4	9.748310	9.918233	9.830077	10.169923	56
5	9.748497	9.918147	9.830349	10.169651	55
6	9.748683	9.918062	9.830621	10.169379	54
7	9.748870	9.917975	9.830893	10.169156	53
8	9.749056	9.917891	9.831165	10.168834	52
9	9.749242	9.917805	9.831437	10.168563	51
10	9.749429	9.917719	9.831709	10.168291	50
11	9.749615	9.917634	9.831981	10.168019	49
12	9.749801	9.917548	9.832253	10.167747	48
13	9.749986	9.917462	9.832525	10.167475	47
14	9.750172	9.917376	9.832796	10.167204	46
15	9.750358	9.917290	9.833068	10.166932	45
16	9.750543	9.917204	9.833339	10.166660	44
17	9.750729	9.917118	9.833621	10.166389	43
18	9.750914	9.917032	9.833882	10.166118	42
19	9.751099	9.916945	9.834154	10.165846	41
20	9.751284	9.916819	9.834425	10.165575	40
21	9.751469	9.916773	9.834696	10.165304	39
22	9.751654	9.916686	9.834967	10.165033	38
23	9.751838	9.916600	9.835238	10.164762	37
24	9.752023	9.916514	9.835509	10.164491	36
25	9.752207	9.916427	9.835780	10.164220	35
26	9.752392	9.916340	9.836051	10.163949	34
27	9.752576	9.916254	9.836322	10.163678	33
28	9.752760	9.916167	9.836593	10.163407	32
29	9.752944	9.916080	9.836864	10.163136	31
30	9.753128	9.915994	9.837134	10.162866	30
		Sine		Tangent	M

Degree 55.

Degree 34.

M	Sine		Tangent		
30	9.753128	9.915994	9.837134	10.162866	30
31	9.753312	9.915907	9.837405	10.162595	29
32	9.753495	9.915820	9.837675	10.162325	28
33	9.753679	9.915733	9.837946	10.162054	27
34	9.753862	9.915646	9.838216	10.161784	26
35	9.754046	9.915559	9.838487	10.161513	25
36	9.754229	9.915472	9.838757	10.161243	24
37	9.754412	9.915385	9.839027	10.160973	23
38	9.754595	9.915297	9.839297	10.160702	22
39	9.754778	9.915210	9.839568	10.160432	21
40	9.754960	9.915123	9.839838	10.160162	20
41	9.755143	9.915035	9.840108	10.159892	19
42	9.755325	9.914948	9.840378	10.159622	18
43	9.755508	9.914860	9.840647	10.159352	17
44	9.755690	9.914773	9.840917	10.159083	16
45	9.755872	9.914685	9.841187	10.158813	15
46	9.756054	9.914597	9.841457	10.158543	14
47	9.756236	9.914510	9.841726	10.158273	13
48	9.756418	9.914422	9.841996	10.158004	12
49	9.756600	9.914334	9.842206	10.157734	11
50	9.756781	9.914246	9.842505	10.157465	10
51	9.756963	9.914158	9.842804	10.157195	9
52	9.757144	9.914070	9.843074	10.156926	8
53	9.757316	9.913982	9.843343	10.156657	7
54	9.757507	9.913894	9.843612	10.156387	6
55	9.757688	9.913806	9.843882	10.156118	5
56	9.757869	9.913718	9.844151	10.155849	4
57	9.758049	9.913630	9.844420	10.155580	3
58	9.758230	9.913541	9.844689	10.155311	2
59	9.758411	9.913453	9.844958	10.155041	1
60	9.758591	9.913361	9.845227	10.154773	0
		Sine		Tangent	M

Degree 55.

Degree 35.

M	Sine		Tangent		
0	9.758591	9.913364	9.845227	10.154774	60
1	9.758772	9.913276	9.845456	10.154504	59
2	9.758952	9.913187	9.845764	10.154235	58
3	9.759132	9.913099	9.846033	10.153967	57
4	9.759312	9.913010	9.846302	10.153698	56
5	9.759492	9.912921	9.846570	10.153429	55
6	9.759672	9.912833	9.846839	10.153161	54
7	9.759851	9.912744	9.847107	10.152892	53
8	9.760031	9.912655	9.847376	10.152624	52
9	9.760210	9.912566	9.847644	10.152356	51
10	9.760390	9.912477	9.847913	10.152087	50
11	9.760569	9.912388	9.848181	10.151819	49
12	9.760748	9.912299	9.848449	10.151551	48
13	9.760927	9.912210	9.848717	10.151283	47
14	9.761106	9.912121	9.848985	10.151015	46
15	9.761285	9.912031	9.849254	10.150746	45
16	9.761464	9.911942	9.849522	10.150478	44
17	9.761642	9.911853	9.849789	10.150214	43
18	9.761821	9.911763	9.850057	10.149943	42
19	9.761999	9.911674	9.850325	10.149675	41
20	9.762177	9.911584	9.850593	10.149407	40
21	9.762356	9.911495	9.850861	10.149139	39
22	9.762534	9.911405	9.851128	10.148872	38
23	9.762712	9.911315	9.851396	10.148604	37
24	9.762889	9.911226	9.851664	10.148336	36
25	9.763067	9.911136	9.851931	10.148069	35
26	9.763245	9.911046	9.852199	10.147801	34
27	9.763422	9.910956	9.852466	10.147534	33
28	9.763599	9.910866	9.852731	10.147267	32
29	9.763777	9.910776	9.853001	10.146999	31
30	9.763954	9.910686	9.853268	10.146732	30
		Sine		Tangent	M

Degree 54.

Degree 35.

M	Sine		Tangent		
30	9.763954	9.910686	9.853208	10.146732	30
31	9.764131	9.910596	9.853532	10.146465	29
32	9.764308	9.910506	9.853802	10.146198	28
33	9.764485	9.910415	9.854069	10.145930	27
34	9.764662	9.910325	9.854336	10.145664	26
35	9.764838	9.910235	9.854603	10.145397	25
36	9.765015	9.910144	9.854870	10.145130	24
37	9.765191	9.910054	9.855137	10.144863	23
38	9.765367	9.909963	9.855404	10.144596	22
39	9.765544	9.909873	9.855671	10.144329	21
40	9.765720	9.909782	9.855937	10.144063	20
41	9.765896	9.909691	9.856204	10.143796	19
42	9.766071	9.909601	9.856471	10.143529	18
43	9.766247	9.909510	9.856737	10.143263	17
44	9.766423	9.909419	9.857004	10.142996	16
45	9.766598	9.909328	9.857270	10.142730	15
46	9.766774	9.909237	9.857537	10.142463	14
47	9.766949	9.909146	9.857803	10.142197	13
48	9.767124	9.909055	9.858069	10.141931	12
49	9.767299	9.908964	9.858336	10.141664	11
50	9.767474	9.908873	9.858602	10.141398	10
51	9.767649	9.908781	9.858868	10.141132	9
52	9.767824	9.908690	9.859134	10.140866	8
53	9.767997	9.908599	9.859400	10.140600	7
54	9.768173	9.908507	9.859666	10.140334	6
55	9.768348	9.908416	9.859932	10.140068	5
56	9.768522	9.908324	9.860198	10.139802	4
57	9.768696	9.908233	9.860464	10.139536	3
58	9.768871	9.908141	9.860730	10.139270	2
59	9.769045	9.908049	9.860995	10.139005	1
60	9.769219	9.907958	9.861261	10.138739	0
	Sine		Tangent		M

Degree 54.

Degree 36.

M	Sine		Tangent		
0	9.769219	9.907958	9.861261	10.138739	60
1	9.769392	9.907866	9.861527	10.138473	59
2	9.769566	9.907774	9.861792	10.138208	58
3	9.769740	9.907682	9.862058	10.137942	57
4	9.769913	9.907590	9.862323	10.137677	56
5	9.770087	9.907498	9.862589	10.137411	55
6	9.770260	9.907406	9.862854	10.137146	54
7	9.770433	9.907314	9.863119	10.136880	53
8	9.770606	9.907221	9.863385	10.136615	52
9	9.770779	9.907129	9.863650	10.136350	51
10	9.770952	9.907037	9.863915	10.136085	50
11	9.771125	9.906945	9.864180	10.135820	49
12	9.771298	9.906852	9.864445	10.135554	48
13	9.771470	9.906760	9.864710	10.135289	47
14	9.771643	9.906667	9.864975	10.135024	46
15	9.771815	9.906574	9.865240	10.134759	45
16	9.771987	9.906482	9.865505	10.134495	44
17	9.772159	9.906389	9.865770	10.134230	43
18	9.772331	9.906296	9.866035	10.133965	42
19	9.772503	9.906203	9.866300	10.133700	41
20	9.772675	9.906111	9.866564	10.133436	40
21	9.772847	9.906018	9.866829	10.133171	39
22	9.773018	9.905925	9.867094	10.132906	38
23	9.773190	9.905832	9.867358	10.132642	37
24	9.773361	9.905738	9.867623	10.132377	36
25	9.773533	9.905645	9.867887	10.132113	35
26	9.773704	9.905552	9.868152	10.131848	34
27	9.773875	9.905459	9.868416	10.131584	33
28	9.774046	9.905365	9.868680	10.131320	32
29	9.774217	9.905272	9.868945	10.131055	31
30	9.774388	9.905179	9.869209	10.130791	30
		Sine		Tangent	M

Degree 53.

Degree 36.

M	Sine		Tangent		
30	9.774388	9.905179	9.869209	10.130791	30
31	9.774558	9.905085	9.864773	10.130527	29
32	9.774729	9.904992	9.867337	10.130263	28
33	9.774899	9.904898	9.870001	10.129999	27
34	9.775070	9.904804	9.870265	10.129735	26
35	9.775240	9.904711	9.870529	10.129471	25
36	9.775410	9.904617	9.870793	10.129207	24
37	9.775580	9.904523	9.871057	10.128943	23
38	9.775750	9.904429	9.871321	10.128679	22
39	9.775920	9.904335	9.871585	10.128415	21
40	9.776090	9.904241	9.871849	10.128151	20
41	9.776259	9.904147	9.872112	10.127888	19
42	9.776429	9.904053	9.872376	10.127624	18
43	9.776598	9.903959	9.872640	10.127360	17
44	9.776768	9.903864	9.872903	10.127097	16
45	9.776937	9.903770	9.873167	10.126833	15
46	9.777106	9.903676	9.873430	10.126570	14
47	9.777275	9.903581	9.873694	10.126306	13
48	9.777444	9.903486	9.873957	10.126043	12
49	9.777613	9.903392	9.874220	10.125780	11
50	9.777781	9.903298	9.874484	10.125516	10
51	9.777950	9.903203	9.874747	10.125253	9
52	9.778119	9.903108	9.875010	10.124990	8
53	9.778287	9.903013	9.875273	10.124727	7
54	9.778455	9.902919	9.875536	10.124464	6
55	9.778623	9.902824	9.875799	10.124201	5
56	9.778792	9.902729	9.876063	10.123937	4
57	9.778960	9.902634	9.876326	10.123674	3
58	9.779129	9.902539	9.876589	10.123411	2
59	9.779295	9.902444	9.876851	10.123149	1
60	9.779463	9.902349	9.877114	10.122886	0
		Sine		Tangent	M

Degree 53.

Degree 37.

M	Sine		Tangent		
0	9.779463	9.902349	9.877114	10.122885	60
1	9.779631	9.902253	9.877377	10.122623	59
2	9.779798	9.902158	9.877640	10.122360	58
3	9.779965	9.902063	9.877903	10.122097	57
4	9.780133	9.901967	9.878165	10.121834	56
5	9.780300	9.901872	9.878428	10.121572	55
6	9.780467	9.901776	9.878691	10.121309	54
7	9.780634	9.901681	9.878953	10.121047	53
8	9.780801	9.901585	9.879216	10.120784	52
9	9.780968	9.901488	9.879478	10.120522	51
10	9.781134	9.901391	9.879741	10.120259	50
11	9.781301	9.901298	9.880003	10.119997	49
12	9.781467	9.901202	9.880265	10.119734	48
13	9.781634	9.901106	9.880528	10.119472	47
14	9.781800	9.901010	9.880790	10.119210	46
15	9.781966	9.900914	9.881052	10.118948	45
16	9.782132	9.900828	9.881314	10.118686	44
17	9.782298	9.900722	9.881576	10.118424	43
18	9.782464	9.900626	9.881839	10.118161	42
19	9.782690	9.900529	9.882101	10.117899	41
20	9.782796	9.900433	9.882363	10.117637	40
21	9.782961	9.900337	9.882625	10.117375	39
22	9.783127	9.900240	9.882886	10.117114	38
23	9.783292	9.900144	9.883148	10.116852	37
24	9.783457	9.900047	9.883410	10.116590	36
25	9.783623	9.899951	9.883672	10.116328	35
26	9.783788	9.899854	9.883934	10.116066	34
27	9.783953	9.899757	9.884195	10.115805	33
28	9.784118	9.899660	9.884457	10.115543	32
29	9.784282	9.899563	9.884719	10.115281	31
30	9.784447	9.899467	9.884980	10.115020	30
		Sine		Tangent	M

Degree 52.

Degree 37.

M	Sine		Tangent		
30	9.784447	9.899467	9.884980	10.115025	30
31	9.784616	9.899370	9.885242	10.114758	29
32	9.784776	9.899273	9.885503	10.114497	28
33	9.784941	9.899175	9.885765	10.114235	27
34	9.785105	9.899078	9.886026	10.113974	26
35	9.785269	9.898981	9.886288	10.113712	25
36	9.785433	9.898884	9.886549	10.113451	24
37	9.785591	9.898787	9.886810	10.113190	23
38	9.785761	9.898689	9.887072	10.112928	22
39	9.785925	9.898592	9.887333	10.112667	21
40	9.786088	9.898494	9.887594	10.112406	20
41	9.786252	9.898397	9.887855	10.112145	19
42	9.786416	9.898299	9.888116	10.111884	18
43	9.786579	9.898201	9.888377	10.111623	17
44	9.786742	9.898104	9.888638	10.111362	16
45	9.786909	9.898006	9.888899	10.111101	15
46	9.787069	9.897908	9.889160	10.110840	14
47	9.787232	9.897810	9.889421	10.110579	13
48	9.787395	9.897712	9.889682	10.110318	12
49	9.787557	9.897614	9.889943	10.110057	11
50	9.787720	9.897516	9.890204	10.109796	10
51	9.787883	9.897418	9.890465	10.109535	9
52	9.788045	9.897320	9.890725	10.109275	8
53	9.788208	9.897222	9.890986	10.109014	7
54	9.788370	9.897123	9.891248	10.108753	6
55	9.788532	9.897025	9.891507	10.108493	5
56	9.788694	9.896926	9.891768	10.108232	4
57	9.788856	9.896828	9.892028	10.107972	3
58	9.789018	9.896729	9.892289	10.107711	2
59	9.789180	9.896631	9.892549	10.107451	1
60	9.789342	9.896532	9.892810	10.107190	0
	Sine		Tangent		M

Degree 52.

Degree 38.

M	Sine		Tangent		
0	9.789342	9.896532	9.892810	10.107190	60
1	9.789504	9.896433	9.893070	10.106930	59
2	9.789665	9.896335	9.893330	10.106669	58
3	9.789827	9.896236	9.893591	10.106408	57
4	9.789988	9.896137	9.893851	10.106149	56
5	9.790149	9.896038	9.894111	10.105889	55
6	9.790310	9.895939	9.894371	10.105628	54
7	9.790471	9.895840	9.894632	10.105368	53
8	9.790632	9.895741	9.894892	10.105108	52
9	9.790793	9.895641	9.895152	10.104844	51
10	9.790954	9.895542	9.895412	10.104588	50
11	9.791115	9.895443	9.895672	10.104328	49
12	9.791275	9.895343	9.895932	10.104068	48
13	9.791436	9.895244	9.896192	10.103808	47
14	9.791596	9.895144	9.896452	10.103548	46
15	9.791756	9.895045	9.896712	10.103288	45
16	9.791917	9.894945	9.896971	10.103028	44
17	9.792077	9.894846	9.897231	10.102769	43
18	9.792237	9.894746	9.897491	10.102509	42
19	9.792397	9.894646	9.897751	10.102249	41
20	9.792557	9.894546	9.898010	10.101990	40
21	9.792716	9.894446	9.898270	10.101730	39
22	9.792876	9.894346	9.898530	10.101470	38
23	9.793035	9.894246	9.898789	10.101211	37
24	9.793195	9.894146	9.899049	10.100951	36
25	9.793354	9.894046	9.899308	10.100692	35
26	9.793513	9.893946	9.899568	10.100432	34
27	9.793673	9.893845	9.899827	10.100173	33
28	9.793832	9.893745	9.900086	10.099913	32
29	9.793991	9.893645	9.900346	10.099654	31
30	9.794149	9.893544	9.900605	10.099395	30
	Sine		Tangent		M

Degree 51.

Degree 38.

M	Sine		Tangent	Tangent	
30	9.794149	9.893544	9.900605	10.099395	30
31	9.794308	9.893444	9.900864	10.099135	29
32	9.794467	9.893343	9.901124	10.098876	28
33	9.794626	9.893243	9.901383	10.098617	27
34	9.794784	9.893142	9.901643	10.098358	26
35	9.794942	9.893041	9.901901	10.098099	25
36	9.795101	9.892940	9.902160	10.097839	24
37	9.795259	9.892839	9.902419	10.097580	23
38	9.795417	9.892738	9.902678	10.097321	22
39	9.795575	9.892637	9.902937	10.097062	21
40	9.795733	9.892536	9.903196	10.096803	20
41	9.795891	9.892435	9.903455	10.096544	19
42	9.796049	9.892334	9.903714	10.096285	18
43	9.796206	9.892233	9.903973	10.096027	17
44	9.796364	9.892132	9.904232	10.095768	16
45	9.796521	9.892030	9.904491	10.095509	15
46	9.796678	9.891929	9.904750	10.095250	14
47	9.796836	9.891827	9.905008	10.094991	13
48	9.796993	9.891726	9.905267	10.094733	12
49	9.797150	9.891624	9.905526	10.094474	11
50	9.797307	9.891522	9.905784	10.094215	10
51	9.797464	9.891421	9.906043	10.093957	9
52	9.797621	9.891319	9.906302	10.093698	8
53	9.797777	9.891217	9.906560	10.093440	7
54	9.797934	9.891115	9.906819	10.093181	6
55	9.798091	9.891013	9.907077	10.092923	5
56	9.798247	9.890911	9.907336	10.092664	4
57	9.798403	9.890809	9.907594	10.092406	3
58	9.798560	9.890707	9.907852	10.092147	2
59	9.798716	9.890605	9.908111	10.091889	1
60	9.798872	9.890503	9.908369	10.091631	0
		Sine			M

Degree 51.

Degree 39.

M	Sine		Tangent		
0	9.798872	9.890503	9.908369	10.091631	60
1	9.799028	9.890400	9.908627	10.091373	59
2	9.799184	9.890298	9.908886	10.091114	58
3	9.799339	9.890195	9.909144	10.090856	57
4	9.799495	9.890093	9.909402	10.090598	56
5	9.799651	9.889990	9.909660	10.090340	55
6	9.799806	9.889888	9.909918	10.090081	54
7	9.799961	9.889785	9.910176	10.089823	53
8	9.800117	9.889682	9.910435	10.089565	52
9	9.800272	9.889579	9.910693	10.089307	51
10	9.800427	9.889476	9.910951	10.089049	50
11	9.800582	9.889374	9.911209	10.088791	49
12	9.800737	9.889271	9.911467	10.088533	48
13	9.800892	9.889167	9.911724	10.088275	47
14	9.801047	9.889064	9.911982	10.088017	46
15	9.801201	9.888961	9.912240	10.087760	45
16	9.801356	9.888858	9.912498	10.087502	44
17	9.801510	9.888755	9.912756	10.087244	43
18	9.801665	9.888651	9.913014	10.086986	42
19	9.801819	9.888548	9.913271	10.086729	41
20	9.801973	9.888444	9.913529	10.086471	40
21	9.802127	9.888341	9.913787	10.086213	39
22	9.802282	9.888237	9.914044	10.085956	38
23	9.802437	9.888133	9.914302	10.085698	37
24	9.802589	9.888030	9.914560	10.085440	36
25	9.802743	9.887926	9.914817	10.085183	35
26	9.802897	9.887822	9.915075	10.084925	34
27	9.803050	9.887718	9.915332	10.084668	33
28	9.803204	9.887614	9.915590	10.084410	32
29	9.803357	9.887510	9.915847	10.084153	31
30	9.803510	9.887406	9.916104	10.083895	30
		Sine		Tangent	M

Degree 50.

Degree 39.

M	Sine		Tangent		
30	9.803510	9.887406	9.916104	10.083895	30
31	9.803664	9.887302	9.916362	10.083638	29
32	9.803817	9.887198	9.916619	10.083381	28
33	9.803970	9.887093	9.916876	10.083123	27
34	9.804123	9.887989	9.917134	10.042866	26
35	9.804276	9.886384	9.917391	10.082609	25
36	9.804428	9.886780	9.917648	10.082352	24
37	9.804581	9.886675	9.917905	10.082094	23
38	9.804734	9.886571	9.918162	10.081837	22
39	9.804886	9.886466	9.918420	10.081580	21
40	9.805038	9.886361	9.918677	10.081323	20
41	9.805191	9.886257	9.918934	10.081066	19
42	9.805343	9.886152	9.919191	10.080809	18
43	9.805495	9.886047	9.919448	10.080552	17
44	9.805647	9.885942	9.919705	10.080295	16
45	9.805799	9.885837	9.919962	10.080038	15
46	9.805951	9.885732	9.920219	10.079781	14
47	9.806103	9.885627	9.920476	10.079524	13
48	9.806254	9.885521	9.920733	10.079267	12
49	9.806406	9.885416	9.920990	10.079010	11
50	9.806557	9.885311	9.921247	10.078753	10
51	9.806709	9.885205	9.921503	10.078496	9
52	9.806860	9.885100	9.921760	10.078240	8
53	9.807011	9.884994	9.922017	10.077983	7
54	9.807162	9.884889	9.922274	10.077726	6
55	9.807314	9.884783	9.922530	10.077469	5
56	9.807464	9.884677	9.922787	10.077213	4
57	9.807615	9.884572	9.923044	10.076956	3
58	9.807766	9.884466	9.923300	10.076699	2
59	9.807917	9.884360	9.923557	10.076443	1
60	9.808067	9.884254	9.923813	10.076186	0
	Sine		Tangent		M

Degree 50.

Degree 40.

M	Sine		Tangent		
0	9.808067	9.884254	9.923813	10.076186	60
1	9.808218	9.884148	9.924070	10.075930	59
2	9.808368	9.884042	9.924327	10.075673	58
3	9.808519	9.883936	9.924583	10.075417	57
4	9.808669	9.883829	9.924839	10.075160	56
5	9.808819	9.883723	9.925096	10.074904	55
6	9.808969	9.883617	9.925352	10.074647	54
7	9.809119	9.883510	9.925609	10.074391	53
8	9.809269	9.883404	9.925865	10.074135	52
9	9.809419	9.883297	9.926121	10.073878	51
10	9.809569	9.883191	9.926378	10.073622	50
11	9.809718	9.883084	9.926634	10.073366	49
12	9.809868	9.882977	9.926890	10.073110	48
13	9.810017	9.882871	9.927147	10.072853	47
14	9.810166	9.882764	9.927403	10.072597	46
15	9.810316	9.882657	9.927659	10.072341	45
16	9.810465	9.882550	9.927915	10.072085	44
17	9.810614	9.882443	9.928171	10.071829	43
18	9.810763	9.882336	9.928427	10.071573	42
19	9.810912	9.882228	9.928683	10.071317	41
20	9.811061	9.882121	9.928940	10.071060	40
21	9.811210	9.882014	9.929196	10.070804	39
22	9.811358	9.881907	9.929452	10.070548	38
23	9.811506	9.881799	0.929708	10.070292	37
24	9.811655	9.881692	9.929964	10.070036	36
25	9.811804	9.881584	9.930219	10.069781	35
26	9.811952	9.881477	9.930475	10.069525	34
27	9.812100	9.881369	9.930731	10.069269	33
28	9.812248	9.881261	9.930987	10.069013	32
29	9.812396	9.881153	9.931243	10.068757	31
30	9.812544	9.881045	9.931499	10.068501	30
		Sine		Tangent	M

Degree 55.

Degree 40.

M	Sine		Tangent		
30	9.812544	9.881045	9.931499	10.068501	30
31	9.812692	9.880937	9.931755	10.068245	29
32	9.812840	9.880829	9.932010	10.067989	28
33	9.812988	9.880721	9.932266	10.067734	27
34	9.813135	9.880613	9.932522	10.067478	26
35	9.813283	9.880505	9.932778	10.067222	25
36	9.813430	9.880397	9.933033	10.066967	24
37	9.813578	9.880289	9.933289	10.066711	23
38	9.813725	9.880180	9.933545	10.066455	22
39	9.813872	9.880072	9.933800	10.066200	21
40	9.814019	9.879963	9.934056	10.065944	20
41	9.814166	9.879855	9.934311	10.065688	19
42	9.814313	9.879746	9.934567	10.065433	18
43	9.814460	9.879637	9.934822	10.065177	17
44	9.814607	9.879529	9.935078	10.064922	16
45	9.814753	9.879420	9.935333	10.064666	15
46	9.814900	9.879311	9.935589	10.064411	14
47	9.815046	9.879202	9.935844	10.064156	13
48	9.815193	9.879093	9.936100	10.063900	12
49	9.815329	9.878984	9.936355	10.063645	11
50	9.815485	9.878875	9.936610	10.063389	10
51	9.815631	9.878766	9.936866	10.063134	9
52	9.815777	9.878656	9.937121	10.062879	8
53	9.815923	9.878547	9.937376	10.062623	7
54	9.816069	9.878438	9.937632	10.062368	6
55	9.816215	9.878328	9.937887	10.062112	5
56	9.816361	9.878219	9.938142	10.061858	4
57	9.816506	9.878109	9.938397	10.061602	3
58	9.816652	9.877999	9.938653	10.061347	2
59	9.816797	9.877890	9.938908	10.061092	1
60	9.816943	9.877780	9.939163	10.060837	0
	Sine		Tangent		M

Degree 49.

Degree 41.

M	Sine		Tangent		
0	9.816943	9.877780	9.939163	10.060837	60
1	9.817088	9.877670	9.939418	10.060582	59
2	9.817233	9.877560	9.939673	10.060327	58
3	9.817378	9.877450	9.939928	10.060072	57
4	9.817523	9.877340	9.940183	10.059816	56
5	9.817668	9.877230	9.940438	10.059562	55
6	9.817813	9.877120	9.940693	10.059307	54
7	9.817958	9.877009	9.940948	10.059052	53
8	9.818103	9.876899	9.941203	10.058797	52
9	9.818247	9.876789	9.941458	10.058542	51
10	9.818392	9.876678	9.941713	10.058287	50
11	9.818536	9.876568	9.941968	10.058032	49
12	9.818681	9.876457	9.942223	10.057777	48
13	9.818825	9.876347	9.942478	10.057522	47
14	9.818969	9.876236	9.942733	10.057267	46
15	9.818113	9.876125	9.942988	10.057012	45
16	9.819257	9.876014	9.943243	10.056757	44
17	9.819401	9.875904	9.943498	10.056502	43
18	9.819545	9.875793	9.943752	10.056248	42
19	9.819689	9.875682	9.944007	10.055993	41
20	9.819832	9.875571	9.944262	10.055728	40
21	9.819976	9.875459	9.944517	10.055483	39
22	9.820119	9.875348	9.944771	10.055229	38
23	9.820263	9.875237	9.945026	10.054974	37
24	9.820406	9.875125	9.945281	10.054719	36
25	9.820549	9.875014	9.945535	10.054464	35
26	9.820693	9.874903	9.945790	10.054210	34
27	9.820836	9.874791	9.946045	10.053955	33
28	9.820979	9.874679	9.946299	10.053701	32
29	9.821122	9.874568	9.946554	10.053446	31
30	9.821264	9.874456	9.946808	10.053192	30
	Sine		Tangent		M

Degree 48.

Degree 41.

M	Sine		Tangent		
30	9.821264	9.874456	9.946808	10.053192	30
31	9.821407	9.874344	9.947063	10.052937	29
32	9.821550	9.874232	9.947317	10.052682	28
33	9.821692	9.874120	9.947572	10.052428	27
34	9.821835	9.874008	9.947826	10.052173	26
35	9.821977	9.873896	9.948081	10.051919	25
36	9.822120	9.873784	9.948335	10.051664	24
37	9.822262	9.873672	9.948590	10.051410	23
38	9.822404	9.873560	9.948844	10.051156	22
39	9.822546	9.873447	9.949099	10.050901	21
40	9.822688	9.873335	9.949353	10.050647	20
41	9.822830	9.873223	9.949607	10.050393	19
42	9.822972	9.873110	9.949862	10.050138	18
43	9.823114	9.872998	9.950116	10.049884	17
44	9.823255	9.872885	9.950370	10.049630	16
45	9.823397	9.872772	9.950625	10.049375	15
46	9.823538	9.872659	9.950879	10.049121	14
47	9.823680	9.872546	9.951133	10.048867	13
48	9.823821	9.872434	9.951388	10.048612	12
49	9.823962	9.872321	9.951642	10.048358	11
50	9.824104	9.872208	9.951896	10.048104	10
51	9.824245	9.872094	9.952150	10.047850	9
52	9.824386	9.871981	9.952404	10.047595	8
53	9.824527	9.871868	9.952659	10.047341	7
54	9.824667	9.871755	9.952913	10.047087	6
55	9.824808	9.871641	9.953167	10.046833	5
56	9.824949	9.871528	9.953421	10.046579	4
57	9.825090	9.871414	9.953675	10.046325	3
58	9.825230	9.871301	9.953929	10.046071	2
59	9.825370	9.871187	9.954183	10.045817	1
60	9.825511	9.871073	9.954437	10.045563	0
		Sine		Tangent	M

Degree 48.

Degree 42.

M	Sine		Tangent		
0	9.825511	9.871073	9.954437	10.045562	60
1	9.825651	9.870960	9.954691	10.045308	59
2	9.825791	9.870846	9.954945	10.045054	58
3	9.825931	9.870732	9.955199	10.044800	57
4	9.826071	9.870618	9.955453	10.044546	56
5	9.826211	9.870504	9.955707	10.044292	55
6	9.826351	9.870390	9.955961	10.044038	54
7	9.826491	9.870275	9.956215	10.043784	53
8	9.826631	9.870161	9.956469	10.043531	52
9	9.826770	9.870047	9.956723	10.043276	51
10	9.826910	9.869933	9.956977	10.043023	50
11	9.827049	9.869818	9.957231	10.042769	49
12	9.827189	9.869704	9.957485	10.042515	48
13	9.827328	9.869589	9.957739	10.042261	47
14	9.827467	9.869474	9.957993	10.042007	46
15	9.827606	9.869360	9.958246	10.041753	45
16	9.827745	9.869245	9.958500	10.041500	44
17	9.827884	9.869130	9.958754	10.041246	43
18	9.828023	9.869015	9.959008	10.040992	42
19	9.828162	9.868900	9.959262	10.040738	41
20	9.828301	9.868785	9.959515	10.040485	40
21	9.828439	9.868670	9.959769	10.040231	39
22	9.828578	9.868555	9.960023	10.039977	38
23	9.828716	9.868439	9.960277	10.039723	37
24	9.828855	9.868324	9.960530	10.039469	36
25	9.828993	9.868209	9.960784	10.039216	35
26	9.829131	9.868093	9.961038	10.038962	34
27	9.829269	9.867978	9.961291	10.038708	33
28	9.829406	9.867862	9.961545	10.038451	32
29	9.829545	9.867747	9.961799	10.038201	31
30	9.829683	9.867631	9.962052	10.037947	30
		Sine		Tangent	M

Degree 47.

Degree 42.

M	Sine		Tangent		
30	9.829683	9.867631	9.962052	10.037977	30
31	9.829821	9.867515	9.962306	10.037694	29
32	9.829959	9.867399	9.962560	10.037440	28
33	9.830096	9.867283	9.962813	10.037187	27
34	9.830234	9.867167	9.963067	10.036933	26
35	9.830372	9.867051	9.963320	10.036680	25
36	9.830509	9.866935	9.963574	10.036426	24
37	9.830646	9.866819	9.963827	10.036173	23
38	9.830784	9.866703	9.964081	10.035919	22
39	9.830921	9.866586	9.964335	10.035665	21
40	9.831058	9.866470	9.964588	10.035412	20
41	9.831195	9.866353	9.964842	10.035158	19
42	9.831332	9.866237	9.965095	10.034905	18
43	9.831469	9.866120	9.965348	10.034652	17
44	9.831606	9.866004	9.965602	10.034398	16
45	9.831742	9.865887	9.965855	10.034144	15
46	9.831879	9.865770	9.966109	10.033891	14
47	9.832015	9.865653	9.966362	10.033638	13
48	9.832152	9.865536	9.966616	10.033384	12
49	9.832288	9.865419	9.966869	10.033131	11
50	9.832425	9.865302	9.967122	10.032878	10
51	9.832561	9.865185	9.967376	10.032624	9
52	9.832697	9.865068	9.967629	10.032371	8
53	9.832833	9.864950	9.967883	10.032117	7
54	9.832969	9.864832	9.968136	10.031864	6
55	9.833105	9.864716	9.968389	10.031611	5
56	9.833241	9.864598	9.968643	10.031357	4
57	9.833376	9.864480	9.968896	10.031104	3
58	9.833512	9.864363	9.969149	10.030851	2
59	9.833648	9.864245	9.969403	10.030597	1
60	9.833783	9.864127	9.969656	10.030344	0
		Sine		Tangent	M

Degree 47.

Degree 43.

M	Sine		Tangent		
0	9.833783	9.864127	9.969656	10.020344	60
1	9.833919	9.864010	9.969909	10.020091	59
2	9.834054	9.863892	9.970162	10.029838	58
3	9.834189	9.863774	9.970416	10.029584	57
4	9.834324	9.863656	9.970669	10.029331	56
5	9.834460	9.863537	9.970922	10.029078	55
6	9.834595	9.863419	9.971175	10.028827	54
7	9.834730	9.863301	9.971428	10.028571	53
8	9.834865	9.863183	9.971682	10.028318	52
9	9.834999	9.863064	9.971935	10.028065	51
10	9.835134	9.862946	9.972188	10.027812	50
11	9.835269	9.862827	9.972441	10.027559	49
12	9.835503	9.862709	9.972694	10.027306	48
13	9.835538	9.862590	9.972948	10.027052	47
14	9.835672	9.862471	9.973201	10.026799	46
15	9.835806	9.862353	9.973454	10.026546	45
16	9.835941	9.862234	9.973707	10.026293	44
17	9.836075	9.862115	9.973960	10.026040	43
18	9.836209	9.861996	9.974213	10.025787	42
19	9.836343	9.861877	9.974466	10.025533	41
20	9.836477	9.861757	9.974719	10.025280	40
21	9.836611	9.861638	9.974973	10.025027	39
22	9.836745	9.861519	9.975229	10.024774	38
23	9.836878	9.861399	9.975479	10.024521	37
24	9.837012	9.861280	9.975732	10.024268	36
25	9.837146	9.861161	9.975985	10.024015	35
26	9.837279	9.861041	9.976238	10.023762	34
27	9.837412	9.860921	9.976491	10.023509	33
28	9.837546	9.860802	9.976744	10.023256	32
29	9.837679	9.860682	9.976997	10.023003	31
30	9.837812	9.860562	9.977250	10.022750	30
		Sine		Tangent	M

Degree 46.

Degree 43.

M	Sine		Tangent		
30	9.837812	9.860562	9.977250	10.022750	30
31	9.837945	9.860442	9.977503	10.022497	29
32	9.838078	9.860322	9.977756	10.022244	28
33	9.838211	9.860202	9.978009	10.021991	27
34	9.838344	9.860082	9.978262	10.021738	26
35	9.838477	9.859962	9.978515	10.021485	25
36	9.838009	9.859842	9.978768	10.021232	24
37	9.838742	9.859721	9.979021	10.020979	23
38	9.838875	9.859601	9.979274	10.020726	22
39	9.839007	9.859480	9.979527	10.020473	21
40	9.839140	9.859360	9.979780	10.020220	20
41	9.839272	9.859239	9.980033	10.019967	19
42	9.839484	9.859118	9.980285	10.019714	18
43	9.839536	9.858998	9.980538	10.019461	17
44	9.839668	9.858877	9.980791	10.019209	16
45	9.839800	9.858756	9.981044	10.018956	15
46	9.839932	9.858639	9.981297	10.018703	14
47	9.840064	9.858514	9.981550	10.018450	13
48	9.840196	9.858398	9.981803	10.018197	12
49	9.840428	9.858272	9.982056	10.017944	11
50	9.840459	9.858150	9.982309	10.017691	10
51	9.840591	9.858029	9.982562	10.017438	9
52	9.840722	9.857908	9.982814	10.017185	8
53	9.840854	9.857786	9.983067	10.016933	7
54	9.840985	9.857665	9.983320	10.016683	6
55	9.841116	9.857543	9.983573	10.016427	5
56	9.841247	9.857421	9.983826	10.016174	4
57	9.841378	9.857300	9.984079	10.015921	3
58	9.841509	9.857178	9.984331	10.015668	2
59	9.841640	9.857056	9.984584	10.015416	1
60	9.841771	9.856934	9.984837	10.015163	0
		Sine		Tangent	M

Degree 46.

Degree 44.

M	Sine		Tangent		
0	9.841771	9.856934	9.984837	10.015162	60
1	9.841902	9.856812	9.985090	10.014910	59
2	9.842033	9.856690	9.985343	10.014657	58
3	9.842163	9.856568	9.985596	10.014404	57
4	9.842294	9.856445	9.985848	10.014151	56
5	9.842424	9.856323	9.986101	10.013899	55
6	9.842555	9.856201	9.986354	10.013646	54
7	9.842685	9.856078	9.986607	10.013393	53
8	9.842815	9.855956	9.986859	10.013140	52
9	9.842945	9.855833	9.987112	10.012888	51
10	9.843076	9.855710	9.987365	10.012635	50
11	9.843206	9.855588	9.987618	10.012382	49
12	9.843336	9.855465	9.987871	10.012129	48
13	9.843465	9.855342	9.988123	10.011877	47
14	9.843595	9.855219	9.988376	10.011624	46
15	9.843725	9.855096	9.988629	10.011371	45
16	9.843855	9.854973	9.988882	10.011118	44
17	9.843984	9.854850	9.989134	10.010866	43
18	9.844114	9.854727	9.989387	10.010613	42
19	9.844243	9.854603	9.989640	10.010360	41
20	9.844372	9.854480	9.989893	10.010107	40
21	9.844502	9.854356	9.990145	10.009855	39
22	9.844631	9.854233	9.990398	10.009602	38
23	9.844760	9.854109	9.990651	10.009349	37
24	9.844889	9.853986	9.990903	10.009096	36
25	9.745018	9.853862	9.991156	10.008844	35
26	9.845147	9.853738	9.991409	10.008591	34
27	9.845276	9.853614	9.991669	10.008338	33
28	9.845404	9.853490	9.991914	10.008086	32
29	9.845533	9.853366	9.992167	10.007833	31
30	9.845662	9.853242	9.992420	10.007580	30
	Sine		Tangent		M

Degree 45.

Degree 44.

M	Sine		Tangent		
30	9.845662	9.853242	9.992420	10.007580	30
31	9.845790	9.853118	9.992672	10.007328	29
32	9.845919	9.852994	9.992925	10.007075	28
33	9.846047	9.852869	9.993178	10.006822	27
34	9.846175	9.852745	9.993430	10.006569	26
35	9.846304	9.852620	9.993683	10.006317	25
36	9.846432	9.852496	9.993936	10.006064	24
37	9.846560	9.852371	9.994189	10.005811	23
38	9.846688	9.852246	9.994441	10.005559	22
39	9.846816	9.852122	9.994694	10.005306	21
40	9.846944	9.851997	9.994947	10.005053	20
41	9.847071	9.851872	9.995199	10.004801	19
42	9.847199	9.851747	9.995452	10.004548	18
43	9.847327	9.851622	9.995701	10.004295	17
44	9.847454	9.851497	9.995957	10.004043	16
45	9.847582	9.851372	9.996210	10.003790	15
46	9.847709	9.851246	9.996463	10.003537	14
47	9.847836	9.851121	9.996715	10.003285	13
48	9.847964	9.850996	9.996968	10.003032	12
49	9.848091	9.850870	9.997220	10.002779	11
50	9.848218	9.850745	9.997473	10.002527	10
51	9.848345	9.850619	9.997726	10.002274	9
52	9.848472	9.850493	9.997979	10.002021	8
53	9.848599	9.850367	9.998231	10.001769	7
54	9.848726	9.850242	9.998484	10.001516	6
55	9.848852	9.850116	9.998737	10.001263	5
56	9.848979	9.849990	9.998989	10.001011	4
57	9.849106	9.849864	9.999242	10.000758	3
58	9.849232	9.849737	9.999495	10.000505	2
59	9.849359	9.849611	9.999747	10.000253	1
60	9.849485	9.849485	10.000000	10.000000	0
	Sine		Tangent		M

Degree 45.

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